Paper

A method for evaluation of uncertainties of noise parameter measurement

Marek Schmidt-Szałowski and Wojciech Wiatr

Abstract — The assessment of uncertainties of a two-port noise parameters measurement, presented in the paper, relies on modeling of sources of errors and an investigation of propagation of the errors through a measurement system. This approach is based on a simplified additive error model and small-change sensitivity analysis. The evaluated uncertainties agrees with those observed in experiments. This method may be implemented in automatic noise measurement systems for on-line uncertainty assessment and for optimization of the design of an experiment.

Keywords — microwaves, noise metrology, measurement system, four noise parameters, sensitivity analysis, error propagation, uncertainty.

1. Introduction

The trade-off between the measurement through-put and the measurement accuracy is a well-known dilemma encountered in the practice of industrial measurements. This problem becomes critical in case of very sensitive instrumentation like radiometers and noise figure meters, whose accuracy can be essentially improved by extending measurement time. In general, two complementary approaches can be useful in such cases: uncertainty analysis, which enables to predict the accuracy of a measurement, and design of experiment, which tells how to conduct a measurement to obtain the highest accuracy. Although both approaches are well established and commonly used in many fields of engineering they are barely exploited in noise parameter measurements [1 − 5]. As a result, even state-of-the-art noise-measurement systems does not provide on-line evaluation of accuracy.

The uncertainty of the two-port noise parameters can be estimated either numerically or analytically. In the first approach, called also perturbation method, the two-port parameters are repeatedly calculated from artificially randomly perturbed values of the system parameters and the observed values. This approach allows for various probability distributions of errors and accounts for non-linearities of the model but is very time-consuming. The latter approach based on small-change sensitivities relies on the investigation of the error propagation through the system. This approach is much faster and sufficiently accurate for typical noise measurement systems [6].

This paper concerns the analytical method of an uncertainty analysis. First, a method for determining of the noise parameters of a microwave receiver is described. Secondly, measurement errors are analyzed and a simple error model is introduced. Then, the error propagation is analyzed and the confidence intervals are calculated for all noise parameters. Finally, the results of total-power radiometer calibrations are presented as exemplary applications of this approach.

2. The model of the measurement system

In this paper an identification of the noise parameters is discussed for the case of a microwave receiver calibration. The receiver is characterized in terms the noise correlation matrix $\boldsymbol{C}_r$, and the small-signal parameters: the input reflection coefficient Γ_r and the power gain g_r. A multi-state noise generator attached to the input of the receiver consist of a bi-state noise source and a multi-state mechanical tuner. The measurement set-up shown in Fig. 1 allows one to observe the output noise temperature T_r as a function of generator parameters: the reflection coefficient Γ_g and the noise temperature T_g.

According to [7] the receiver parameters may be determined from a seven-term model

$$\boldsymbol{\beta}^T \boldsymbol{A}\boldsymbol{\beta} + \boldsymbol{a}^T \boldsymbol{\beta} + a = 0\,, \tag{1}$$

where seven-element real vector $\boldsymbol{\beta}$ contains unknown receiver parameters, while matrix $\boldsymbol{A}$, vector $\boldsymbol{a}$, and coefficient a depend on the generator state

$$\boldsymbol{\beta} = \begin{bmatrix} \Re\Gamma_r \\ \Im\Gamma_r \\ g_r \\ g_r\, c_{r11} \\ g_r\, c_{r22} \\ g_r\, \Re c_{r21} \\ g_r \Im c_{r21} \end{bmatrix},\ \boldsymbol{A} = T_r \begin{bmatrix} |\Gamma_g|^2 & 0 & \dots & 0 \\ 0 & |\Gamma_g|^2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \end{bmatrix},$$

$$\boldsymbol{a} = \begin{bmatrix} -2T_r\Re\Gamma_g \\ 2T_r\Im\Gamma_g \\ -T_g(1-|\Gamma_g^2|) \\ -|\Gamma_g^2| \\ -1 \\ 2\Re\Gamma_g \\ 2\Im\Gamma_g \end{bmatrix}, \quad a = T_r\,. \tag{2}$$

Equation (1) is real, quadratic with respect to β_1 and β_2 but linear with respect to the other unknowns. It is worth noting that a separate measurement of Γ_r is not required since its value is to be determined simultaneously with the other receiver parameters [7].

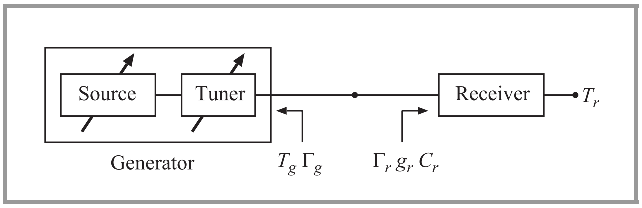

Fig. 1. The measurement set-up.

The deterministic model (1) does not fit reality because $\boldsymbol{A}, \boldsymbol{a}$, and a are affected by various errors. Typically for small and moderate errors an additive error model may be used

$$\boldsymbol{\beta}^T \boldsymbol{A}\boldsymbol{\beta} + 2\boldsymbol{a}^T\boldsymbol{\beta} + a = \varepsilon\,, \tag{3}$$

where ε is a random variable representing errors of Γ_g, T_g, and T_r. It should be emphasised that error ε does not account for all types of errors emerging in the system but only those that produce random dispersion of $\boldsymbol{\beta}$, i.e. related to type-A uncertainty [8]. Usually, it is assumed that ε is distributed according to a normal distribution with null mean and variance σ^2. Evaluation of σ is the most difficult part of the the uncertainty analysis. To simplify this task, it is assumed that error ε results from two independent sources:

- Errors related to tuner calibration errors and tuner nonrepeatability, which are modeled as uncorrelated random errors of Γ_g with constant variance $\operatorname{var}\Gamma_g$ and random errors of T_g with variance $\operatorname{var} T_g$ proportional to $T^2_{g\mathrm{exc}}$, where $T_{g\mathrm{exc}} = T_g - T_0$.

- Errors caused by finite time of integration of noise power T_r, which are modeled as random errors of T_r with variance $\operatorname{var} T_r$ proportional to T_r^2 .

The values of $\operatorname{var}\Gamma_g$, $\operatorname{var} T_g$, and $\operatorname{var} T_r$ can be estimated for each frequency of interest on the basis of long term monitoring and general knowledge of the measurement system.

Owing to the assumed independence of errors the variance σ^2 can be expressed in an additive form

$$\sigma^2 = \left|\frac{\partial\varepsilon}{\partial\Gamma_g}\right|^2 \operatorname{var}\Gamma_g + \left(\frac{\partial\varepsilon}{\partial T_g}\right)^2 \operatorname{var} T_g + + \left(\frac{\partial\varepsilon}{\partial T_r}\right)^2 \operatorname{var} T_r\,, \tag{4}$$

where sensitivities $\partial\varepsilon/\partial\,\cdot$ are calculated from Eqs. (1) and (2)

$$\frac{\partial\varepsilon}{\partial\Gamma_g} = 2\left(c_{r21} + \Gamma_g\left(g_r T_g - c_{r11}\right) - T_r\Gamma_r^*\zeta\right),$$

$$\frac{\partial\varepsilon}{\partial T_g} = -g_r\left(1 - |\Gamma_g|^2\right),$$

$$\frac{\partial\varepsilon}{\partial T_r} = |\zeta|^2, \quad \zeta = 1 - \Gamma_r\Gamma_g\,.$$

3. Identification method

The value of $\boldsymbol{\beta}$ can be determined on the basis of a series of measurements of T_r made for several states of the generator through solving a system of equations

$$\boldsymbol{\beta}^T\boldsymbol{A}\boldsymbol{\beta} + \boldsymbol{a}^T\boldsymbol{\beta} + a = \varepsilon \quad \text{for} \quad i = 1, \ldots, M\,. \tag{5}$$

Since these equations are nonlinear (quadratic) the deterministic case $M = 7$ usually leads to more than one solution $\boldsymbol{\beta}$ [7] and additional knowledge is needed to choosing the proper one. For this reason the system (5) should be overdetermined which also benefits in better immunity to random errors.

According to the Gauss-Markov theorem a generalized least squares estimator $\boldsymbol{b}$ is the best linear unbiased estimator of $\boldsymbol{\beta}$ defined as in model (3) [9]. If errors ε, related to different states of the generator, are statistically independent $\boldsymbol{b}$ can be obtained by minimizing the sum of squared residuals

$$\phi(\boldsymbol{\beta}) = \sum_{i=1}^{M} \sigma_i^{-2} e_i^2(\boldsymbol{\beta})$$

$$\text{with} \quad e_i(\boldsymbol{\beta}) = \boldsymbol{\beta}^T\boldsymbol{A}_i\boldsymbol{\beta} + \boldsymbol{a}_i^T\boldsymbol{\beta} + a_i\,. \tag{6}$$

Since the normal equation $\partial/\partial\boldsymbol{\beta}\,\phi(\boldsymbol{\beta}) = 0$ is nonlinear (as the model (3) is) and it can be solved using the Gauss-Newton method. Once the estimator $\boldsymbol{b}$ is known the covariance matrix $\operatorname{cov}\boldsymbol{b}$ can be calculated [9]

$$\operatorname{cov}\boldsymbol{b} = s^2\boldsymbol{M}^{-1} \ \text{with} \ \boldsymbol{M} = \sum_{i=1}^{M}\sigma_i^{-2}\frac{\partial e_i(\boldsymbol{b})}{\partial\boldsymbol{\beta}}\left(\frac{\partial e_i(\boldsymbol{b})}{\partial\boldsymbol{\beta}}\right)^T,$$

$$\boldsymbol{s}^2 = \frac{1}{M-7}\sum_{i=1}^{M}\sigma_i^{-2}e_i^2(\boldsymbol{b})\,. \tag{7}$$

Factor s^2 can be treated as a measure of goodness of the error model and should equals 1 for the the best model.

The receiver noise parameters $T_{\min}$, T_N, and Γ_{opt} can be calculated from $\boldsymbol{\beta}$. To properly account for their correlations an auxiliary vector $\boldsymbol{t}$ may be introduced

$$\boldsymbol{t} = \begin{bmatrix} T_{\min} \\ T_N \\ \Re\Gamma_{\mathrm{opt}} \\ \Im\Gamma_{\mathrm{opt}} \end{bmatrix} = \frac{1}{2\beta_3}\begin{bmatrix} \beta_5 - \beta_4 + \beta_3 T_N \\ \sqrt{\left(\beta_4 + \beta_5\right)^2 - 4\beta_6^2 - 4\beta_7^2} \\ \vartheta\beta_3\beta_6 \\ \vartheta\beta_3\beta_7 \end{bmatrix}$$

$$\text{with} \ \vartheta = \frac{2}{\beta_4 + \beta_5 + \beta_3 t_2}\,. \tag{8}$$

Hence

$$\mathrm{cov}\,\boldsymbol{t} = \partial \boldsymbol{t}/\partial \boldsymbol{\beta} \;\; \mathrm{cov}\,\boldsymbol{\beta} \;\; (\partial \boldsymbol{t}/\partial \boldsymbol{\beta})^T \,, \tag{9}$$

where

$$\frac{\partial \boldsymbol{t}}{\partial \boldsymbol{\beta}} = \frac{1}{\beta_3} \times$$

$$\times \begin{bmatrix} 0 & 0 & -t_1 & \frac{\nu-1}{2} & \frac{\nu+1}{2} & -2\frac{\beta_6}{t_2} & -2\frac{\beta_7}{t_2} \\ 0 & 0 & -t_2 & \nu & \nu & -4\frac{\beta_6}{t_2} & -4\frac{\beta_7}{t_2} \\ 0 & 0 & -t_3 & -\vartheta t_3 \frac{\nu+1}{2} & -\vartheta t_3 \frac{\nu+1}{2} & 2\frac{t_3^2}{t_2} + \vartheta & 2\frac{t_3 t_4}{t_2} \\ 0 & 0 & -t_4 & -\vartheta t_4 \frac{\nu+1}{2} & -\vartheta t_4 \frac{\nu+1}{2} & 2\frac{t_4 t_3}{t_2} & 2\frac{t_4^2}{t_2} + \vartheta \end{bmatrix}$$

$$\text{with } \nu = \frac{\beta_4 + \beta_5}{t_2} \,. \tag{10}$$

Finally the uncertainties of the noise parameters can be determined as

$$\mathrm{unc}\,T_{\min} = 2\sqrt{\mathrm{var}\,T_{\min}}\,, \quad \mathrm{unc}\,T_N = 2\sqrt{\mathrm{var}\,T_N}\,,$$

$$\mathrm{unc}\,\Gamma_{\mathrm{opt}} = 2\sqrt{\mathrm{var}\,\Re\,\Gamma_{\mathrm{opt}} + \mathrm{var}\,\Im\,\Gamma_{\mathrm{opt}}}\,. \tag{11}$$

The correlation coefficient, needed in some applications, can be also determined from cov $\boldsymbol{b}$.

4. Experimental results

The described method for identification of the noise parameters was used to calibrate a total-power radiometer (TPR) with various two-ports attached to its input. In the first case the TPR with the noise injection circuit (NIC, a part of the multi-state radiometer [7]) was tested in several states of the circuit (Table 1). For each state the output noise temperature T_r was measured for twelve values of Γ_g and two values of T_g thus $M = 24$. It was assumed that the standard deviation of T_{gexc} was equal to 0.002 dB, the standard deviation of Γ_g was equal to 0.001, and the standard deviation of T_r was equal to 0.01 dB. For each state the mean square error (MSE) was calculated as the rms value of differences between the observed values of T_r and those predicted by the model. MSE may be treated as a measure of the quality of the fit. The values of s show whether the assumed levels of errors were underestimated $(s > 1)$ or overestimated $(s < 1)$. Since the values of s approach to one the error model may be considered accurate.

The obtained uncertainties of $T_{\min}$ and T_N significantly depend on the state of the NIC, from 0.5% for the states ln and ma up to 9% for the state Bs. This effect results from the distribution of Γ_g on the Smith chart. For the first two states the value of Γ_{opt} approximately lay in the center of the Γ_g constellation so the system of Eqs. (5) was well-conditioned. Since the same constellation was used for all states of NIC, for some of them the value of Γ_{opt} departed far from the center of the Γ_g constellation which may have deteriorated the conditioning.

In the next example two independent calibrations of TPR with NIC are compared (Fig. 2). The confidence intervals of both sets of parameters are in a good agreement. Similar results were obtained for other frequencies.

Moreover, the uncertainties obtained using this method agreed with ones calculated using perturbation method.

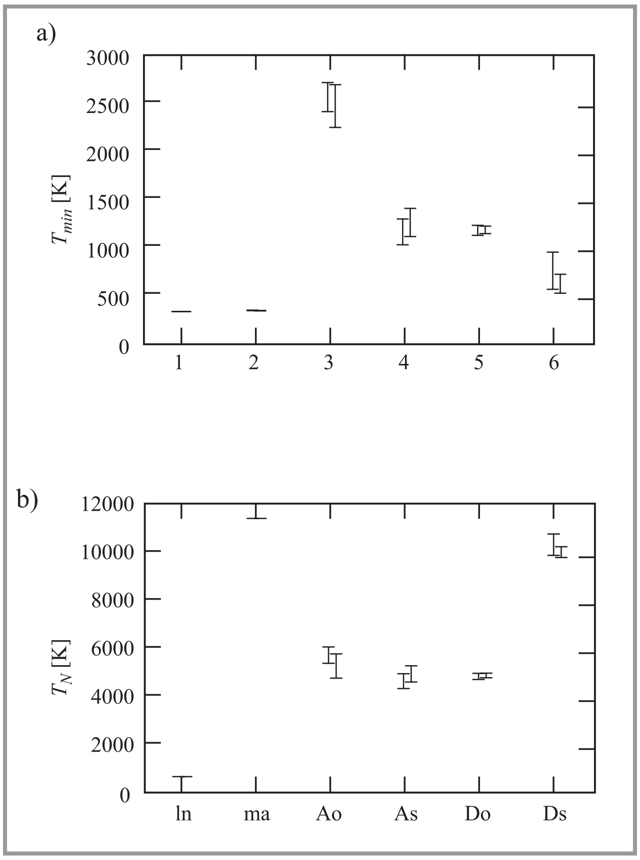

Fig. 2. Comparison of two calibrations of the TPR with NIC: (a) $T_{\min}$ and (b) T_N, $f = 1$ GHz.

As the final case the TPR with an FET at the input was measured at several operating points (Table 2). For each bias point the output noise temperature T_r was measured for twelve cold states of the generator with $\Gamma_g < 0.85$ and two hot ones with $\Gamma_g \sim 0$. The values of var T_{gexc}, var Γ_g, and var T_r were like those in the first case but they seem to be underestimated (s is up to 1.7). It is also worth noting that unc $T_{\min}$ and unc T_N get smaller when I_D increases. This can be explained as an improvement of conditioning for Γ_{opt} approaching the center of the Γ_g constellation.

5. Conclusions

The presented method for two-port noise parameter identification allows one to estimate the uncertainties of the parameters and their correlations. A simplified additive error model, which the method is based on, very well agrees with errors observed in experiments. The procedures for

Table 1
The results of calibration of a TPR with a noise injection circuit, $f = 2$ GHz

State	T_{min} [K]	unc T_{min} [K]	T_N [K]	unc T_N [K]	$\|\Gamma_{opt}\|$ [mag]	$\angle\Gamma_{opt}$ [deg]	unc Γ_{opt} [mag]	$\|\Gamma_r\|$ [mag]	$\angle\Gamma_r$ [deg]	g_r [dB]	MSE [dB]	s
ln	482.5	2.0	731.6	4.0	0.1068	-139.0	0.0039	0.0895	163.7	-19.631	0.010	0.9
ma	488.0	5.2	8895.5	44.3	0.1112	-113.7	0.0021	0.0845	161.6	-19.687	0.015	1.1
Ao	2463.1	68.4	6044.8	131.1	0.7223	40.6	0.0044	0.0792	152.9	-19.719	0.008	0.7
As	1281.1	76.9	3760.8	150.2	0.7133	-150.2	0.0081	0.0850	160.7	-19.647	0.012	1.0
Bo	1404.0	55.4	4025.8	104.1	0.6543	142.4	0.0069	0.0881	160.3	-19.646	0.014	1.3
Bs	1502.9	137.9	7505.2	323.6	0.6692	-64.7	0.0095	0.0820	161.0	-19.721	0.015	1.3
Co	1308.9	58.2	5148.7	114.9	0.6000	97.9	0.0065	0.0921	157.6	-19.665	0.014	1.3
Cs	927.1	21.1	5965.8	96.8	0.6166	-108.5	0.0039	0.0844	161.2	-19.676	0.013	1.0
Do	870.6	26.1	5765.6	95.4	0.6037	-117.6	0.0044	0.0850	160.9	-19.671	0.013	1.1
Ds	997.7	34.6	7702. 7	66.4	0.5290	38.2	0.0031	0.0863	156.4	-19.707	0.011	0.8

Table 2
The results of calibration of TPR with a FET (MGF1412), $f = 2$ GHz, V_{DS} = 3 V

I_D [mA]	T_{min} [K]	unc T_{min} [K]	T_N [K]	unc T_N [K]	$\|\Gamma_{opt}\|$ [mag]	$\angle\Gamma_{opt}$ [deg]	unc Γ_{opt} [mag]	$\|\Gamma_r\|$ [mag]	$\angle\Gamma_r$ [deg]	g_r [dB]	MSE [dB]	s
2	79.6	19.4	143.5	34.2	0.8746	-163.3	0.0223	0.9692	157.8	-15.068	0.007	1.1
5	50.9	15.2	95.3	31.0	0.8458	-162.0	0.0357	0.9622	153.2	-11.258	0.012	1.6
10	45.9	11.8	83.1	25.5	0.8181	-161.1	0.0392	0.9499	149.5	-9.070	0.012	1.7
20	45.4	8.2	83.3	18.3	0.7967	-158.8	0.0314	0.9424	146.0	-7.520	0.010	1.4
50	65.8	7.0	121.5	16.0	0.7655	-152.8	0.0223	0.9311	141.4	-6.207	0.007	1.0

calculating the uncertainties are tightly connected with the general least squares algorithm of noise parameters estimation. The numerical overheads related to the presented method are much smaller than in the perturbation method. The uncertainties of the two-port noise parameters calculated using the presented method may be used as a quick validation of the quality of a chosen Γ_g constellation. If implemented in the automatic measurement systems it may be utilised as a criterion for on-line optimization of the design of experiment, which may be extended or shrunk as needed for a given measurement accuracy. It may also facilitate finding out the most critical elements to be improved in order to minimise the sensitivity of the system to errors.

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