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THE EFFECT OF ERRORS IN SETTING BEARING HOUSINGS OF A THREE-BEARING SHAFT ON THE BASIC RATING LIFE OF A BEARING SYSTEM

ZNACZENIE BŁĘDÓW USTAWIENIA OPRAW ŁOŻYSKOWYCH WAŁU TRZYPODPOROWEGO DLA TRWAŁOŚCI ŁOŻYSKOWANIA

Key words:

rolling bearing system, bearing load, bearing deformation, shaft deflection, basic rating life

Słowa kluczowe:

łożyskowanie, łożysko kulkowe, niewspółosiowość, obciążenie łożyska, trwałość łożyska

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INTRODUCTION

A system consisting of a shaft and three bearings is statically undetermined. In line with external loads, shaft rigidity and bearing rigidity also influence the scale of loading of supports (i.e. bearings).

It is taken into consideration that fulcrums of a conventional shaft line (determined by inner bearing rings) may not be located along one straight line due to errors in setting of bearing cases.

Moreover, these points can get dislocated during the bearing system work as a result of

a) Internal clearances of bearings, and
b) Elastic shifts in bearings.

Elastic shifts in bearings result from elastic deformations at the contact zones of rolls with rings. **Figure 1a**) presents the shaft line in a nominal position (at the axle of perfectly coaxial cases). The clearances in bearings Δr_A, Δr_B and Δr_C are marked too. In **Fig. 1b**), a simplified deflection line of a loaded shaft is presented. As it can be seen, the shift of a shaft line in every bearing consists of ½ of bearing clearance and of elastic deformation of the bearing elements.

The described working shifts overlap with preliminary shifts resulting from an error in the setting of bearing cases. This error is included in further considerations in the form of the misalignment of the middle bearing with the line determined by end bearings.

Bearings modelled in **Fig. 1** behave like articulations, i.e., they do not counteract shaft bending. In reality, angular deflections of a shaft, occurring in supports, are the reason for angular misalignment of inner rings in relation to outer rings.

Shifts in shaft and bearings are coupled: radial shifts in bearings correspond with shaft radial deflections on supports; whereas, angular misalignment of the inner ring in relations to outer ring are equal to angles of shaft deflection on supports. As a result of these angular misalignments in the bearings' bending reactive moments appear and are presented in simplified form in **Fig. 2**.

These reactive bending moments, usually higher in the end bearings, counteract shaft angular deflection. Thus, in the general approach, the shaft is a subject to the following loads:

– External forces (in accordance with **Fig. 1**),

– Reaction forces of supports *Rxa*, *Rya*, *Ryb*, *Ryc*, and
– Reactive moments of supports $M_g a$, $M_g b$, $M_g c$.

These are illustrated in **Fig. 3**.

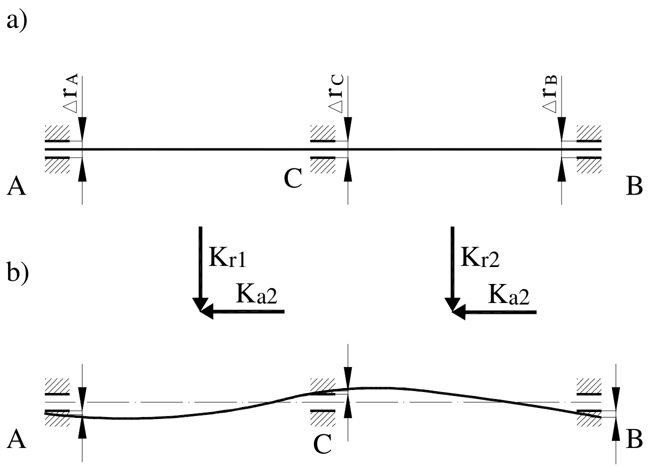

Fig. 1. Simplified graph of a shaft deflection line
Rys. 1. Uproszczona ilustracja linii ugięcia wału

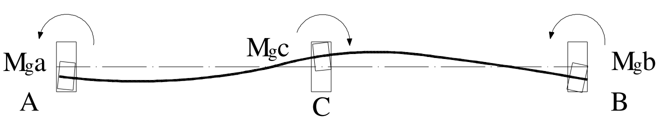

Fig. 2. Simplified graph of angular misalignments in bearings
Rys. 2. Uproszczona ilustracja odchyleń kątowych w łożyskach

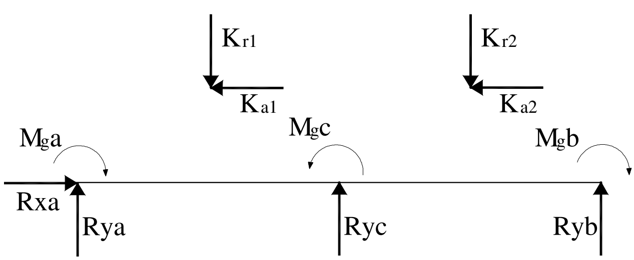

Fig. 3. Illustration of the discussed shaft loadings
Rys. 3. Ilustracja rozważanych obciążeń wału

All the enumerated loads are taken into consideration in static equations, which are the basis for calculation application.

Values of reactive radial forces *Ry* of every support depend not only on the distribution of external loads and shaft stiffness, but also on radial elasticity of supports, because displacements of supports cause a change in the shaft deflection line. For the same reason, the values of reactive bending moments also depend on the radial elasticity of supports.

Radial elasticity can be defined as the relationship of radial force influencing the bearing to radial elastic shift of the inner ring in relation to the outer one starting with removing clearance. Radial elastic shifts of inner rings in relation to outer rings are synonymous with the previously mentioned notion of elastic shifts in bearings. Axial elasticity is defined analogically.

It is difficult to find a solution, because bearing elasticity is not a constant parameter nor even in linear dependence on the loading, especially in case of complex loading. Thus, it is not possible to determine displacements analytically in the function of loads in a general case.

PHYSICAL MODEL

A machine shaft made of steel is supported by three roller bearings. The shaft can have variable diameter, i.e. it can consist of a number of segments with a cylindrical shape. The shaft is elastic, and the relation between the load and the deflection is a result of a commonly known equation of the shaft deflection line. Deep groove ball bearings are the shaft support. The bearings are elastic too, and the relation between load and the displacement of an inner ring in relation to the outer one results from geometrical summing of local loads and Hertz deformation at the point where the balls contact the race. Axial loading of the shaft is taken over by the first bearing to the left being a locking bearing.

Simplifying assumptions:

- The problem is considered statically (forces and changes of forces caused by rotary motion of the shaft and inner parts of bearings are omitted).
- Shaft loading occurs in one axial plane and is represented by component forces in radial x and axial y direction.
- No errors in balls and bearing ring shape are considered.

– Elastic deformations of bearings are considered only at the zones of contact of rolling parts with rings (free surfaces of rings maintain cylindrical shape).
– Clearances associated with bearing fits are not considered.

PROBLEM SOLVING METHOD

Due to mutual relations of the discussed values (deformations and loadings in shaft and in bearings) and no possibility of straight calculating of a component shift in the bearing depending on the load, it has been decided to follow an iterative procedure in which the following steps can be enumerated:

1. In the first step, preliminary (initial) values of supports reactions and preliminary (initial) values of angles of shaft deflection in supports are calculated without taking into consideration bearing elasticity and bending moments in bearings (**Fig. 4**). For determining the reaction of supports and tentative lines of shaft deflection a superposition method is used **[L. 1, 2]**.

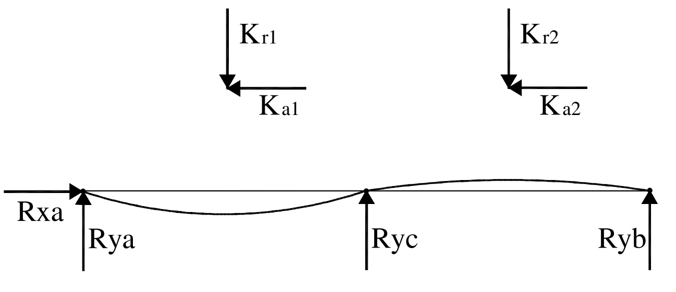

Fig. 4. Shaft loading and its bending line assumed in the first step
Rys. 4. Obciążenie wału i jego linia ugięcia, przyjmowana w pierwszym kroku

2. In the second step, elastic shifts in bearings and reactive moments of bearings are determined for preliminary loadings and preliminary parameters of the shaft deflection line (according to point 1). It is not possible to directly assign elastic shift in a bearing on the basis of its loading, in particular with complex loading. It is due to the fact that the number of balls carrying the load and the contact angle of particular balls is not known. With regard to these difficulties, the following process of the problem solving presented in **[L. 3, 4]** and used in **[L. 5**, **6**, **7**, **8]** has been accepted:

– Trial values of shifts in a bearing have been assumed (i.e. values of mutual shifts and angular misalignments of inner rings in relation to the outer ones).
– On the basis of shifts, elastic deformations at the contact of balls with the race are determined.
– With the knowledge of these shifts, the forces acting between the balls and the races are assigned.
– These forces, together with their directions, are the basis for calculating the resultants of forces and moments operating between the bearing rings, which are equivalent to external forces and moments influencing the bearing.
– On the basis of forces operating at the contact of balls with the races, the substitute load of the bearing is calculated.
– Correlating appropriate values of external forces and moments with the appropriate substitute loads, a dependency of substitute load on external loads of the bearing is received.

3. In the third step, final values of the reaction of supports and final parameters of the shaft deflection line are determined with respect to elastic shifts in the bearings and the reactive moments of bearings as well as misalignment of the middle bearing (**Fig. 5**).

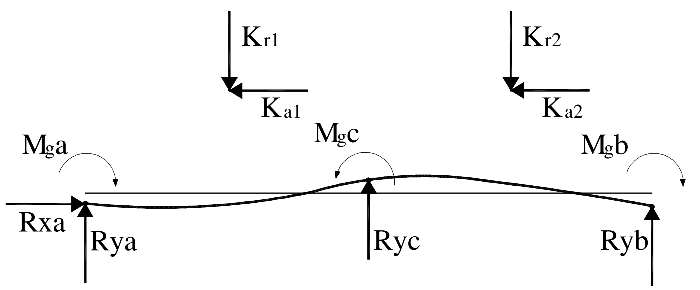

Fig. 5. Shaft loading and its deflection line assumed in the first step of calculations
Rys. 5. Obciążenie wału i jego linia ugięcia, przyjmowane w trzecim kroku obliczeń

Determining the loading of "C" support and angles of deflections of a shaft at the "A" and "B" support is done using the rule of a fictitious reverse shift of the deflection of "C" support, the value of which results from the following:

1) Compensating y_L deflection,
2) Correction $\Delta'y$, equal to misalignment of C bearing (positive when misalignment is compliant with the direction of axle y (**Fig. 7**)),

3) Corrections *Δ"y*, leading into consideration the difference between inner clearances in end bearings (**Fig. 6**).

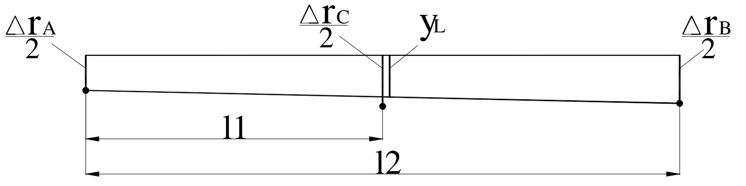

Fig. 6. Graphic presentation of *Δ"y* correction
Rys. 6. Graficzne przedstawienie poprawki *Δ"y*

$$y_L = \frac{\Delta r_A}{2} + \frac{l1}{l2} \cdot \frac{\Delta r_B - \Delta r_A}{2} \tag{1}$$

$$\Delta y'' = \frac{\Delta r_C}{2} - y_L \tag{2}$$

$$y_2 = y_1 - \Delta y' - \Delta y'' \tag{3}$$

Through the iterative process by the method of successive approximations, the conformity of loads and deflections in the shaft and in bearings is reached. With successive corrections of all component shifts of inner rings in relation to the outer ones in bearings, the values of forces and reactive moments of bearings are determined in the analysis of shaft statics and they correspond with shaft deflections in accordance with shifts in bearings.

On the basis of the above presented procedure, a computer application is proposed for calculating contact angles of the balls, contact deformations, contact loads and stresses in particular locations of a ball, and basic rating life of a bearing with respect to actual spectrum of the loading of the balls in a three-bearing shaft.

To provide an example, calculations of the influence of the misalignment of the middle bearing on the durability of bearings have been made for a simple bearing system presented in **Fig. 7**. In calculations, bearings with clearance in group N have been used (assumed value Δ_r = 13 µm).

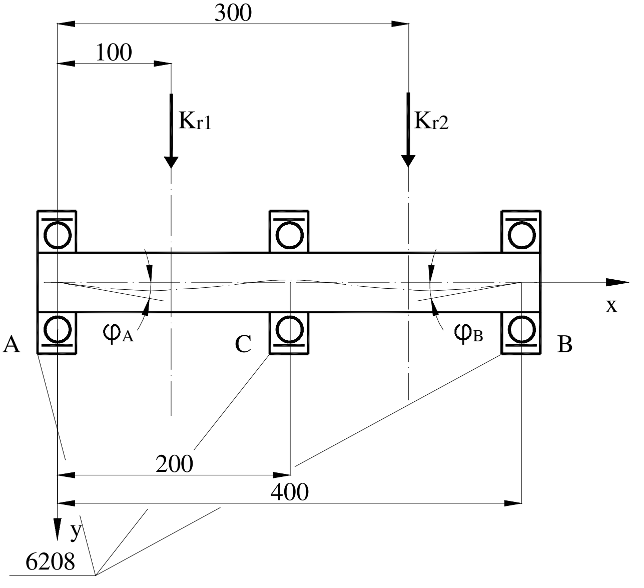

Fig. 7. An example of a bearing used for calculations
Rys. 7. Przykład łożyskowania przyjętego do obliczeń

RESULT

The influence of the misalignment of the inner bearing being accordingly -60 µm, -20 µm, -10 µm, 0 µm, 10 µm, 20 µm, 60 µm, is presented in **Fig. 8**.

On the basis of the results, it can be stated that misalignment of the middle ring has a great influence on the basic rating life of a bearing system in the assumed calculation system. Basic rating lives of bearings calculated with respect to misalignment differ considerably from the durability calculated without this parameter. These differences depend on the direction of misalignment. We assume that a positive sense of misalignment is the one corresponding with the sense of deflection line caused by radial loading of a shaft. Regarding this assumption, with a positive rise of misalignment of the middle bearing (in the assumed coordinate system) its durability grows, and durability of other bearings decreases; with negative rise of misalignment of the middle bearing its durability falls and durability of other bearings grows. Naturally, this scheme applies to minor misalignments at a technically realistic level.

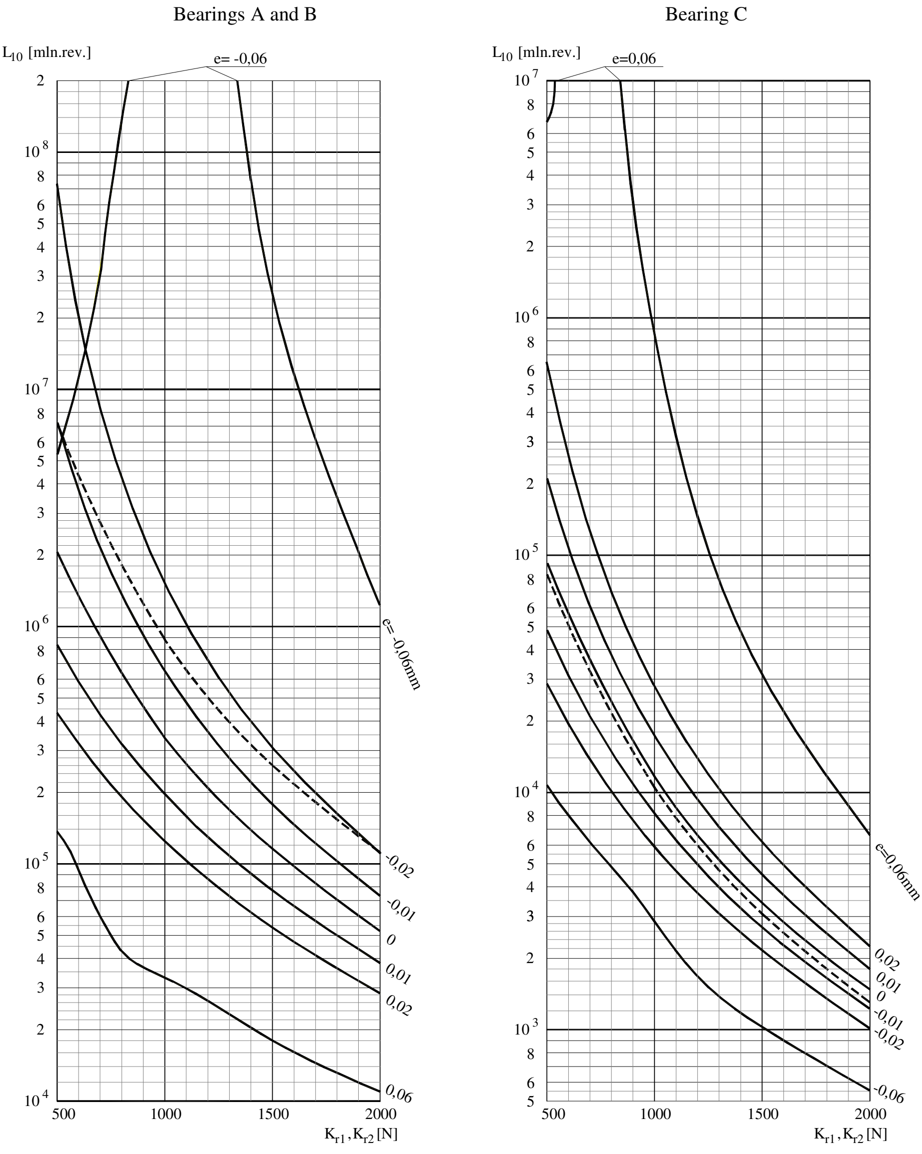

Fig. 8. Basic rating life L_{10} of roller bearings with different misalignments "e" of C bearing. Solid line – according to the author's calculations, dashed line – according to catalogue calculations (i.e. without consideration of elasticity of shaft nor bearings)

Rys. 8. Trwałość L_{10} łożysk kulkowych dla różnych niewspółosiowości łożyska C. Linia ciągła – wg obliczeń własnych, linia kreskowa – wg obliczeń katalogowych

Computer application, the essence of which has been described above, can be used for calculating basic rating life of a three-bearing shaft for any geometrical parameters and loads.

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Streszczenie

W układach napędowych maszyn i urządzeń bardzo często występują długie wały napędowe, które wymagają osadzenia na kilku podporach. Dlatego też ustalenie wpływu dokładności wykonania i montażu oraz warunków pracy łożysk na ich trwałość jest bardzo istotne.

W artykule tym podjęto próbę określenie wpływu niewspółosiowości łożyska środkowego na trwałość łożyskowania wału trzypodporowego, uwzględniając przy tym luzy w łożyskach i sprężyste przemieszczenia w łożyskach. Opisano model fizyczny ułożyskowanego wału oraz sposób rozwiązania zagadnienia. Wyniki obliczeń przedstawiono w postaci wykresów.