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APPLICATION OF THE PEARSON'S DISTRIBUTIONS FOR CHARACTERIZING THE SURFACE ROUGHNESS OF MACHINE COMPONENTS

ZASTOSOWANIE ROZKŁADÓW PEARSONA DO CHARAKTERYZOWANIA CHROPOWATOŚCI POWIERZCHNI ELEMENTÓW MASZYN

Key words:

probability distribution, Pearson's distributions, surface roughness

Słowa kluczowe:

rozkład prawdopodobieństwa, rozkłady Pearsona, chropowatość powierzchni

Summary

The family of the Pearson's distributions of probability has been presented. These distributions have been defined as the functions of the

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asymmetry and concentration of the random variable, i.e. the dimensionless skewness parameter Sk and the dimensionless kurtosis parameter Ku. The probability distributions of the seven Pearson's distributions have been given. The connecting point of these distributions is a normal one. The results of the measurements of the surface roughness of a number of the unworn and worn machine elements have been presented, for which the Pearson's analysis has been applied.

INTRODUCTION

The Pearson's distributions **[L. 1, 2]** were originally devised in an effort to model the unsymmetrical distributions. In the case of the symmetrical probability distributions the adjusting a theoretical model is based on the two first moments of the observed data, that means the expected value (the first common moment) and variance (the second central moment) are to be taken into consideration. It was not known how to construct the probability distrbutions, in which the skewness (*i.e.* the standardised third central moment) and kurtosis (*i.e.* the standardised fourth central moment) could be adjusted freely.

The tribological applications of the Pearson's distributions have been utilized by this author [b. In this paper the Pearson's approach has been focused on modeling the surface roughness.

MOMENTS OF THE DISTRIBUTION

The random variables can be characterized by the moments of their distributions. The common n-th moment defines the following equation

$$m_n = EZ^n = \int_{-\infty}^{\infty} z^n \cdot f(z) \cdot dz \quad (n = 0,\ 1,\ 2...) \tag{1}$$

The expected value EZ (or the centre of gravity) for any distribution is the first moment m_1.

$$EZ = m_1 \tag{2}$$

The central n-th moment, for which the origin of the coordinates is located in the centre of gravity of the distribution of the random variable, describes the formula

$$\mu_n = E(Z - m_1)^n = \int_{-\infty}^{\infty} (z - m_1)^n \cdot f(z) \cdot dz \quad (n = 0,\ 1,\ 2...) \tag{3}$$

The first central moment is 0. The second one characterizes the dispersion of the achieved data as their variance VZ

$$VZ = E\left[(Z - m_1)^2\right] = \mu_2 \tag{4}$$

The positive square root of the variance is the standard deviation σ

$$\sigma = \sqrt{\mu_2} \tag{5}$$

Skewness, the third standardized central moment, is written as Sk and defined as

$$Sk = \frac{\mu_3}{\sigma^3} \tag{6}$$

In case of the symmetric distribution the skewness is zero. The positive skewness occurs when the mass of the distribution is concentrated on the left hand side and for negative skewness the mass concentration is on the right hand side.

Kurtosis Ku is the fourth standardized central moment of the distribution

$$Ku = \frac{\mu_4}{\sigma^4} \tag{7}$$

For the normal distribution the kurtosis is 3. The higher values of this parameter concern the distributions with a sharper peak, while a low kurtosis has a more rounded peak.

THE PEARSON'S SYSTEM

The family of the Pearson's distributions defines the following differential equation

$$\frac{dz}{z} = \frac{\xi + b}{c_0 + c_1 \cdot \xi + c_2 \cdot \xi^2} \tag{8}$$

where

$$c_0 = -\sigma^2 \cdot \frac{s+1}{s-2} \tag{9}$$

$$c_1 = -b = -0.5 \cdot \sigma \cdot Sk \cdot \frac{s+2}{s-2} \tag{10}$$

$$c_2 = \frac{1}{s-2} \tag{11}$$

$$s = 6 \cdot \frac{Ku - Sk^2 - 1}{3 \cdot Sk^2 - 2 \cdot Ku + 6} \tag{12}$$

$$t = 2 \cdot \sqrt{(s+1) \cdot (1-\kappa)} \tag{13}$$

The type of the Pearson's distribution can be identified by means of the criterion parameter

$$\kappa = \frac{c_1^2}{4 \cdot c_0 \cdot c_2} = -\frac{Sk^2 \cdot (s+2)}{16 \cdot (s+1)} \tag{14}$$

The another deciding parameters are the skewness and kurtosis. There are seven types of the Pearson's distributions that have been summarized in **Tab. 1**.

Table 1. Criteria for the type of the Pearson's distribution
Tabela 1. Kryteria dla typu rozkładu Pearsona

Type of the Pearson's distribution	Criteria	Related distributions
1	$\kappa < 0$	beta distribution
2	$\kappa = 0;\ Sk = 0;\ Ku < 3$	beta distribution
3	$\kappa = \pm\infty$	gamma, exponential, chi-square distribution
4	$0 < \kappa < 1$	Cauchy distribution, Student's t-distribution
5	$\kappa = 1$	inverse-chi-square, inverse gamma distribution
6	$1 < \kappa < \infty$	F-distribution
7	$\kappa = 0;\ Sk = 0;\ Ku > 3$	distribution introduced by Pearson

The connecting point of the all seven Pearson's distributions is the Gaussian one, for which we have κ = 0; Sk = 0; Ku = 3. The density functions of the Pearson's distributions are given in **Table 2**.

Table 2. Summary of the Pearson's distributions

Tabela 2. Zestawienie zbiorcze rozkładów Pearsona

Type 1	$f = f_0 \cdot \left(1+\frac{z}{l_1}\right)^{q1} \cdot \left(1-\frac{z}{l_2}\right)^{q2} \quad for \quad -l_1 < z < l_2$ $q_{1,2} = \frac{(s-2) \mp s \cdot (s+2) \cdot \frac{Sk}{t}}{2}; \quad l_{1,2} = \frac{q_{1,2} \cdot l}{s-2}; \quad l = \frac{\sigma \cdot t}{2}$	(15)
Type 2	$f = f_0 \cdot \left[1-\left(\frac{z}{l}\right)^2\right]^q \quad for \quad -l < z < l$ $q = \frac{2.5 \cdot Ku - 4.5}{3 - Ku}; \quad l = \sigma \cdot \sqrt{\frac{2 \cdot Ku}{3 - Ku}}$	(16)
Type 3	$f = f_0 \cdot \left(1+\frac{z}{l}\right)^q \quad for \quad -1 \le z < \infty \quad (-\infty < z \le 1)$ $q = \left(\frac{2}{Sk}\right)^2 - 1; \quad l = \sigma \cdot \left(\frac{2}{Sk} - \frac{Sk}{2}\right)$	(17)
Type 4	$f = f_0 \cdot \left[1+\left(\frac{z}{l}\right)^2\right]^{-q} \cdot \exp\left[-v \cdot \tan^{-1}\left(\frac{z}{l}\right)\right] \quad for \quad -\infty < z < \infty$ $q = \frac{2-s}{2}; \quad v = \frac{s \cdot (2-s) \cdot Sk}{p}; \quad l = \frac{\sigma \cdot p}{4}; \quad p =$	(18)
Type 5	$f = f_0 \cdot z^{-q} \cdot \exp\left(-\frac{v}{z}\right) \quad for \quad 0 \le z < \infty$ $q = 4 + \frac{8 + 4 \cdot \sqrt{4 + Sk^2}}{Sk^2}; \quad v = \sigma \cdot (q-2) \cdot \sqrt{q-3}$	(19)
Type 6	$f = f_0 \cdot z^{q1} \cdot (z-l)^{q2} \quad for \quad l < z < \infty \quad (-\infty < z < l)$ $q_{1,2} = \frac{s \cdot (s+2) \cdot \frac{Sk}{t} \mp (s-2)}{2}; \quad l = \frac{\sigma \cdot t}{2}$	(20)
Type 7	$f = f_0 \cdot \left[1+\left(\frac{z}{l}\right)^2\right] \quad for -\infty < z < \infty$ $q = \frac{2.5 \cdot Ku - 4.5}{Ku - 3}; \quad l = \sigma \cdot \sqrt{\frac{2 \cdot Ku}{Ku - 3}}$	(21)

The type of the Pearson's distribution depends only on the two dimensionless parameters characterizing the random variable: the skewness Sk and kurtosis Ku (the parameter κ is a function of Sk and Ku).

The theoretical background for the Pearson's distributions has been given by Markov **[L. 4]**.

MEASUREMENTS OF THE SURFACE ROUGHNESS

The measurements of the parameters Sk and Ku of the surface texture have been performed for more than 50 machine components, both the machined and worn ones. The results are summarized in **Fig. 1**.

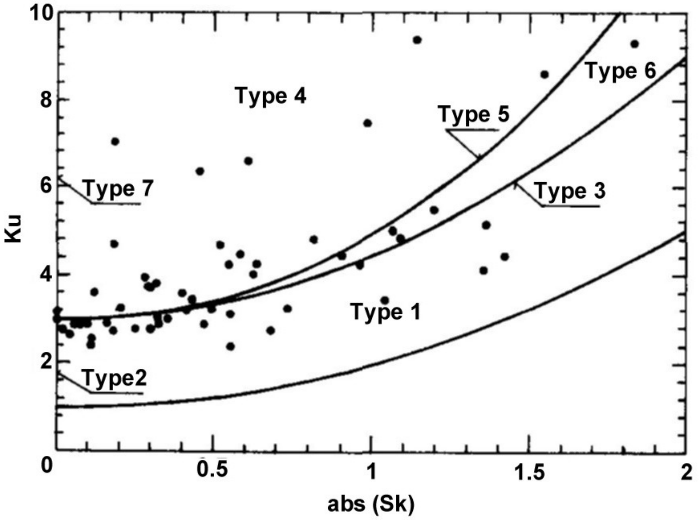

Fig. 1. Summary of the skewness and kurtosis parameters for a number of the measured machine components

Rys. 1. Zestawienie zbiorcze parametrów skośności i kurtozy dla zestawu pomierzonych elementów maszyn

It should be emphasized that the most of the measured surfaces represent the Type 1 from the family of the Pearson's distributions (the Type 1 is equivalent to the beta distribution). The opinion that the surface roughness of machine components can be described as a normal distribution has been not confirmed: the only Gaussian distribution was for the unworn components that have been ground centreless.

An example of the modeling of the measured utilizing the Pearson's system and the least squares method has been shown in **Fig. 2**.

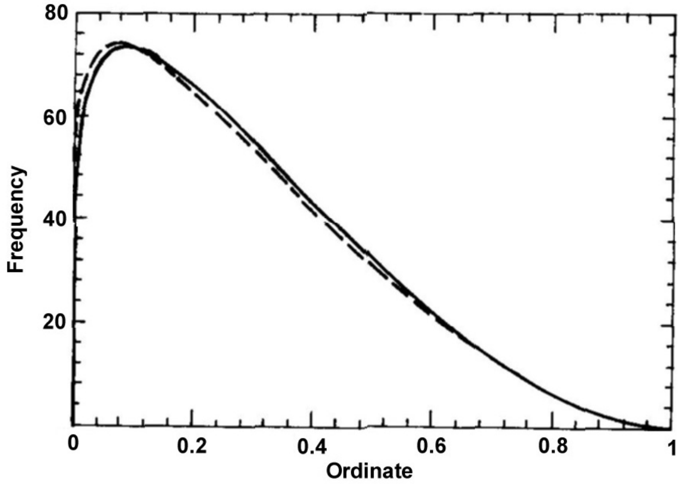

Fig. 2. Approximation of the profile of the surface roughness measurements by means of the Pearson's distributions (the full line) and utilizing the least mean square method (the dotted line)

Rys. 2. Aproksymacja profilu pomiarów chropowatości powierzchni z zastosowaniem rozkładów Pearsona (linia ciągła) oraz za pomocą metody najmniejszych kwadratów (linia przerywana)

Fig. 2 confirms the efficacy of the Pearson's approach; the both approximations shown are very similar. Then, the distribution functions of random variables can be easily produced by the application of the Pearson's system, without the loss of the accuracy of these approximations.

CONCLUSIONS

1. The surface roughness parameters for a number of the unworn and worn machine components have been identified, for which the Pearson's system has been applied.
2. The most of investigated surfaces was very far from the normal distribution and symmetry.
3. The Pearson's distribution enable the accurate approximation of the statistic parameters of any distribution of the random variable.

LITERATURE

1. Pearson K.: Methods of moments and method of maximum likelihood. Biometrica, vol. 28 (1936), p. 34–59.1991.
2. Mitropolski A.K.: Technika statističeskich wyčislenij. Moskwa: Nauka 1971.
3. Wiśniewski M.: Tribologische Auswirkungen der Oberflächenfeinstruktur in konzentrierten Kontakten. Fortschr.-Ber. VDI, Reihe 1, Nr. 204. Düsseldorf: VDI-Verlag 1991.
4. Markov A.: O ellipsoidach dispersii i korrelacii. Izvestia AN SSSR, ser. 6, vol. 18 (1924), p. 117–126.

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Streszczenie

Przedstawiona została rodzina rozkładów Pearsona. Rozkłady te zostały zdefiniowane jako funkcje współczynników asymetrii i skupienia rozkładu zmiennej losowej, tj. bezwymiarowych parametrów skośności Sk oraz kurtozy Ku. Podano gęstości prawdopodobieństwa dla siedmiu typów rozkładów Pearsona z punktem łączącym, będącym rozkładem normalnym. Przedstawiono wyniki pomiarów chropowatości powierzchni dla zestawu niezużytych i zużytych elementów maszyn, dla których przeprowadzono analizy zgodności według systemu Pearsona.