Paper

The possible mechanism for a frequency shift by a time-varying of medium features

Aleksander G. Nerukh, Igor V. Scherbatko, and Marian Marciniak

Abstract — The optical frequency shift of an electromagnetic wave reflecting from a boundary of a medium is studies for two cases: for temporal variations of medium permittivity, and for a moving plasma boundary. It has been shown that a simultaneous occurrence of both cases leads to an enhanced frequency shift.

Keywords — frequency shift, moving plasma boundary.

Introduction

As it's known, temporal variations of medium parameters cause alterations in the frequency and amplitude of the electromagnetic wave propagating in the medium. The wave reflection from a moving medium boundary results in the same effect. A combination of these two phenomena may produce a qualitatively new effect that consists of an enhanced frequency shift.

Frequency transformation by time altering of medium permittivity

It is well known [1-4] that a plane wave $E_0(t,r) = E_0 e^{i\omega t} e^{-i\omega sr}$, as an initial field, maintains a wave number $s = \omega/v$ with a jump changing of a medium permittivity but exhibits a transformation of a frequency and an amplitude. Wave splitting into direct and inverse waves comes about also. For example, for a dissipative dielectric, when a medium goes to a state with a permittivity ε_1 and a conductivity σ_1 at some moment of time, $\varepsilon \to \varepsilon_1, \sigma_1$, the initial field transforms to the form $E_1(t,r) = A(t)e^{-isr}$, where

$$A(t) = \frac{\varepsilon}{2\varepsilon_1}\left[\left(1+\frac{\omega + i\frac{\overline{\sigma}_1}{2\varepsilon_1}}{\overline{\omega}_1}\right) e^{\left(-\frac{\overline{\sigma}_1}{2\varepsilon_1}+i\overline{\omega}_1\right)t} + \left(1-\frac{\omega + i\frac{\overline{\sigma}_1}{2\varepsilon_1}}{\overline{\omega}_1}\right) e^{\left(-\frac{\overline{\sigma}_1}{2\varepsilon_1}-i\overline{\omega}_1\right)t}\right] \quad (1)$$

and $\overline{\omega}_1 = \sqrt{\omega_1^2 - \left(\frac{\overline{\sigma}_1}{2\varepsilon_1}\right)^2}$, $\omega_1 = \frac{v_1}{v}\omega$, $\overline{\sigma}(t) = \sigma(t)/\varepsilon_0$, $v_1 = c/\sqrt{\varepsilon_1}$, ε_0 is the electric permittivity of vacuum.
For abrupt ionisation of a medium, when a cold plasma is created and the permittivity becomes equal to $\varepsilon = 1 - \omega_{e1}^2/\omega^2$, where a plasma frequency takes a value ω_{e1} the transformation has an analogues form

$$A(t) = \frac{1}{2}\left[\left(1+\frac{\omega}{\sqrt{\omega^2+\omega_{e1}^2}}\right) e^{i\sqrt{\omega^2+\omega_{e1}^2}t} + \left(1-\frac{\omega}{\sqrt{\omega^2+\omega_{e1}^2}}\right) e^{-i\sqrt{\omega^2+\omega_{e1}^2}t}\right]. \quad (2)$$

Wave splitting is connected with time variation of a medium parameter and occurs not only with an abrupt change of parameter but with continuous changing of ones [5] as well as for an electromagnetic impulse [6].
When parameters change continuously exact solutions can be derived only in unique cases [5]. But it can be made numerically by virtue of the recursion method [7, 8] that based on the evolutionary approach [9]. The field is determined by means of the equations that for the n-th time step have the form

$$E_n(t,x) = F_n(t,x) - \frac{1}{2v_n}\left\{\frac{\varepsilon_n - \varepsilon}{\varepsilon_n}\frac{\partial^2}{\partial t^2} + \frac{\overline{\sigma}_n}{\varepsilon_n}\frac{\partial}{\partial t}\right\} \times \int_{t_{n-1}}^{\infty} dt' \int_{-\infty}^{\infty} e^{-\frac{\overline{\sigma}_n}{2\varepsilon_n}(t-t')} \times \theta\left(t-t'-\frac{|x-x'|}{v_n}\right) \times I_0\left(\frac{\overline{\sigma}_n}{2\varepsilon_n}\sqrt{(t-t')^2 - \frac{(x-x')^2}{v_n^2}}\right) F_n(t',x')dx', \quad (3)$$

$$F_n(t,x) = E_0(t,x) - \frac{1}{2v}\sum_{k=1}^{n-1}\int_{t_{k-1}}^{t_k} dt' \int_{-\infty}^{\infty} dx' \left(\overline{\sigma}_0(t') + \frac{\varepsilon(t')-\varepsilon}{\varepsilon}\frac{\partial}{\partial t}\right)\delta\left(t-t'-\frac{|x-x'|}{v}\right) E_k(t',x'). \quad (4)$$

Here, I_0 is the modified Bessel Function, δ is the Dirac function. It is convenient to calculate not electric field but an electric flux density which remains continuous with time jumps of medium parameters: $D_n(\tau,\xi) = \varepsilon_0\varepsilon_n E_n(t,x)$, $L_n(\tau,\xi) = \varepsilon_0\varepsilon F_n(t,x)$, where $\tau = tv\kappa$, $\xi = x\kappa$ are dimensionless variables, and κ is the factor with a wave number dimension.
For example, a transformation of the harmonic primary field $L_0(\tau,\xi) = \cos(\tau - \xi)$ for various time dependencies

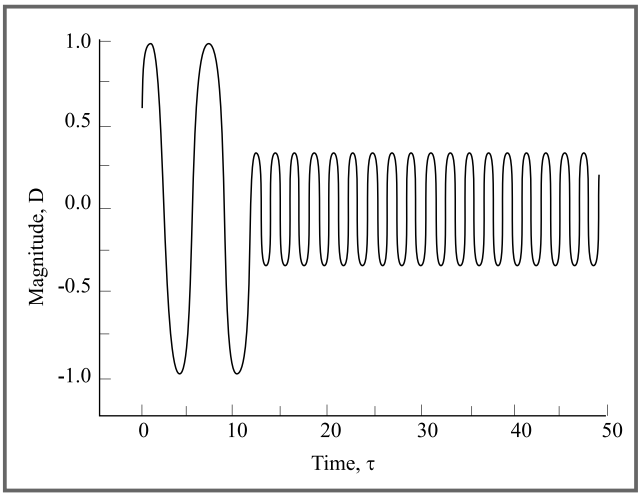

Fig. 1. The frequency shift with time jump of permittivity at $\tau_0 = 12$ when $\varepsilon_1/\varepsilon = 9$

of the permittivity is given below. The coefficients for this calculation are chosen from the data refraction index $n = n' + in''$ for semiconductor of kind InGaAsP [10] that has magnitudes $n' \approx 3.6$, $n'' \approx 0.01$. For an abrupt change of permittivity $\varepsilon(\tau) = \frac{\varepsilon_1}{\varepsilon}\,\theta(\tau - \tau_0)$, the result is shown in Fig. 1.

For periodic modulation of the permittivity $\varepsilon(\tau) = [1 + b\sin(g\tau)]^{-1}$ a transformed field is shown in Fig. 2 and 3.

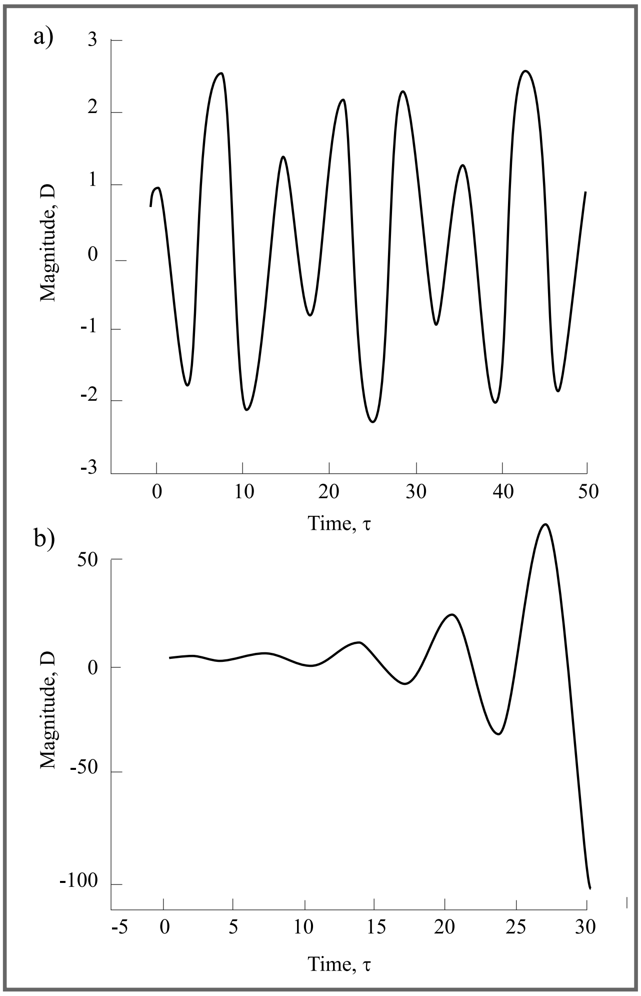

Fig. 2. The field transformation for small modulation depth of the permittivity: (a) $g = \sqrt{2}, b = 0.15$, (b) $g = 1.9, b = 0.2$

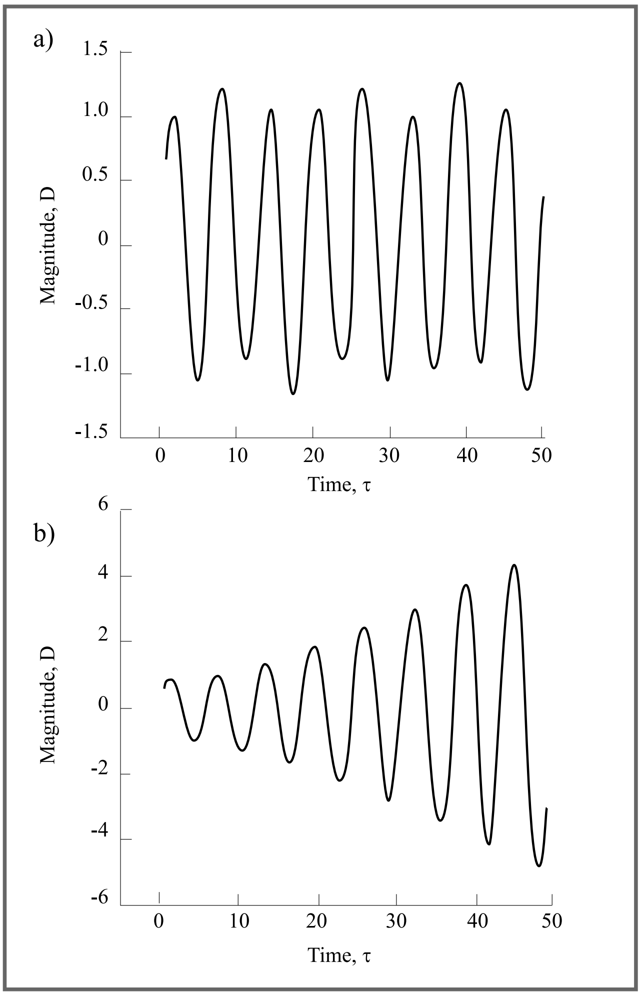

Fig. 3. The field transformation for great modulation depth of the permittivity: (a) $g = \sqrt{2}, b = 0.9$, (b) $g = 1.9, b = 0.7$

Enhanced reflection of electromagnetic wave from a plasma moving in a waveguide structure

Another way to shift a wave frequency and to amplify its amplitude is a double Doppler effect when an electromagnetic wave reflects from a moving medium boundary [11-18].

It is a common practice to characterise the efficiency of such a wave reflection by the ratio of a boundary velocity to a wave phase velocity. However, in a dispersive structure the ratio of a boundary velocity to a wave group velocity is of prime importance [19, 20]. It appears most clearly in a waveguide structure when a double dispersion mechanism is in existence.

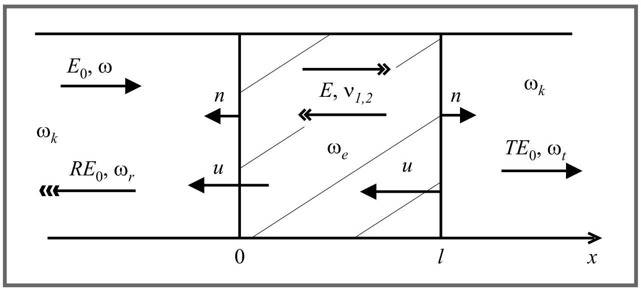

Fig. 4. A wave reflection from a plasma cluster moving in a waveguide

Simulation of such an interaction is a cluster of a homogeneous cold plasma that moves along the waveguide with velocity u (Fig. 4). Let us ω_e is the Lorentz-covariant plasma frequency, $\delta = \omega_e/\omega$ is the plasma factor, $\gamma^2 = (1-\beta^2)^{-1}$ is the relativistic factor, $\beta = u/c$.

The incident electromagnetic wave is of the TE type

$$E_0(t,x) = b_\perp E_0 \exp\left(i(\omega t - k_0 x)\right), k_0(\omega) = \frac{1}{c}\sqrt{\omega^2 - \omega_k^2}$$

is the wave number in an empty waveguide, ω_k - the waveguide frequency, $\Lambda^{-1} = \left(\sqrt{1-(\omega_k/\omega)^2}\right)^{-1}$ is a waveguide factor for the considered mode.

The frequencies and the wavenumbers of the interior waves are

$$\nu_{1,2} = \gamma^2\left(\Omega \pm \beta\sqrt{\Omega^2 - \omega_{ke}^2\gamma^{-2}}\right),$$
$$k_{1,2} = \frac{1}{c}\gamma^2\left(\pm\beta\Omega + \sqrt{\Omega^2 - \omega_{ke}^2\gamma^{-2}}\right), \tag{5}$$

where $\Omega = \omega - uk_0(\omega)$ and $\omega_{ke}^2 = \omega_k^2 + \omega_e^2$.

For the plasma velocity $\beta > \beta_1$, where β_1 defined by

$$\beta_1 = \left(\Lambda - \delta\sqrt{\delta^2 - \Lambda^2 + 1}\right)\left(\delta^2 + 1\right)^{-1} \tag{6}$$

depends strongly on waveguide and plasma factors (what is illustrated in Table 1, the interior field consists of the damped waves as the expressions under the roots are negative.

The frequency multiplication coefficient for the reflected wave is determined by the movement velocity and the waveguide factor by virtue of the formula

$$P = \omega_r/\omega = \left(1 - 2\beta\Lambda + \beta^2\right)/\left(1-\beta^2\right), \tag{7}$$

and does not depend on the interior parameters of the cluster (length and plasma frequency).

Table 1
The β_1 values

Λ^{-1}	$\delta = 0.5$	$\delta = 1.0$	$\delta = 1.5$
10	0.365	0.665	0.800
2.5	-0.097	-0.472	-0.688
1.25	0.327	-0.183	-0.499
1.001	0.584	-0.090	-0.390

Reflectance and external transmittance are given by the formula

$$R = \frac{Pf2i\sin\alpha}{(1-f^2)\cos\alpha + i(1+f^2)\sin\alpha},$$
$$T = \frac{(1-f^2)\exp(i\gamma\delta\,\omega c^{-1} g l)}{(1-f^2)\cos\alpha + i(1+f^2)\sin\alpha}, \tag{8}$$

where $q = \gamma\left(\Lambda - \beta\right)/\delta$, $f = \left(q - \sqrt{q^2-1}\right)^2$ and $\alpha = \gamma\delta\,\omega c^{-1} l\sqrt{q^2-1}$.

When the plasma cluster length tends to infinity one has a reflectance of a half-infinite plasma cluster

$$R_0 = Pf = P\left(q - \sqrt{q^2-1}\right)^2. \tag{9}$$

This reflectance peaks at $\beta \approx \beta_1$.

Reflectivity and transmittancy of a cluster are determined by the known equations

$$\overline{R} = S_R/S_O = RR^*\nu_{gR}/\nu_{gO}, \quad \overline{T} = S_T/S_O = TT^*, \tag{10}$$

where S_O, S_R, S_T are the energy fluxes of incident, reflected and passed waves, respectively.

For $\beta_1 \le \beta$

$$\overline{R} = \frac{\overline{R}_0 sh^2\alpha^*}{4q^2(1-q^2) + sh^2\alpha^*},$$
$$\overline{T} = \frac{4q^2(1-q^2)}{4q^2(1-q^2) + sh^2\alpha^*}, \tag{11}$$

$$\overline{R}_0 = \frac{(1+\beta^2)\Lambda - 2\beta}{\Lambda(1-2\beta\Lambda+\beta^2)}P^2. \tag{12}$$

For $-1 < \beta < \beta_1$

$$\overline{R} = \frac{\overline{R}_0 4\sin^2\alpha}{\left[1-\left(q-\sqrt{q^2-1}\right)^4\right]^2 + 4\left(q-\sqrt{q^2-1}\right)^4\sin^2\alpha},$$
$$\overline{T} = \frac{\left[1-\left(q-\sqrt{q^2-1}\right)^4\right]^2}{\left[1-\left(q-\sqrt{q^2-1}\right)^4\right]^2 + 4\left(q-\sqrt{q^2-1}\right)^4\sin^2\alpha}, \tag{13}$$

$$\overline{R}_0 = \frac{(1+\beta^2)\Lambda - 2\beta}{\Lambda(1-2\beta\Lambda+\beta^2)}P^2\left(q-\sqrt{q^2-1}\right)^4. \tag{14}$$

The maximal reflectivity of the electromagnetic wave in the waveguide can take very great magnitudes and is observed not for relativistic values of the cluster velocity but for smaller values as it is noticed in Table 2. The value of this plasma cluster velocity depends on the parameters of the plasma and the waveguide and can be done very small. A strong influence of the waveguide is explained by the fact that the group velocity of the incident wave tends to zero when $\Lambda^{-1} \to \infty$ but the group velocity of the reflected wave $\nu_{gR} = c\left(2\beta - (1+\beta^2)\Lambda\right)\left(1-2\beta\Lambda+\beta^2\right)^{-1}$ does not tend to zero.

Table 2
The reflectivity of the half-infinite cluster

Freq. multipl. coeff. P	The relativistic factor γ	The waveguide factor Λ^{-1}	The plasma factor δ	Reflectiv. of the half-inf. clust. $\overline{R}_0$
2	1.05	1.25	1.25	4.5
2	1.12	2.5	1.1	7.2
2	1.21	20	0.073	70.0
10	1.84	1.25	3.0	125.0
10	1.96	2.5	2.55	225.0
10	2.3	20	2.18	2000.0

The shift of the reflectivity maximum to smaller values of the cluster velocity owes to the existence of a double dispersion mechanism, a plasma dispersion and a waveguide dispersion.

Combination of two mechanisms for a frequency shift

Combination of an effect of electromagnetic wave frequency changing caused by time variation of permittivity and a similar one caused by reflection from a moving boundary gives a new effect. It is shown at an example of a flat dielectric slab whose boundaries move beginning from zero moment of time and meet through any time interval (Fig. 5).

The equation for electromagnetic field inside the slab as well as outside one is analogues to Eq. (5)

$$E = E_0 - \frac{1}{2\varepsilon\nu}\times \\ \times\frac{\partial^2}{\partial t^2}\int_{-\infty}^{\infty}\mathrm{d}t'\int_{-\infty}^{\infty}\mathrm{d}x'\left[\varepsilon_1-\varepsilon+\left(\varepsilon_2-\varepsilon_1\right)\theta\left(t'\right)\right]\times \\ \times\chi\theta\left(t-t'-\frac{|x-x'|}{\nu}\right)E\ . \quad (15)$$

Here, ε is the permittivity outside a slab; ε_1 and ε_2 are the permittivity inside a slab before zero time moment and after it, respectively; $\chi(t,x)$ - characteristic function that equals to one inside a slab and zero outside of it; $\theta(t)$ - the Heaviside unit function.

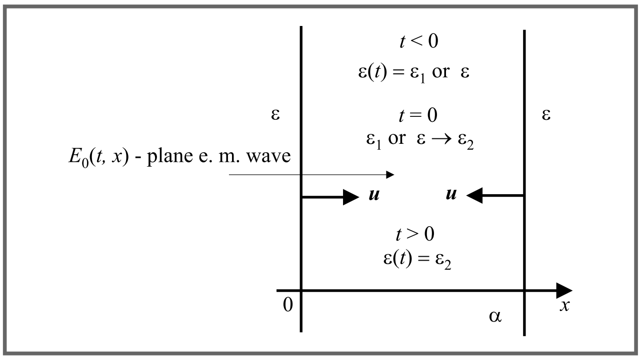

Fig. 5. The geometry of the problem

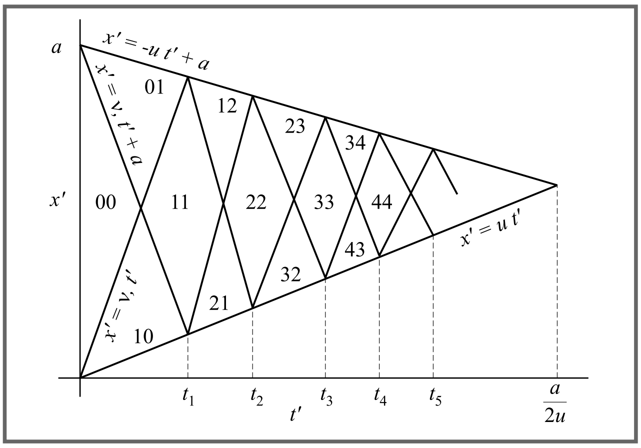

Fig. 6. Formation of time-spatial zones a slab that is collapsing

Collapsing slab is created after zero moment of time when slab boundaries begin to move with a velocity u and meet at a moment $t_c = a/2u$. The electromagnetic field has qualitatively dissimilar forms in the different zones on the time-spatial diagram, Fig. 6.

A distance between zones decreases by the law (for the case $u < \nu_2$)

$$t_n - t_{n-1} = p^{1-n}\frac{a}{\nu_2+u}\,, \quad (16)$$

so that infinitely many zones are packing up in a finite interval. Here, $p = \frac{\nu_2+u}{\nu_2-u}$.

If $u > \nu_2$ the slab boundaries do not influence on a field. The field in the 00 zone consists of two splitting waves [21]

$$E = C_1\mathrm{e}^{i\omega_2(t-x/\nu_2)} + C_2\mathrm{e}^{-i\omega_2(t+x/\nu_2)}\,, \\ C_{1,2} = \frac{\nu_2}{\nu}\,\frac{\nu_2\pm\nu}{2\nu}\,,\ \omega_2 = \omega\frac{\nu_2}{\nu}\,. \quad (17)$$

The field in the mm zone has more complicated structure

$$E_{mm} = \frac{\omega^{(+)}}{\omega}C_0\left\{\mathrm{e}^{i\omega^{(+)}(t-x/\nu_2)} + \sum_{k=1}^{m-1}R_1^k\exp\left[i\omega^{(+)}\times\right.\right. \\ \left.\left.\times\, p^k\left(t-(-1)^k x/\nu_2\right) - i\frac{1-p^{\frac{k+i_k}{2}}}{1-p^2}q\frac{\omega^{(+)}}{\nu_2}a\right]\right\} + \\ +R_1^m C_1\exp\left[i\omega_2 p^m\left(t-(-1)^m x/\nu_2\right)+\right. \\ \left.-\,i\frac{1-p^{\frac{m+i_m}{2}}}{1-p^2}q\frac{\omega_2}{\nu_2}a\right] + R_1^m C_2\exp\left[-i\omega_2 p^m\left(t+\right.\right. \\ \left.\left.+(-1)^m x/\nu_2\right) + i\frac{1-p^{\frac{m+i_m}{2}}}{1-p^2}\,pq\frac{\omega_2}{\nu_2}a\right] + \\ +R^m C_1\mathrm{e}^{i\omega_2(t-(-1)^m x/\nu_2)}\Phi^{\frac{m+i_m}{2}\frac{\nu_2}{\nu}} + \\ +R^m C_2\mathrm{e}^{-i\omega_2(t+(-1)^m x/\nu_2)}\Phi^{-\frac{m-i_m}{2}\frac{\nu_2}{\nu}}\,, \quad (18)$$

where

$$\begin{aligned} C_0 &= \frac{2\nu_2}{\nu+\nu_2}, \quad q = \frac{2\nu_2}{\nu_2 - u}, \\ \omega^{(\pm)} &= \omega\frac{\nu_2}{\nu}\frac{\nu-u}{\nu_2 \pm (-1)u}, \quad R_1 = pR, \quad R = \frac{\nu-\nu_2}{\nu+\nu_2}, \\ \Phi &= \mathrm{e}^{-i\omega\frac{2a}{\nu_2}}, \quad i_m = \frac{1}{2}\left(1-(-1)^m\right). \end{aligned} \tag{19}$$

Inside the slab there are two waves caused by splitting waves C_1 and C_2 owing to a permittivity jump, but frequencies of these waves rise with a zone number. The set of the waves that are proportional to C_0 and raised by a field that incidences upon a slab after zero moment of time has a descrete frequency spectrum. Frequencies of all waves grows up with a zone number and with time consequently. A frequency multiplication coefficient equals $p = \frac{\nu_2+u}{\nu_2-u}$ and grows with $u \to \nu_2$.

Behavior of the secondary waves amplitudes is determined by a relation between wave phase velocity and a boundary velocity. If $R_1 > 0$, that is $\nu u - \nu_2^2 > 0$, the amplitudes grow infinitely during a finite time interval. The region with such a relation between velocities is shown in Fig. 7 as a single crosshatched region.

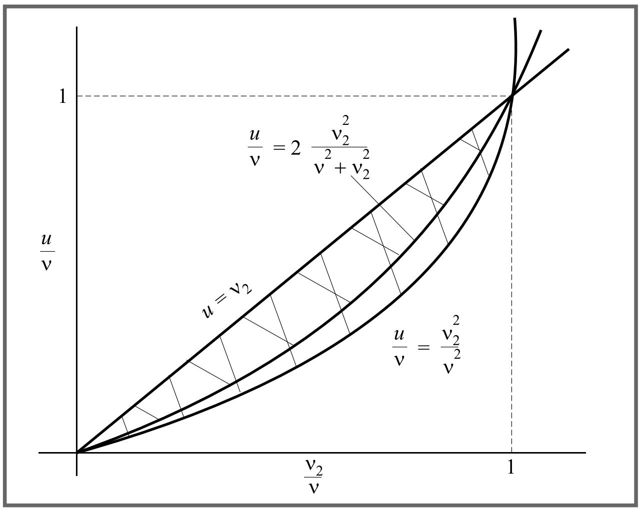

Fig. 7. Regions where amplitudes grow and field energy accumulates

The energy balance for waves raised by the waves C_1 and C_2 in the slab in the time interval $[t_{m-1}, t_m]$ is determined by

$$\begin{aligned} \frac{\mathrm{d}W}{\mathrm{d}t} &= \varepsilon_2\left(\nu_2 - u\right)\left(R_1^2 - p\right)R_1^{2(m-1)} \times \\ &\times \left\{C_1^2\cos^2\left(\omega_m t - \phi_1\right) + C_2^2\cos^2\left(\omega_m t - \phi_2\right)\right\}, \end{aligned} \tag{20}$$

where $\omega_m = p^{m-1}\frac{\nu_2+u}{\nu}\omega$.

When a movement is absent $u = 0$ then $\mathrm{d}W/\mathrm{d}t < 0$. The waves with the frequency ω_2 are shone out.

When boundaries move and $R_1^2 - p > 0$ or

$$\frac{u}{\nu} > \frac{2\nu_2^2}{\nu^2 + \nu_2^2} \tag{21}$$

then $\mathrm{d}W/\mathrm{d}t > 0$.

A region where a field energy accumulates is shown in Fig. 7 as a double crosshatched region.

Outside the slab the field represents a sequence of waves packages that are devided by fronts $x_m = \nu t_m$. The field in such a package within planes νt_{m+1} and νt_m has the form

$$\begin{aligned} E_m &= \frac{2\nu^2}{\nu_2(\nu+\nu_2)}\frac{\nu_2+u}{\nu+u}C_1^{i_m}C_2^{1-i_m} \times \\ &\times R_1^m\,\mathrm{e}^{-i\omega p^m\frac{\nu_2+u}{\nu+u}(t+x/\nu) + (-1)^m i\omega\eta_m\frac{a}{\nu}}, \end{aligned} \tag{22}$$

where $\eta_m = -\frac{\nu_2-u}{2u}p^{1-i_m}\left(1-p^{m+i_m}\right)$. This wave frequency rises by a factor p^m. A field energy within the package is proportional to

$$W_{ex} \approx \left[\left(\frac{\nu-\nu_2}{\nu+\nu_2}\right)^2 p\right]^m. \tag{23}$$

If $\frac{u}{\nu} > \frac{2\nu_2^2}{\nu^2+\nu_2^2}$ a field energy within the package outside the slab grows infinitely when $m \to \infty$, that corresponds to approaching to a collapse moment.

References

[1] F. R. Morgenthaler, „Velocity modulation of electromagnetic waves", *IRE Trans. Microw. Theory Technol.*, vol. MTT-6, no. 4, pp. 167–172, 1958.

[2] L. B. Felsen and G. M. Whitman, „Wave propagation in time-varying media", *IEEE Trans. Anten. Propag.*, vol. AP-18, no. 2, pp. 242–253, 1970.

[3] R. L. Fante, „Transmission of electromagnetic waves into time-varying media", *IEEE Trans. Anten. Propag.*, vol. AP-19, no. 3, pp. 417–424, 1971.

[4] L. A. Ostrovsky and N. S. Stepanov, „Nonresonant parametric phenomena in distributed system", *Radiophys. Quant. Electron.* (Eng. transl.), vol. 14, no. 4, pp. 489–529 (in Russian pagination), 1971.

[5] S. I. Averkov and V. P. Boldin, „Waves in nondispersive nonstationary inhomogeneous media", *Radiophys. Quant. Electron.* (Eng. transl.), vol. 23, no. 9, pp. 1060–1066 (in Russian pagination), 1980.

[6] A. G. Nerukh and I. Yu. Shavorikina, „Splitting of electromagnetic impulse under a conductivity jump of restricted medium", *J. Technich. Fis.*, vol. 62, no. 5, pp. 108–118 (in Russian), 1992.

[7] A. G. Nerukh, I. V. Scherbatko, and D. A. Nerukh, „Using the evolutionary recursion for solving electromagnetic problem with time-varying parameter", *Microw. Opt. Technol. Lett.*, vol. 14, no. 1, pp. 31–36, 1997.

[8] A. G. Nerukh, I. V. Scherbatko, and O. N. Rybin, „The direct numerical calculation of an integral Volterra equation for an electromagnetic signal in time-varying dissipative medium", *J. Electromagn. Wav. Appl.*, vol. 12, pp. 163–176, 1998.

[9] A. G. Nerukh and N. A. Khizhnjak, *Modern problems of transient macroscopic electrodynamics*. Kharkov: Test-radio Publ., 1991.

[10] T. D.Visser, H. Block, and D. Lenstra, „Modal analysis of a planar waveguide with gain and losses", *IEEE J. Quant. Electron.*, vol. 31, no. 10, pp. 1803–1810, 1995.

[11] K. Landecker, „Possibility of frequency multiplication and wave amplification by means of some relativistic effects", *Phys. Rev.*, vol. 86, no. 6, pp. 852–855, 1952.

[12] M. A. Lampert, „Reflection of electromagnetic waves by Cerenkov electron gas", *Phys. Rev.*, vol. 102, no. 2, pp. 299-304, 1956.

[13] J. B. Fainberg and V. S. Tkalich, „Electromagnetic wave reflection from a plasma moving through dielectric medium in a constant magnetic field", *Sov. Phys.-Tech. Phys.* (Eng. transl.), vol. 4, no. 4, 1959.

[14] O. G. Zagorodnov, J. B. Fainberg, A. M. Egorov, and B. M. Bolotovsky, „Frequency multiplication by means of a plasma «collapse»", *Sov. Phys.-Tech. Phys.* (Eng. transl.), vol. 6, no. 3, 1961.

[15] V. I. Kurilko and V. I. Miroshnichenko, „Electromagnetic wave reflection by a moving plasma", *Sov. Phys.-Tech. Phys.* (Eng. transl.), vol. 7, no. 7, 1962.

[16] C. Yeh, „Reflection and transmission of electromagnetic by waves by a moving plasma medium", *J. Appl. Phys.*, vol. 37, no. 8, pp. 3079–3084, 1966.

[17] S. N. Stoljarov, *Selected papers by A. Einstein.* Moscow: 1975–1976; *„Nauka"*, pp. 152–215 (in Russian), 1978.

[18] W. B. Mori, „Generation of tunable radiation using an underdense ionization front", *Phys. Rev. A.*, vol. 44, no. 8, 1991.

[19] A. G. Nerukh and N. A. Khizhnjak, „Energy relations for the interaction of an electromagnetic wave with a plasma cluster moving in a waveguide", *Radiophys. Quant. Electron.* (Eng. transl.), vol. 26, no. 12, 1983.

[20] A. G. Nerukh, N. A. Khizhnjak, and P. E. Minko, „Electromagnetic wave reflection from stratified plasma cluster moving in a waveguide", *Radiophys. Quant. Electron.* (Eng. transl.), vol. 33, no. 5, 1990.

[21] A. G. Nerukh, „Electromagnetic waves in the dielectric layer with time-dependent parameters", *Zhurn. Tekhnich. Fiz.*, vol. 57, no. 11, pp. 2078–2087, 1987.

Aleksander G. Nerukh, Igor V. Scherbatko
Techn. Univ. of Radio Electronics,
Lenin Av. 14, 310726, Kharkov, Ukraine,

Marian Marciniak
Technical University of Kielce
Al. 1000-lecia P.P. 7, 25-314 Kielce, Poland;
National Institute of Telecommunications,
1 Szachowa Str., 04-894 Warsaw, Poland,
e-mail: M.Marciniak@itl.waw.pl