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CALCULATION OF JOURNAL BEARINGS WITH CAVITATION

OBLICZANIE ŁOŻYSK ŚLIZGOWYCH Z UWZGLĘDNIENIEM KAWITACJI

Key words:

journal bearings, cavitation, calculations

Słowa kluczowe:

łożysko ślizgowe, kawitacja, obliczanie

Summary

The design and calculations of hydrodynamic journal bearings often requires taking into account the fact that in the bearing lubricating gap occurs the phenomena of cavitation. The literature on the cavitation problem can be found in many publications, however they do not give the practical method of calculation. The analysis of the oil flow on the boundary of cavitated zone and the conditions that allow solving the described problem, has be formulated. The results of carried out calculations were compared to the results of experimental investigation.

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The conclusions recommend the developed method as well as they point on the need to add to this method an experimental determination of cavitation coefficient.

INTRODUCTION

The parameters of radial journal bearings such as the load capacity, power losses, the requirements of lubricating oil, position of journal with regard to the sleeve and the stability of this position depend on the oil flow in the lubricating gap. In general case this flow is considered as isothermal one and mostly the cavitation phenomena occurs in it. The cavitation means the generation of gas bubbles at the rapid drop of the pressure in the divergent part of lubricating gap.

The complex description of such flow is very complicated and the equations that are applied in the practice require many simplifications **[L. 1].** The limits of the applications of these equations are difficult to determine. Adding to this non-repeatable conditions of bearings operation that occurs in the designing practice, one can say that the known calculations methods are not fully universal. These methods cannot give correct basis for the design of bearings **[L. 2, 3]**. The existing situation requires that at the formulating of assumptions in the theoretical considerations, the results of experimental investigation have to be considered.

The task of this paper is to describe the flow of lubricating fluid in the journal bearing that undergoes cavitation and to determine the effect of these phenomena on the bearing operation. The development of the practical method of the load capacity calculation and the position of journal axis with regard to the sleeve was considered, too.

MODEL OF FLUID FLOW IN THE LUBRICATING GAP

It was assumed that in the lubricating gap of the thickness $h(\varphi, z)$ **(Fig. 1),** of finite length bearing there is isothermal flow of non-compressible Newtonian fluid of viscosity η and pressure $p(\varphi, z)$.

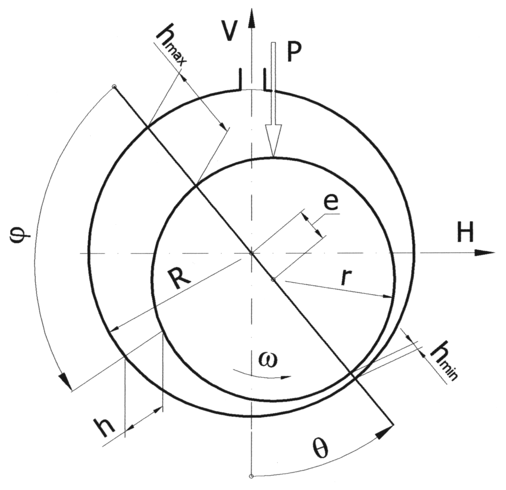

Fig. 1. Geometry of lubricating gap
Rys. 1. Geometria szczeliny smarowej

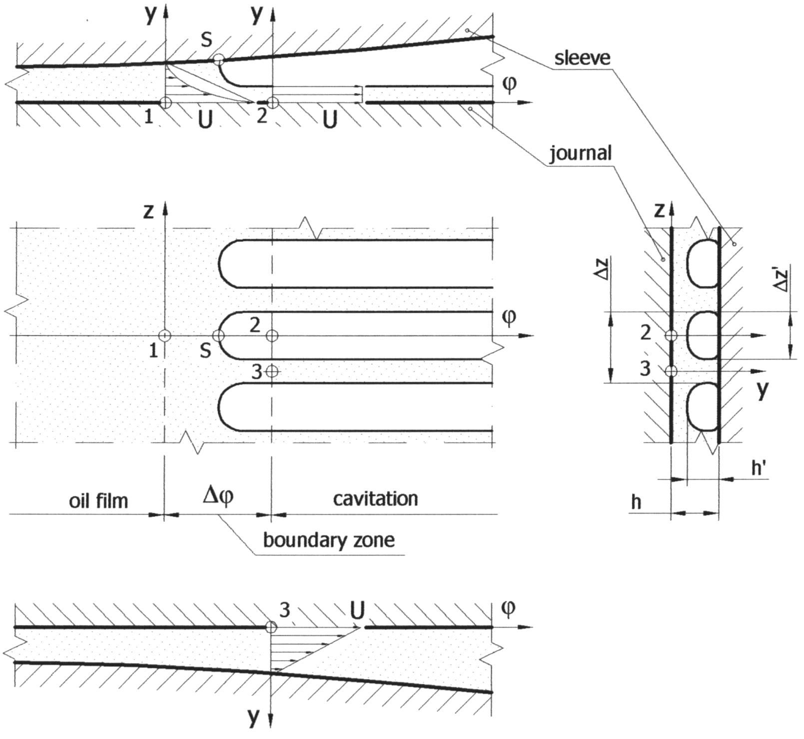

Fig. 2. Flow of fluid in the lubricating gap
Rys. 2. Przepływ cieczy w szczelinie smarowej

There are following zones in the lubricating gap (**Fig. 2):**

1. Oil film – the zone where the oil film pressure distribution describes the equation:

$$\frac{\delta}{\delta\varphi}\left(H^3\frac{\delta\overline{p}}{\delta\varphi}\right)+\frac{\delta}{\delta\overline{z}}\left(H^3\frac{\delta\overline{p}}{\delta\overline{z}}\right)=\frac{\delta H}{\delta\varphi} \tag{1}$$

where: dimensionless oil film thickness

$$H=\frac{h}{R-r}=1-\varepsilon\cos\varphi\ , \tag{2}$$

relative eccentricity

$$\varepsilon=\frac{e}{R-r}\ , \tag{3}$$

dimensionless oil film pressure

$$\overline{p}=\frac{p(R-r)^2}{\eta\cdot\omega\cdot r^2}\ . \tag{4}$$

2. Boundary zone - the space where the fluid flows around the tops of bubbles. Analysis of flow in this zone was the subject of many theoretical reports (Table 1). In all these reports, the pressure equations describing the form of bubbles were obtained. They were valuable for the better knowledge of cavitation phenomena but were not convenient in engineering calculations of oil film pressure distribution.
3. Cavitated zone – the space with the underpressure part of oil film where the uniform, longitudinal gas bubbles, equally distributed on the bearing width are generated. In this zone there is the flow of fluid separating the bubbles that are consisted in the layer bound to the surface of rotating journal.

Table 1. Boundary conditions with the gas cavitation
Tabela 1. Warunki graniczne kawitacji gazu

Author	Bubble surface	Top of bubble „S"
Swift **[L. 4]** – Stieber **[L. 5]**	$\frac{\delta p}{\delta\varphi}=\frac{\delta p}{\delta z}=0$	$\frac{dp}{d\varphi}=0$
Floberg **[L. 6]**	$\frac{\partial p}{\partial\varphi}-\frac{\delta p}{\delta z}\cdot\frac{d\varphi}{dz}=\frac{6\eta U}{h^2}$	$\frac{dp}{d\varphi}=\frac{6\eta U}{h^2}$
Birkhoff, Hays **[L. 7]**	$\frac{\delta p}{\delta\varphi}=\frac{6\eta U}{h^3}\left(h-h_o\right)$	$\frac{dp}{d\varphi}=\frac{2\eta U}{h^2}$
Coyne, Elrod **[L. 8]**		$\frac{dp}{d\varphi}=\frac{6\eta U}{h_s^2}\left(1-\frac{2h_\infty}{h_s}\right)$
Crosby, Badawy **[L. 9]**	$\frac{\delta p}{\delta\varphi}-\frac{\delta p}{\delta z}\cdot\frac{d\varphi}{dz}=\frac{2\eta}{h}\left[\frac{yU_2+\left(h-y\right)U_1}{y\left(h-y\right)}\right]$	$\frac{dp}{d\varphi}=\frac{2\eta}{h^2}\left(\sqrt{U_1}+\sqrt{U_2}\right)^2$

BOUNDARY CONDITION

The end edge of oil film is called the line that separates the zone totally filled up by oil from the zone where the stable longitudinal gas bubbles, separated by oil strips, occur. This line goes approximately through the tops (point S) of finger shaped, bubbles (Fig. 2).

Velocities distribution in the oil film, on the gap thickness (point „1") is described by the equation

$$\upsilon\left(y\right)=\frac{1}{2\eta R}\cdot\frac{\delta\overline{p}}{\delta\varphi}(y^2-hy)+\frac{U}{h}\cdot y\,. \tag{5}$$

In the cavitated zone and on the axis of gas bubble (point "2") the oil sticks to the surface of rotating journal with the layer of thickness ($h-h'$), and on its full length the oil velocity is constant and namely

$$\upsilon\left(y\right)=U \tag{6}$$

Among the bubbles (point "3") the oil wets both, the surface of journal and the sleeve and its velocity varies as follows

$$\upsilon\left(y\right)=\frac{U}{h}y\,. \tag{7}$$

The oil flow on the unit of the bearing width is calculated as

$$q = \int_0^h \upsilon dy\,. \tag{8}$$

After taking into account the equations (5), (6), (7) in (8), the oil flow is determined by dimensionless dependencies in the peripheral direction in point "1", "2" and "3"respectively:

$$\overline{q}_1 = \frac{H}{2} - \frac{H^3}{12} \cdot \frac{\delta\overline{p}}{\delta\varphi}, \tag{9}$$

$$\overline{q}_2 = \frac{H}{2}, \tag{10}$$

$$\overline{q}_3 = H - H'. \tag{11}$$

On the assumption that the width of limit zone tends to nil $\Delta\varphi \to 0$ on the oil film boundary, the equation of flow continuity in the form (12) should be fulfilled

$$\overline{q}_1 \cdot \Delta z = \overline{q}_2 (\Delta z - \Delta z') + \overline{q}_3 \cdot \Delta z'. \tag{12}$$

Considering equations (9), (10) and (11), equation (12) obtains the form

$$\frac{\delta\overline{p}}{\delta\varphi} = \frac{6}{H^2}(2\frac{H'}{H} - 1) \cdot \frac{\Delta z'}{\Delta z}. \tag{13}$$

This condition has to be fulfilled at the end of oil film boundary, too.

By comparing of The right side of above equation can be compared to the results of authors that are mentioned in Table 1, and the view on the magnitude of bubbles (width $\Delta z'$ and thickness H') can be formulated. The magnitude of bubbles depends on the surface tension, viscosity and bulk density of oil (lubricant fluid) as well as on the velocity of operating surface of journal and sleeve.

The cavitation coefficient A is determined from the relation

$$A = 6(2\frac{H'}{H} - 1) \cdot \frac{\Delta z'}{\Delta z}, \tag{14}$$

and it allows writing the boundary condition at the end of oil film:

$$\frac{\delta \overline{p}}{\delta \varphi} = \frac{A}{H^2}. \tag{15}$$

When $\Delta z' / \Delta z = 0$ (the lack of bubbles) then $A = 0$, i.e. $\delta \overline{p} / \delta \varphi = 0$ (Reynolds condition). However, when the thickness of oil layers that is bound to the journal surface $h - h' = 0$, then the bubble fills up the total width of bearing, $\Delta z' / \Delta z = 1$, $H' / H = 1$ and $\delta \overline{p} / \delta \varphi = 6$ (Floberg model **[L. 4]**). It results from the carried out analysis and from the content of **Table 1**, that:

$$0 \le A \le 6. \tag{16}$$

CRITICAL ECCENTRICITY

Relative bearing eccentricity, at which the gas bubbles occur in the sub pressure zone of oil gap is called the critical eccentricity – ε_{kr}.

It was assumed, that in the boundary zone and in the neighbourhood of single bubble, the resultant flow of fluid in the axial direction (z) is equal nil. Based on this assumption, the pressure distribution at the end boundary of oil film is described with correct exactness by following equation:

$$\frac{\partial}{\partial \varphi}\left(H^3 \frac{\partial \overline{p}}{\partial \varphi} \right) = \frac{\partial H}{\partial \varphi}. \tag{17}$$

Integration of equation (17) gives

$$\frac{\partial \overline{p}}{\partial \varphi} = 6 \frac{1}{H^2} + \frac{C_1}{H^3}. \tag{18}$$

The constant of integration was determined from equations (15) and (18)

$$C_1 = H(A - 6). \tag{19}$$

Integrating for the second time the equation (18) and introducing additional variable

$$\gamma = arc\cos\frac{\varepsilon + \cos\varphi}{1 + \varepsilon\cos\varphi}, \tag{20}$$

the equation of oil film pressure distribution in the boundary zone was obtained

$$\overline{p} = 6J_2 + C_1 J_3 + C_2 \tag{21}$$

where the members of equation (21) are:

$$J_2 = \frac{\gamma - \varepsilon\sin\gamma}{\sqrt{(1-\varepsilon^2)^3}}, \tag{22}$$

$$J_3 = \frac{\gamma - 2\varepsilon\sin\gamma + 0{,}5\varepsilon^2\gamma + 0{,}25\varepsilon^2\sin 2\gamma}{\sqrt{(1-\varepsilon^2)^5}}, \tag{23}$$

and C_2 is the constant of the second integration.

On the assumption that the bearing eccentricity $\varepsilon \longrightarrow \varepsilon_{kr}$, the pressure and additional variable γ, on the begin and end of oil film, obtain the following values:

$$\overline{p}_{(\varphi=0)} = 0 \quad \text{and} \quad \gamma_{(\varphi=0)} = 0, \tag{24}$$

$$\overline{p}_{(\varphi=360^o)} = 0 \quad \text{and} \quad \gamma_{(\varphi=360^0)} = 2\pi. \tag{25}$$

Considering the equations (22), (23) as well as (24) and (25) in equation (21) the following relations were obtained:

$$C_1 = -\frac{12(1-\varepsilon_{kr}^2)}{2+\varepsilon_{kr}^2}, \tag{26}$$

$$C_2 = 0. \tag{27}$$

The comparison of the right sides of equations (19) and (26) and taking into account the equation (2) on the assumption that $\varphi = 360^o$ it was obtained

$$\varepsilon_{kr} = \frac{6 - \sqrt{36 - 2A(A-6)}}{A-6}. \tag{28}$$

This is the criterion of oil film rupture that depends on the cavitation coefficient A (**Fig. 3**).

At small eccentricities ε and almost in the full range of the variations of coefficient A, the oil film is on the full peripheral of the bearing (full oil film). When at assumed value of cavitation coefficient A, the relative eccentricity overcomes the critical value ε_{kr}, then the cavitation occurs in the lubricating gap.

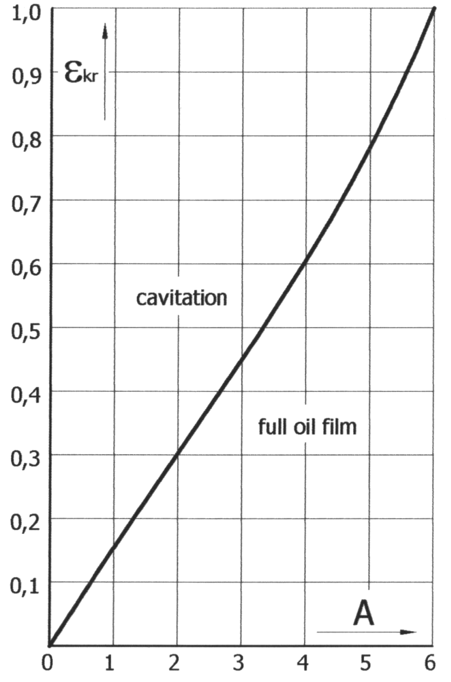

Fig. 3. Critical eccentricity (ε_{kr}) versus cavitation coefficient (A)

Rys. 3. Mimośrodowość krytyczna (ε_{kr}) w zależności od współczynnika kawitacji (A)

CALCULATIONS

The oil film pressure was determined from the solution of equation (1) on the assumption of respective boundary conditions.

For the range of relative eccentricities $0 \le \varepsilon \le \varepsilon_{kr}$ the following conditions were assumed:

$$\overline{p}_{(\varphi=0)} = 0 \quad \text{and} \quad \overline{p}_{(\varphi=360^o)} = 0 . \tag{29}$$

However, for the range $\varepsilon_{kr} \le \varepsilon \le 1$ the conditions:

$$\overline{p}_{(\varphi=0)} = 0 \quad \text{and} \begin{cases} \overline{p}_{(\varphi=\varphi_k)} = 0 \\ \left(\dfrac{d\overline{p}}{d\varphi}\right)_{(\varphi=\varphi_k)} = \dfrac{A}{H^2} \end{cases} . \tag{30}$$

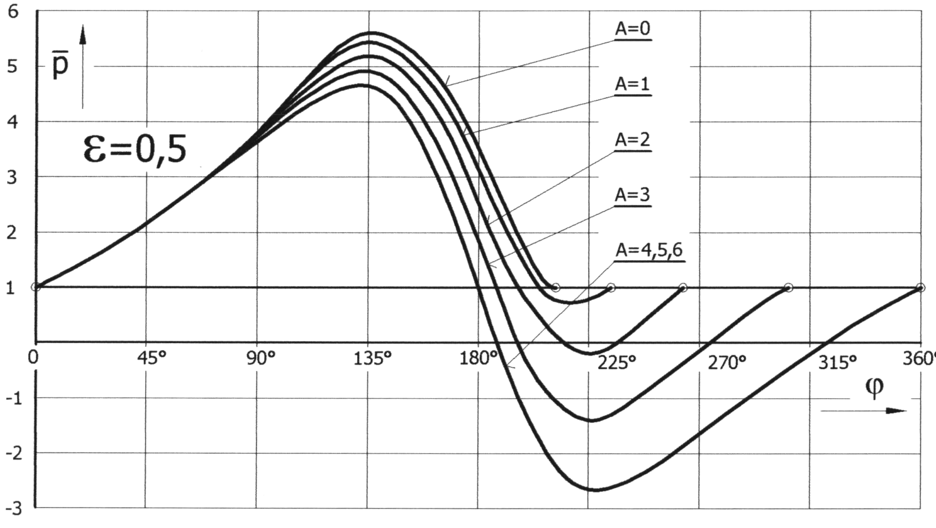

Fig. 4. Pressure distributions for the cavitation coefficient $0 \le A \le 6$ and relative eccentricity $\varepsilon = 0.5$

Rys. 4. Rozkład ciśnienia dla współczynnika kawitacji $0 \le A \le 6$ i mimośrodowości względnej $\varepsilon = 0,5$

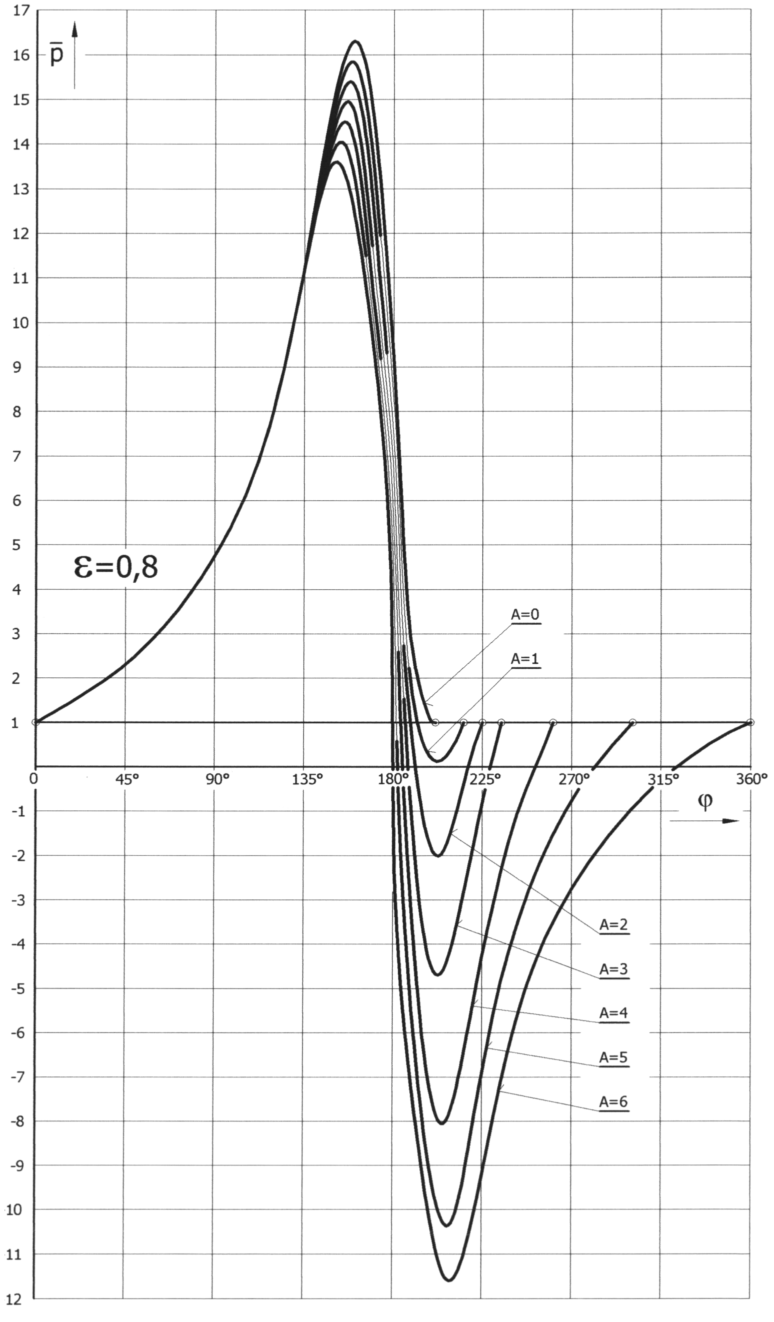

Fig. 5. Pressure distributions for the cavitation coefficient $0 \leq A \leq 6$ and relative eccentricity $\varepsilon = 0.8$

Rys. 5. Rozkład ciśnienia dla współczynnika kawitacji $0 \leq A \leq 6$ i mimośrodowości względnej $\varepsilon = 0{,}8$

In the full range of relative eccentricity variations ε $(0 \le \varepsilon \le 1)$ and on the side edges of calculation area the condition (31) was assumed:

$$\overline{p}_{(z=1)} = \overline{p}_{(z=-1)} = 0 . \tag{31}$$

The calculations were carried out for different values ε and the chosen cavitation coefficient A. The results were showed in the form of bunch of curves and exemplary were showed in **Fig. 4 and 5.**

The load capacity of bearing was determined by the dependency

$$P = \frac{\overline{P} \cdot \eta \cdot \omega \cdot r^3}{(R-r)^2}, \tag{32}$$

where the dimensionless value of load capacity $\overline{P}$ determines the equation

$$\overline{P} = \left[\left(\int_{-1}^{1} \int_{0}^{\varphi_k} \overline{p} \sin \varphi d\varphi dz \right)^2 + \left(\int_{-1}^{1} \int_{0}^{\varphi_k} \overline{p} \cos \varphi d\varphi dz \right)^2 \right]^{0,5} . \tag{33}$$

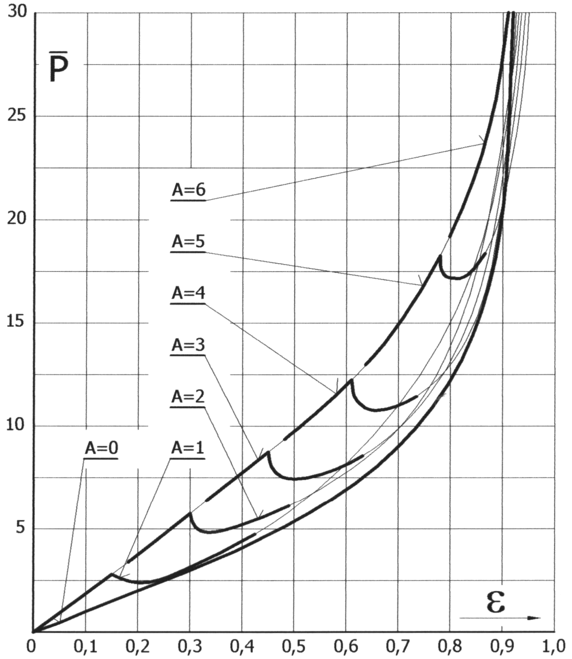

Fig. 6. Bearing load capacity for the cavitation coefficient $0 \le A \le 6$
Rys. 6. Nośność łożyska dla współczynnika kawitacji $0 \le A \le 6$

The angle of journal centre line Θ determines the equation

$$\Theta = arctg\frac{\int_{-1}^{1}\int_{0}^{\varphi_k}\overline{p}\sin\varphi d\varphi dz}{\int_{-1}^{1}\int_{0}^{\varphi_k}\overline{p}\cos\varphi d\varphi dz}. \tag{34}$$

The results of the calculations of load capacity $\overline{P}$ are showed in the form of overlapping characteristics for different values of cavitation coefficient A (Fig. 6). Association of the angle value Θ with the respective eccentricity ε, is showed in the form of map of journal displacements with regard to the sleeve for the chosen values of A **(Fig. 7).**

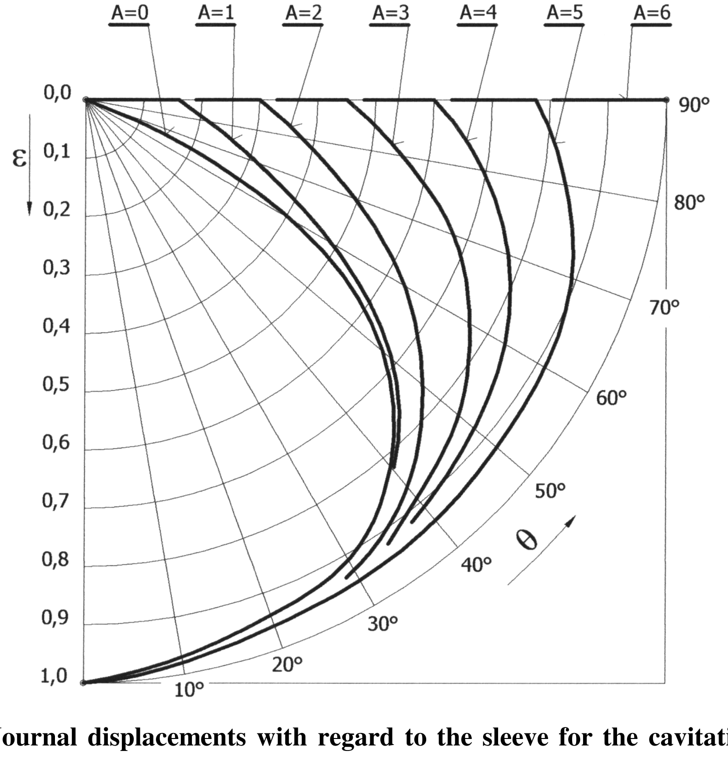

Fig. 7. Journal displacements with regard to the sleeve for the cavitation coefficient $0 \leq A \leq 6$

Rys. 7. Mapa przemieszczeń środka czopa względem panewki dla współczynnika kawitacji $0 \leq A \leq 6$

EXPERIMENTAL INVESTIGATION

Experimental investigations were carried out on the model bearing with the glass sleeve that was operated on the steel journal **[L. 10, 11].** Geometrical parameters of bearing are given in **Table 2**.

Table 2. Parameters of model bearing
Tabela 2. Parametry wzorcowego łożyska

Sleeve diameter	D_p=2R	63,525[mm]
Journal diameter	D_c=2r	63,270[mm]
Bearing length to diameter ratio	L_p/D_c	1
Radial clearance	Δr=R-r	0,1275[mm]
Diameter of oil supplying bore	d	6[mm]

Design of the test rig allows varying the load, rotational speed of journal and feeding oil pressure.

Measuring equipment has allowed to:

- visual observation of oil flow and the phenomena occurring in the cavitated area (an example of high speed camera),
- determination of journal position with regard to the sleeve (relative eccentricity, plane and misalignment angle),
- determination of oil film pressure in the lubricating gap, on the full peripheral and in any of transverse plane,
- the measurements of journal resistant torque.

An example of graphic interpretation of oil flow in the lubricating gap with the confrontation of oil film pressure oscillogram and the photos of peaks and ends of gas bubbles are showed in **Fig. 8.**

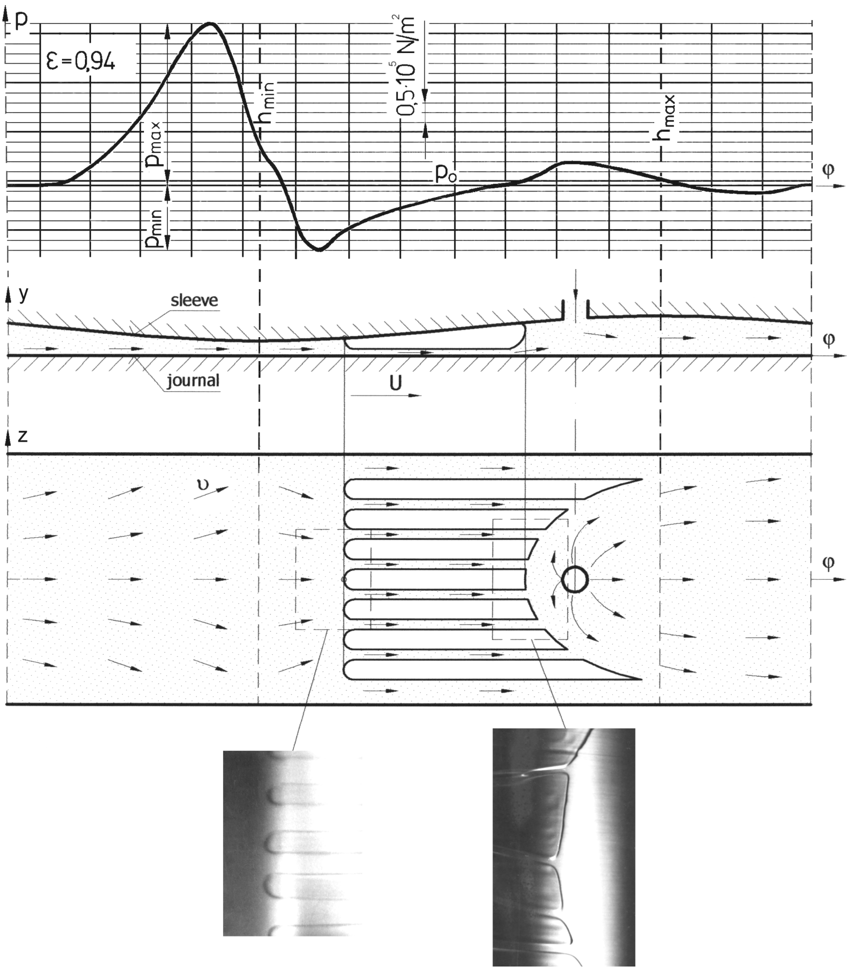

Fig. 8. Fluid flow in the lubricating gap on the basis of experimental investigation
Rys. 8. Przepływ cieczy w szczelinie smarowej na podstawie badań eksperymentalnych

RESULTS

The analysis of oil film pressure distributions $\overline{p} = f(\varphi)$, at different values of coefficient A, are illustrated by the diagrams for assumed values of relative eccentricity ε (**Fig. 4 and 5**) allows separating two ranges of relative eccentricity ε.

1. In the bearing operating range and at $\varepsilon < \varepsilon_{kr}$ the oil film (area where $\overline{p} \neq 0$) stretches over full length of bearing peripheral (the coordinate of film end $\varphi_k = 360^o$ – Sommerfeld condition **[L. 1]**), and absolute

values of maximum oil film positive and negative pressure are the same. The journal displacement with regard to the sleeve is carried on the straight line that is perpendicular to the direction of load **(Fig. 7).** The load capacity of bearing $\overline{P}$ **(Fig. 6)** is determined by the curve for $A = 6$ in full range of the variation of relative eccentricity.

2. At exceeding of critical eccentricity $\varepsilon > \varepsilon_{kr}$) the cavitated area and the shortening of oil film ($\varphi_k < 360^o$) are generated. The film is as shorter as larger is relative eccentricity, but always there is $\varphi_k > 180^o$. Absolute value of negative pressure undergoes decreasing and the journal centre moves on the arc that is convergent to the direction of the bearing load. The load capacity curve $\overline{P}$ **(Fig. 6)** undergoes visible collapse, the load capacity decreases and only at further increase in the eccentricity ε it comes to its fast increase. The places of the collapse of load capacity curves depend on the cavitation coefficient A. The occurrence of collapses and their profile, certify that the generation of cavitation is associated with the rapid variation of oil flow conditions and the loss of balance in the equilibrium forces of the bearing.

The comparison of theoretical calculations and experimental results in the range of eccentricity **(Fig. 9)** and journal position with regard to the sleeve **(Fig. 10)** shows qualitative agreement. However, there is a large in the quantitative aspect.

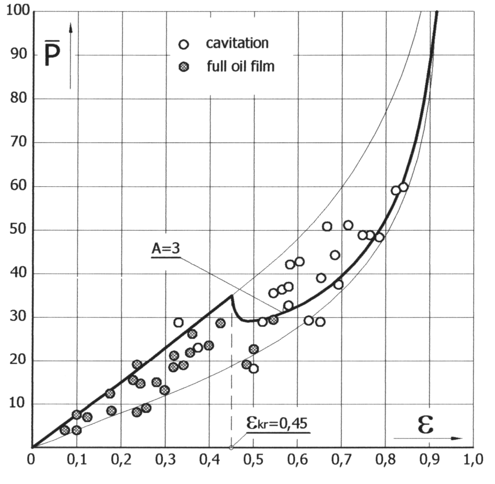

Fig. 9. Bearing load capacity according to theoretical calculations and experimental measurements (A = 3)

Rys. 9. Nośność łożyska wg obliczeń teoretycznych i pomiarów eksperymentalnych (A = 3)

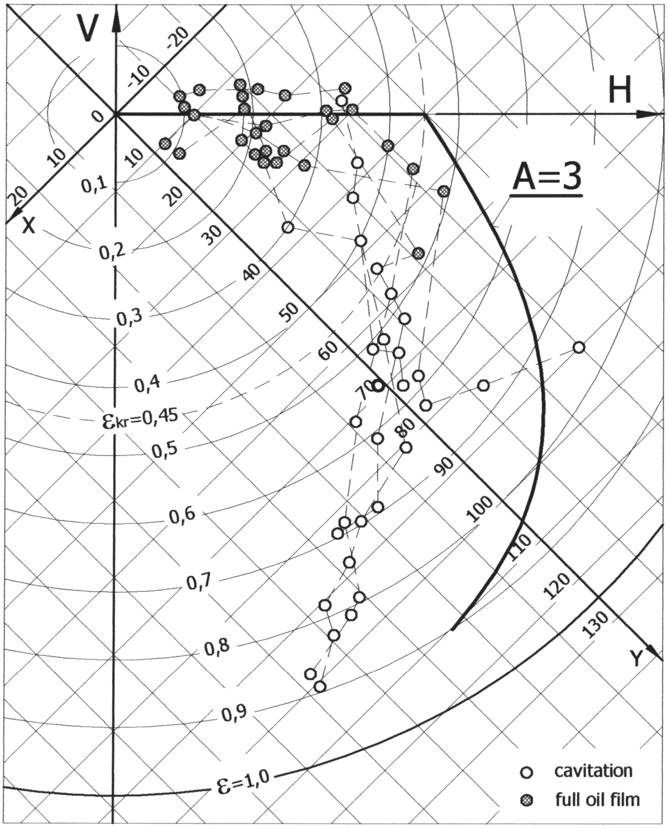

Fig. 10. Journal position with regard to the sleeve according to the theoretical calculations and experimental measurements (*A* = 3)

Rys.10. Położenie czopa względem panewki wg obliczeń teoretycznych i pomiarów eksperymentalnych (*A* = 3)

The gas bubbles that occur in the lubricating gap are the structures of the stiffness of some orders smaller than the oil and they unfavourable influence the forecast ability of journal position. The above facts and the visual observations of oil flow confirm and reveal that the generation of cavitation bubbles is in some measure of random process.

CONCLUSIONS

1. The calculations of the oil film pressure distribution of bearings operating at the relative eccentricities $\varepsilon \leq \varepsilon_{kr}$ should apply the following boundary conditions:

$$\overline{p}_{(\varphi=0)}=0\,,\qquad \overline{p}_{(\varphi=360^o)}=0\,,$$

as well as $\overline{p}_{(z=1)}=\overline{p}_{(z=-1)}=0\,.$

2. When the bearing operates at the eccentricity $\varepsilon \geq \varepsilon_{kr}$ (the occurrence of cavitation) the conditions given below are applied:

$$\overline{p}_{(\varphi=0)}=0\,,\qquad \begin{cases} \overline{p}_{(\varphi=\varphi_k)}=0 \\ \left(\dfrac{d\overline{p}}{d\varphi}\right)_{(\varphi=\varphi_k)}=\dfrac{A}{H^2} \end{cases},$$

and $\overline{p}_{(z=1)}=\overline{p}_{(z=-1)}=0\,.$

3. The presented method of calculation allows considering in the calculations of bearings with the cavitation phenomenon, one synthetic coefficient $A=f(\varepsilon_{kr})$.
4. The cavitation coefficient $A=f(\varepsilon_{kr})$ is the function of the physical properties of oil, the operating conditions and the bearing design and it can obtain the values from the range $\langle 0\ 6\rangle$.
5. As the supplement of this method the preparation of experimental procedure for determination of critical eccentricity (ε_{kr}) for specific bearings and theirs conditions of operation should be developed.

REFERENCES

1. Sommerfeld A.: Zur hydrodynamischen Theorie der Schmiermittelreibung, Oswalds Klassiker der exakten Wissenschaften, nr 218, Lipsk 1927.
2. Gerlach T.: Próba modyfikacji równania Reynoldsa dla potrzeb związanych z obliczaniem łożysk ślizgowych. Szkoła „Inżynieria łożyskowania”, Politechnika Gdańska, Czarlin 1976.
3. Kiciński J.: Teoria i badania hydrodynamicznych poprzecznych łożysk ślizgowych. Ossolineum, Wrocław 1994.
4. Swift H. W.: The stability of lubricating films in journal bearings, J. Inst. Civ. Engrs, 233, (Pt 1), 1931.
5. Stieber W.: Das Schwimmlager (V.D.I., Berlin), 1933.

6. Floberg L.: On Hydrodynamic Lubrication with Special Reference to Sub-cavity Pressures and Number of Streamers in Cavitation Regions, Acta polytechnica scandinavica, ME 19, 1965.
7. Birkhoff G., Hays D.: Free boundaries in partial lubrication, Journal of Mathematics and Physics, Vol. 42, No. 2, 1963.
8. Coyne J.C., Elrod H.G.: Conditions for the rupture of a lubricating film, Part I – Theoretical model, Journal of Lubrication Technology, 451, 1970.
9. Crosby A.W., Badawy E.M.: On the Performance of Journal Bearings Under Conditions of Film Rupture. Part I, II Journal of Lubrication Technology. 585, 1975.
10. Merc T.: Przepływ oleju w nieroboczej części poprzecznego łożyska ślizgowego. Praca doktorska. Politechnika Łódzka, Łódź 1981.
11. Merc T., Stasiak J.M.: Kawitacja w łożyskach ślizgowych obciążonych statycznie. Przegląd Mechaniczny, No. 20/1983.

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Streszczenie

Parametry pracy poprzecznych łożysk ślizgowych takie jak nośność, opory ruchu, zapotrzebowanie oleju, położenie czopa względem panewki i stabilność tego położenia są funkcją przepływu oleju w szczelinie smarowej.

Projektowanie hydrodynamicznych łożysk ślizgowych często wymaga uwzględnienia w obliczeniach faktu, że w szczelinie smarowej występuje zjawisko kawitacji, które na ogół ma negatywny wpływ na nośność i stabilność pracy łożyska. W wykazach literatury przedmiotu wymieniane są liczne prace, ale żadna nie podaje praktycznej metody obliczeniowej. Po analizie przepływu oleju na granicy strefy skawitowanej sformułowano warunki brzegowe pozwalające rozwiązać przedstawiony problem. Wyniki obliczeń porównano z rezultatami badań eksperymentalnych. We wnioskach podano zalecenia do stosowania proponowanej metody i wskazano potrzebę uzupełnienia jej procedurą eksperymentalnego wyznaczania współczynnika kawitacji.