Paper

Stop criteria for retransmission termination in soft-combining algorithms

Hans-Jürgen Zepernick and Manora Caldera

Abstract — Soft-combining algorithms use retransmissions of the same codeword to improve the reliability of communication over very noisy channels. In this paper, soft-outputs from a maximum *a posteriori* (MAP) decoder are used as *a priori* information for decoding of retransmitted codewords. As all received words may not need the same number of retransmissions to achieve satisfactory reliability, a stop criterion to terminate retransmissions needs to be identified. As a first and very simple stop criterion, we propose an algorithm which uses the sign of the soft-output at the MAP decoder. The performance obtained with this stop criterion is compared with the one assuming a genius observer, which identifies otherwise undetectable errors. Since this technique needs always a particular number of initial retransmissions, we exploit cross-entropy between subsequent retransmissions as a more advanced but still simple stop criterion. Simulation results show that significant performance improvement can be gained with soft-combining techniques compared to simple hard or soft decision decoding. It also shows that the examined stop criteria perform very close to the optimistic case of a genius observer.

Keywords* — *MAP decoder, soft-combining, retransmission termination, cross-entropy.

1. Introduction

In order to provide efficient and reliable data transmission over a noisy communication channel, error control coding techniques are employed as an essential part in almost every modern digital communication system and in particular in mobile radio systems. In addition, hybrid automatic repeat request (ARQ) schemes are used to further improve reliability in a noisy channel. A hybrid ARQ scheme basically uses an error-correcting code to detect and if necessary to correct transmission errors. If the error pattern is detectable but not correctable, the receiver discards the received word and asks for a retransmission of that particular codeword. Unfortunately in this case, the whole effort put into the decoding process and the information gained from that failed decoding attempt is completely lost. This can be avoided by combining several repeated codewords at the output of a noisy channel, for example, using a maximum likelihood decoder [1]. Soft-combining algorithms incorporate reliability information into the decoding process and use soft values on a symbol-by-symbol basis. In this case, it is beneficial to exploit soft-input/soft-output decoding algorithms and this may follow the work presented in [2, 3]. These algorithms aim at minimising the probability of symbol or bit error and play a crucial part in iterative decoding.

In this paper, we focus on soft-combining techniques, which preserve the information obtained with each decoding attempt and incorporate this with retransmitted copies of a codeword. We are specifically interested in the post-decoding bit error probability when symbol-by-symbol MAP decoding is applied to linear block codes. For that purpose, we suggest a trellis-based decoding approach to be fit into the automata theory setting presented in [4, 5] rather than using generating functions [6, 7].

Moreover, this paper investigates two simple stop criteria to terminate retransmissions. The first criterion uses the sequence of signs of the soft-outputs at the MAP decoder and performs essentially a mapping of soft decisions onto hard decisions. The second approach for terminating retransmission exploits the cross-entropy between two subsequent retransmissions of a codeword which gives an indirect measure of the performance improvement that may be gained from the information contained in the latest retransmission.

The paper is organised as follows. Section 2 describes fundamentals of MAP decoding and introduces the principal soft-combining algorithm. In Section 3, the simple stop criterion, which uses the sign of the soft-output at the MAP decoder is presented. The stop criterion based on the cross-entropy between subsequent retransmissions is introduced in Section 4. Numerical examples are presented in Section 5. Finally, conclusions of the paper are given in Section 6.

2. Soft-combining algorithm

Let C denote an (n, k) block code over the Galois field $F = GF(2)$. The code C defines a one-to-one mapping of the k-dimensional information space F^k onto the n-dimensional vector space F^n. Let $\boldsymbol{u} = [u_1, u_2, \ldots, u_n] \in C$ be a codeword in the linear block code C and let $\boldsymbol{v} = [v_1, v_2, \ldots, v_n]$ denote a noisy observation at the output of a demodulator. In addition, we assume statistically independent source bits, systematic codes, and a memoryless channel. Eventually, the systematic linear block code C shall be defined by a parity check matrix $\mathbf{H}$. Then, the soft-output of a symbol-by-symbol MAP decoder for the estimate $\hat{u}_i$ of the ith transmitted bit u_i is given by the log-likelihood ratio [7]:

$$L(\hat{u}_i) \triangleq L(u_i \mid \mathbf{v}) = \log \frac{P(u_i = 0 \mid \mathbf{v})}{P(u_i = 1 \mid \mathbf{v})} = \\ = L(u_i, v_i) + L_e(\hat{u}_i) \qquad (1)$$

with joint log-likelihood ratio:

$$L(u_i, v_i) = \begin{cases} L(u_i) + L(v_i \mid u_i) & \text{for} \quad 1 \le i \le k\,, \\ L(v_i \mid u_i) & \text{for} \quad k < i \le n\,, \end{cases} \qquad (2)$$

where $L(u_i)$ denotes the *a priori* value of the transmitted information bit u_i and $L(v_i \mid u_i)$ represents the soft-output value of the channel. The so-called extrinsic log-likelihood value $L_e(\hat{u}_i)$ is based on the indirect information about u_i due to the particular code in use and is given by

$$L_e(\hat{u}_i) = \log \left\{ \frac{\sum\limits_{s=o}^{2^{n-k}-1} \prod\limits_{\substack{j=1 \\ j \neq i}}^{n} \left(\frac{1-e^{-L(u_j, v_j)}}{1+e^{-L(u_j, v_j)}} \right)^{u^{\perp}_{s,j}}}{\sum\limits_{s=0}^{2^{n-k}-1} (-1)^{u^{\perp}_{s,i}} \prod\limits_{\substack{j=1 \\ j \neq i}}^{n} \left(\frac{1-e^{-L(u_j, v_j)}}{1+e^{-L(u_j, v_j)}} \right)^{u^{\perp}_{s,j}}} \right\} \qquad (3)$$

with $u^{\perp}_{s,j}$ being the jth bit of the sth codeword $\mathbf{u}^{\perp}_s = \mathbf{s} \cdot \mathbf{H}$ in the dual code $C^{\perp}$ and $\mathbf{s} = bin(s)$ denotes the binary representation of the decimal number s. Note that the sign of the log-likelihood ratio $L(\hat{u}_i)$ provides a hard decision, i.e. $sign\{L(\hat{u}_i)\} = +1 \rightarrow \hat{u}_i = 0$ and $sign\{L(\hat{u}_i)\} = -1 \rightarrow \hat{u}_i = 1$ whereas the magnitude $abs\{L(\hat{u}_i)\}$ represents the reliability of the decision.

Since a MAP decoder processes soft-inputs and releases soft-outputs, the decoding outcome may be used along with a suitable ARQ scheme as the input for subsequent decoding attempts. To be more specific, a soft-combining algorithm can incorporate the information of each retransmission into the decoding procedure and this will be done on a symbol-by-symbol basis. The principal soft-combining procedure can be scheduled according to the following steps.

First transmission

1. Initialise $L^{(0)}(u_i)$ with *a priori* value of u_i.
2. Process soft-output value $L^{(1)}(v_i \mid u_i)$ of channel.
3. Compute extrinsic value $L_e^{(1)}(\hat{u}_i)$.
4. Compute log-likelihood ratio $L^{(1)}(\hat{u}_i)$.
5. Release decoded word if stop criterion is satisfied, otherwise continue with step 6.

Subsequent retransmissions

6. Request retransmission of codeword.
7. Use log-likelihood ratio $L^{(m-1)}(\hat{u}_i)$ obtained from $(m-1)$th retransmission as new *a priori* value of mth retransmission.
8. Process mth soft-output value $L^{(m)}(v_i \mid u_i)$ of channel.
9. Compute extrinsic value $L_e^{(m)}(\hat{u}_i)$ of mth retransmission.
10. Compute log-likelihood ratio $L^{(m)}(\hat{u}_i)$.
11. Release decoded word if stop criterion is satisfied, otherwise continue with step 6.

All the transmitted blocks may not need the same number of retransmissions to achieve reliable decoding. Some blocks may be detected with satisfactory reliability after the first decoding whereas some may need a few retransmissions. Also, some blocks would not even show any improvement in the post-decoding bit error rate (BER) performance with an increase in the number of retransmissions. Therefore, it is needed to observe whether improvement in the BER is possible to achieve, before requesting for another retransmission. If further performance improvement is not feasible, the retransmission of the same block can be stopped. This can be attained by specifying a criterion to terminate the retransmissions.

In previous publications, e.g. [8], availability of a genius observer has been assumed and was used to identify the undetectable errors. In this case, the genius observer requests a retransmission until a block is decoded error free or a specified maximum number of retransmissions is reached. Albeit this approach is rather idealistic, the performance characteristics obtained by employing a genius observer may serve as a benchmark for more realistic stop criteria.

3. Simple stop criterion based on hard decisions

A simple criterion for terminating retransmissions may be obtained by mapping the sequence of soft-outputs of the MAP decoder onto a sequence of hard decisions. A stop criterion can then be based on the hard decision for a complete word and may be defined as follows:

1. The hard decision for the complete word of the current transmission may be compared with that of any previous transmission and retransmissions will continue until a predefined number of those hard decisions, say three or four, are the same. This option requires sufficiently large storage space to buffer retransmissions until the examined word can finally be released.
2. Another option is to compare hard decisions of the current transmission only with that of the most recent retransmissions and terminate the soft-combining algorithm once they are the same. Since the reliability of a potential decision may be expected to increase with each retransmission, it is likely that subsequent decoding attempts result in the same hard decision.

The computer simulations have shown that the performance improvement obtained using the stop criterion based on the hard decisions of the current with any previous transmission is negligible compared to the results obtained based on only the most recent transmission. Therefore, considering the complexity and storage requirements needed to compare all the previous hard decisions, only the most recent transmissions were considered for further study. This concept can also be extended to compare the current transmission with more than one previous retransmissions. However, with this criterion, at least two transmissions are required before stopping the retransmissions.

4. Stop criterion based on the cross-entropy

Moher has shown that cross-entropy provides a useful theoretical framework for iterative decoding [9]. Those ideas have been extended in [10] to show that cross-entropy can also be useful as a stop criterion for iterative decoding. Since the simple stop criterion introduced in Section 3 always needs a particular number of initial retransmissions, we follow the ideas presented in [9, 10] and exploit cross-entropy here between subsequent retransmissions as a more advanced stop criterion.

The cross-entropy between two subsequent retransmissions, which gives a measure of the closeness of two suitable distributions, is considered as a stop criterion in the decoding process. With this approach, the cross-entropy between the distributions corresponding to consecutive transmissions of the soft-combining process is estimated and compared to a threshold. Whenever the cross-entropy drops below the specified threshold, indicating a small change in the distribution from one retransmission to the next, the soft-combining algorithm is terminated.

The cross-entropy of two distributions $P(\hat{\boldsymbol{u}})$ and $Q(\hat{\boldsymbol{u}})$ over an alphabet F^n is defined as [9]

$$\boldsymbol{E}_P\left\{\log\frac{P(\hat{\boldsymbol{u}})}{Q(\hat{\boldsymbol{u}})}\right\} = \sum_{\hat{\boldsymbol{u}}\in F^n} P(\hat{\boldsymbol{u}})\log\frac{P(\hat{\boldsymbol{u}})}{Q(\hat{\boldsymbol{u}})}\,, \tag{4}$$

where $\boldsymbol{E}_p$ denotes the expectation operation over the distribution $P(\hat{\boldsymbol{u}})$ and F is the Galois field $GF(2)$. Assuming statistical independence of the symbol probabilities, we obtain

$$\log\frac{P(\hat{\boldsymbol{u}})}{Q(\hat{\boldsymbol{u}})} = \sum_i \log\frac{P(\hat{u}_i)}{Q(\hat{u}_i)}\,. \tag{5}$$

Let us assume that $P(\hat{\boldsymbol{u}})$ and $Q(\hat{\boldsymbol{u}})$ represent the distributions corresponding to the log-likelihood values of the $(m-1)$th and the mth retransmission, respectively. Then, the cross-entropy between the distributions corresponding to the $(m-1)$th and the mth retransmissions of the decoding process can be expressed as

$$\boldsymbol{E}_P\left\{\log\frac{P(\hat{\boldsymbol{u}})}{Q(\hat{\boldsymbol{u}})}\right\} = \sum_i\left[P(\hat{u}_i=0)\log\frac{P(\hat{u}_i=0)}{Q(\hat{u}_i=0)} + \right.$$
$$\left. + P(\hat{u}_i=1)\log\frac{P(\hat{u}_i=1)}{Q(\hat{u}_i=1)}\right]. \tag{6}$$

In Eq. (6), $P(\hat{u}_i)$ and $Q(\hat{u}_i)$ are the probabilities corresponding to the $(m-1)$th and the mth retransmission, respectively.

Using the log-likelihood value corresponding to the estimate $\hat{u}_i$ of the ith bit u_i in codeword $\mathbf{u}$, we can obtain the probabilities related to the $(m-1)$th retransmission as

$$P(\hat{u}_i=0) = \frac{e^{L^{(m-1)}(\hat{u}_i)}}{1+e^{L^{(m-1)}(\hat{u}_i)}} = \frac{1}{1+e^{-L^{(m-1)}(\hat{u}_i)}}\,, \tag{7}$$

$$P(\hat{u}_i=1) = \frac{1}{1+e^{L^{(m-1)}(\hat{u}_i)}} = \frac{e^{-L^{(m-1)}(\hat{u}_i)}}{1+e^{-L^{(m-1)}(\hat{u}_i)}}\,. \tag{8}$$

Here, $L^{(m-1)}(\hat{u}_i)$ represents the log-likelihood value for the estimate $\hat{u}_i$ which has been obtained after processing the information from the $(m-1)$th retransmission. Similarly, the probabilities related to the mth retransmission can be expressed as

$$Q(\hat{u}_i=0) = \frac{e^{L^{(m)}(\hat{u}_i)}}{1+e^{L^{(m)}(\hat{u}_i)}} = \frac{1}{1+e^{-L^{(m)}(\hat{u}_i)}}\,, \tag{9}$$

$$Q(\hat{u}_i=1) = \frac{1}{1+e^{L^{(m)}(\hat{u}_i)}} = \frac{e^{-L^{(m)}(\hat{u}_i)}}{1+e^{-L^{(m)}(\hat{u}_i)}}\,. \tag{10}$$

Using Eqs. (7) to (10), we obtain

$$P(\hat{u}_i{=}0)\log\frac{P(\hat{u}_i{=}0)}{Q(\hat{u}_i{=}0)} = \frac{1}{1+e^{-L^{(m-1)}(\hat{u}_i)}}\log\frac{1+e^{-L^{(m)}(\hat{u}_i)}}{1+e^{-L^{(m-1)}(\hat{u}_i)}}\,, \tag{11}$$

$$P(\hat{u}_i{=}1)\log\frac{P(\hat{u}_i{=}1)}{Q(\hat{u}_i{=}1)} = \frac{e^{-L^{(m-1)}(\hat{u}_i)}}{1+e^{-L^{(m-1)}(\hat{u}_i)}}\log\frac{1+e^{L^{(m)}(\hat{u}_i)}}{1+e^{L^{(m-1)}(\hat{u}_i)}}\,. \tag{12}$$

Let the difference between the two log-likelihood values corresponding to the two retransmissions be

$$\Delta L^{(m)}(\hat{u}_i) = L^{(m)}(\hat{u}_i) - L^{(m-1)}(\hat{u}_i). \tag{13}$$

Combining Eqs. (11), (12) and (13), we can express Eq. (6) as

$$\boldsymbol{E}_P\left\{\log\frac{P(\hat{\boldsymbol{u}})}{Q(\hat{\boldsymbol{u}})}\right\} = \sum_i\left[\log\frac{1+e^{-L^{(m)}(\hat{u}_i)}}{1+e^{-L^{(m-1)}(\hat{u}_i)}} + \right.$$
$$\left. + \frac{1}{1+e^{L^{(m-1)}(\hat{u}_i)}}\Delta L^{(m)}(\hat{u}_i)\right]. \tag{14}$$

With the proposed stop criterion, the request for a retransmission is terminated once the cross-entropy value as given in Eq. (14) falls below a specified threshold. In the present paper, however, the threshold has been set to a value of 10^{-3}.

5. Numerical example

In order to show the potential of performance improvement that can be gained from the soft-combining techniques, the computer simulations have been carried out considering a (7,4) Hamming code. Although the considered code is very simple, it can be used in constructing more powerful codes such as product codes. The examined (7,4) Hamming code can be defined by the parity check matrix:

$$\mathbf{H} = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}. \qquad (15)$$

Further, we assume that the source bits are equally likely to appear, i.e. $P(u_i = 0) = P(u_i = 1) = 0.5$ giving an *a priori* value of $L(u_i) = 0$. The Hamming code is used with binary phase shift keying (BPSK) over additive white Gaussian noise (AWGN) and Rayleigh fading channels. Soft-output of the channel $L(v_i \mid u_i)$ is specified by [11]:

$$L(v_i \mid u_i) = 4\frac{E_s}{N_0} a_i v_i . \qquad (16)$$

Here, a_i and v_i are the Rayleigh distributed fading amplitude and the ith output of a matched filter, respectively, and E_s/N_0 represents the signal-to-noise ratio (SNR). The Rayleigh fading channel is assumed being perfectly interleaved to ensure uncorrelated fading amplitudes a_i. Moreover, for an AWGN channel, amplitude $a_i = 1$.

The simulation results obtained for the (7,4) Hamming code using simple hard or soft decision decoding are compared with soft-combining on AWGN and perfectly interleaved Rayleigh fading channels as shown in Figs. 1 and 2, respectively. Since MAP decoding aims at minimising the symbol or bit error probability [2], performance has been evaluated in terms of post-decoding bit error rate. The obtained BER curves of this code using the proposed stop criteria are also included in Figs. 1 and 2. In the case

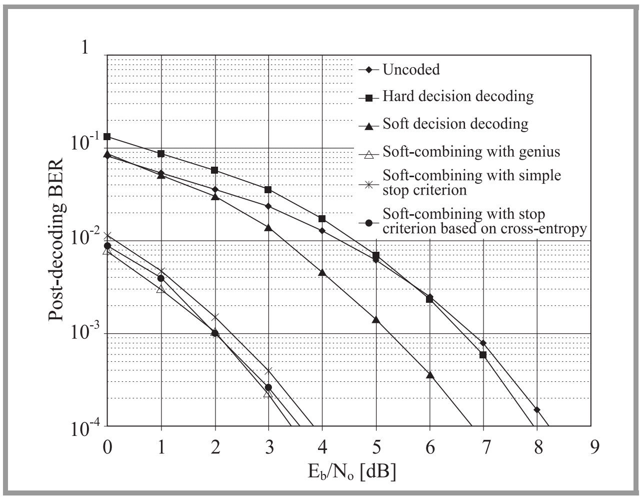

Fig. 1. BER performance of (7,4) Hamming code with soft-combining over AWGN channel.

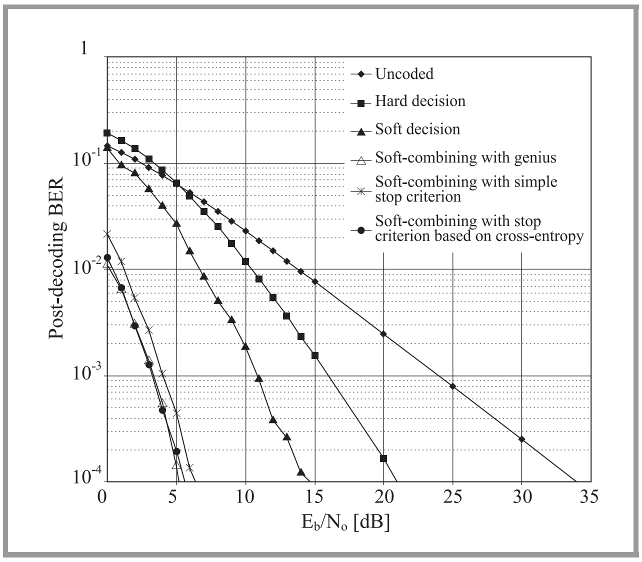

Fig. 2. BER performance of (7,4) Hamming code with soft-combining over Rayleigh channel.

of simple stop criterion, the hard decision of the current transmission is compared with that of the two most recent previous transmissions. Moreover, in these simulations, the maximum number of retransmissions has been set to 10.

It can be observed from the simulation results that error control coding using the (7,4) Hamming code provides a large performance improvement compared to an uncoded transmission, especially over fading channels. For example, the soft decision decoding of the (7,4) Hamming code gives coding gains of approximately 1.5 dB and 18 dB at a BER of 10^{-4} over AWGN and Rayleigh fading channels, respectively. Moreover, the soft-combining approach with a maximum of 10 retransmissions using the Hamming code over AWGN and Rayleigh fading channels provides additional gains of around 3 dB and 7 dB at the BER of 10^{-4}, respectively. It can also be observed that the proposed stop criteria perform very close to the one assuming a genius observer.

6. Conclusions

This paper presented soft-combining techniques for linear block codes using realistic stop criteria for retransmission termination. Two suitable stop criteria have been considered, firstly a very simple criterion based on hard decisions and secondly a more advanced criterion based on cross-entropy. It has been shown using an example that a significant performance improvement may be achieved with soft-combining techniques compared to simple soft decision decoding. The performance of the proposed stop criteria based on either the hard decisions of the current transmission with the two most recent retransmissions or the cross-entropy between the successive retransmissions was very close to the one assuming a genius observer over AWGN and Rayleigh fading channels. In addition, the stop criterion based on cross-entropy has the advantage of requiring only two subsequent transmissions before terminating the retransmissions compared to at least three required for the one with hard decisions.

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