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MODELLING THE TROCHOIDAL GEARING WITH RESPECT TO THE UNIFORM TEETH WEAR

MODELOWANIE PRZEKŁADNI TROCHOIDALNEJ Z UWAGI NA RÓWNOMIERNE ZUŻYCIE ZĘBÓW

Key-words:

trochoidal gearing, specific sliding, uniform wear

Słowa kluczowe:

przekładnia trochiodalna, poślizg względny, równierne zużycie

Summary

This paper presents a theoretical model of gearing with a modified trochoidal profile applied to Gerotor pumps. Gerotor pumps belong to the category of rotating machines whose kinematics is based on the principles of a planetary mechanism with internal gearing. Using geometrical and kinematical models of the trochoidal gearing, we provide a detailed analysis of specific sliding at the contact points of the profiles, as well as the relations for its determination. Specific sliding is one of the most sig-

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nificant constraining factors in the selection of the geometrical parameters of the gearing profile, which indicates conditions for appearing friction and wear of the tooth contact surface. The key objective in optimal gerotor pump design is to provide the appropriate selection of geometrical parameters of the gearing, leading to the uniform wear of meshing gears teeth profiles. This requires ensuring the uniform wear of meshing profiles at the point with highest value of sliding velocity, i.e. at the contact points most distant from the pitch point. Consequently, the methodology described in the paper defines the constraints to enable uniform wear of meshing gear teeth profiles from the kinematical point of view.

INTRODUCTION

Gerotor pumps belong to the group of the planetary rotating machines whose kinematics is based on the principle of the planetary mechanism with the internal gearing. Teeth number of the external gear is always one more of the teeth number of the internal gear. By this kind of gearing, the moving circle is rolling without sliding along the other stationary circle providing a chosen point that describes the trochoidal tooth profile **[L. 1, 5]**. The stationary circle is, conditionally, the kinematical circle of the gear. The meshing profile can be presented as the envelope of the based profile successive position by its relative movement. In the general case, the meshing envelope has peaks, which are unwanted because the intensive wear and the phenomena that is introduced by the modification of the based trochoid. The trochoidal curves are modified through the increase in constant value of r_c, which is along the normal section on the given curve. The obtained curve is equidistant, and the constant increase r_c can be defined as the radius of equidistant.

On the basis of geometrical and kinematical models, which are developed in the references **[L. 2–9]**, this paper defines the formulae for the calculation of the specific sliding and the conditions with them supply the same values of the specific sliding in the points with the most sliding velocity. To this aim the following coordinate systems are introduced: generating the coordinate system, the connection for the generating point, the coordinate system of trochoid, the coordinate system of envelope, and the stationary coordinate system. For the kinematical analysis of the meshing profiles, we considered the moving of the contact point of the meshing profiles.

GEOMETRICAL AND KINEMATICAL RELATIONS OF THE TROCHOIDAL GEARING

The based geometrical relations of the generating of the unmodified and modified epitrochoid are presented in the **Fig. 1**. The epitrochoid describes the point *D* fixed in the circle plane with the radius r_a when the circle with its internal side is rolling along the external side of the stationary circle with its radius r_t. The equidistant epitrochoid equations are given on the coordinate system of the trochoid $O_t x_t y_t$ in **Fig. 1**.

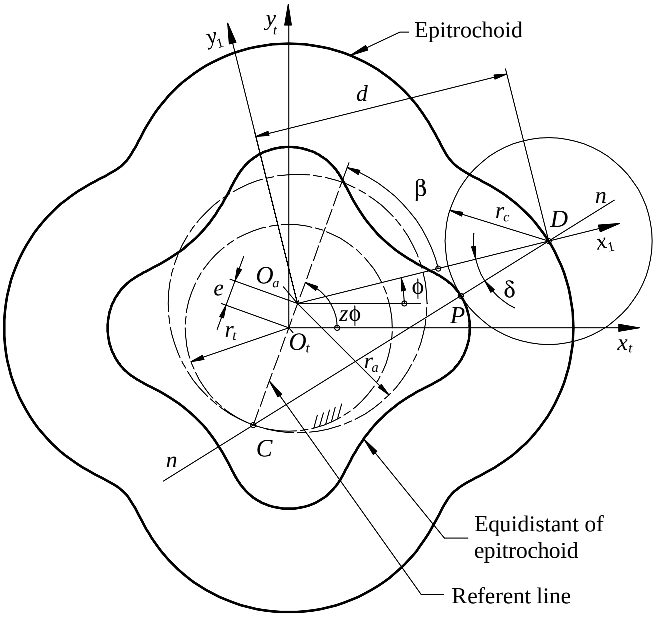

Fig. 1. Generating of the unmodified and modified epitrochoids
Rys. 1. Generowanie niezmodyfikowanej i zmodyfikowanej epitrochoidy

Fig. 1 indicates that, during the relative movement of the kinematical circles when the point *D* is generating epitrochoid at the same time, the point P is generating equidistant. The angle signified with δ presents the angle between normal *n-n* and radius vector of the point *D* and can be defined as the leaning angle. Coordinates of the point *P* in the coordinate system of the epitrochoid can be written in the following form:

$$x_t = e(\cos z\phi + \lambda z \cos\phi) - r_c \cos(\phi + \delta)$$
$$y_t = e(\sin z\phi + \lambda z \sin\phi) - r_c \sin(\phi + \delta) \qquad (1)$$

where λ is the coefficient of the trochoid, which is defined as the relationship between the values of trochoid radius and the radius of the moving circle $\lambda = d/ez$.

Based on geometrical relations from the **Fig. 1**, the formula for determination of angle δ can be obtained as follows:

$$\delta = \arctan\frac{\sin(z-1)\phi}{\lambda + \cos(z-1)\phi} \qquad (2)$$

For kinematic analysis of the meshing profiles, the moving of the point P_t on the profile of the internal gear and the point P_a on the profile of the external gear (**Fig. 2**) is considered.

During the meshing, the trochoidal gearing profiles are rolling simultaneously and sliding one to each other. The sliding of the profiles of the contact point is the consequence of the difference of the intensity of the relative velocities of the points on the profiles of the internal and the external gear, respectively.

From gearing theory, it is known that only pitch circles can realise rolling without sliding. Based on this, it is revealed that the profile sliding is inevitable, because they are made with the curves, which are different than the pitch circles. In this case, the sliding velocity of the meshing profiles in the observed contact point is the velocity of the contact point by the movement of the relative profiles.

In the **Fig. 2** is given the distribution of the velocity in the contact point of the two meshing profiles, where are **[L. 2, 5]**: $\vec{v}$ is the vector of the absolute velocity of the meshing profiles in the contact point *P*; $\vec{v}_{pt}$, $\vec{v}_{pa}$ are the vectors of the transfer velocities of the contact point P_t, P_a; $\vec{v}_{nt}$, $\vec{v}_{na}$ are the projections of the transfer velocity on the common normal and $\vec{v}_{\tau t}$, $\vec{v}_{\tau a}$ on the tangent in the contact point P_t, P_a; $\vec{v}_{ra}$ are vectors of the relative velocities of the contact point P_t, P_a; $\omega_r = \omega_t - \omega_a$ is the angular velocity of the epitrochoid in relation to the envelope; $\vec{v}_{ta}$ is the vector of the sliding velocity of the internal gear profiles in relation to external gear profiles; and, $\vec{v}_{at}$ is the vector of the sliding velocity of the external gear profiles in relation to internal gear profiles.

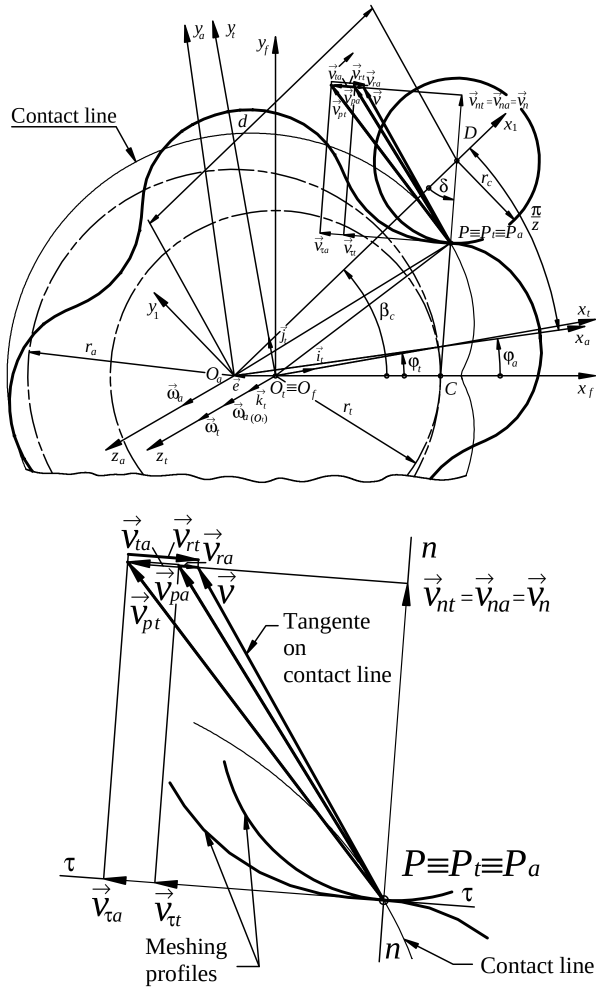

Fig. 2. Kinematical parameters of the trochoidal gear pair
Rys. 2. Parametry kinematyczne zazębienia trochoidalnego

The intensity of the relative velocity vector in the contact point P_t of the profile is as follows:

$$v_{rt} = \left\{ ez\left(1 + \lambda^2 + 2\lambda\cos\beta\right)^{\frac{1}{2}} - r_c\left(1 + \delta'\right) \right\}\omega_r \tag{3}$$

and the intensity of the relative velocity vector in point P_a is equal to:

$$v_{ra} = r_c \delta' \omega_r \tag{4}$$

where

$$\delta' = \frac{d\delta}{d\phi} = \frac{(z-1)(1+\lambda\cos\beta)}{1+\lambda^2+2\lambda\cos\beta} \tag{5}$$

The intensity of the profile sliding velocity in the contact point is

$$v_r = \left|\vec{v}_{ta}\right| = \left\{ ez\left(1+\lambda^2+2\lambda\cos\beta\right)^{\frac{1}{2}} - r_c \right\}\omega_r \tag{6}$$

On the basis of the obtained equations, we can define the formulae to determination the specific sliding of the meshing profiles in the contact point.

SPECIFIC PROFILE SLIDING

The sliding existence in the sliding in the meshing profiles process comes to their wear by that the sliding velocities that define the friction force directions and intensities, which take effect on the meshing profiles of the gears. The friction force is in the direction opposite to the relative motion velocity in the contact point. So the direction of the sliding velocity $\vec{v}_{ta}$ is in agreement with the direction of the friction forces, which take effect on the profile of the external gear profile, but the direction of $\vec{v}_{at}$ is the same the direction of the friction force on the trochoidal profile (**Fig. 3**).

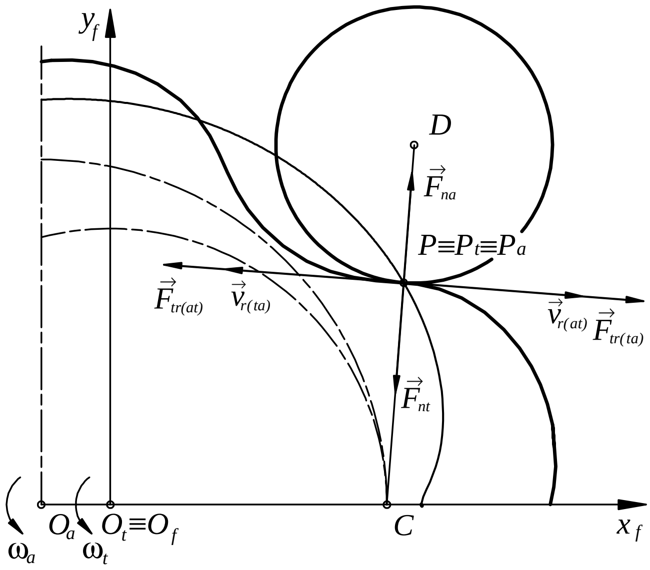

Fig. 3. Sliding velocities, friction forces and normal forces in the contact point of the meshing profiles

Rys. 3. Prędkości poślizgu, siły tarcia i siły normalne w punkcie styku współpracujących profili

For the analysis of the meshing profile sliding, it is necessary to know, except the sliding velocity in the contact point, the distribution of their change in relation to the corresponding relative velocity of the contact point. The relationship between the sliding velocity and relative velocity of the contact point of the meshing profiles is specific sliding **[L. 8]**. After the substitution of the corresponding formulae for the velocities, we obtain the formulae for the specific sliding on the tooth profile of the internal gear:

$$\xi_t = \frac{z\left(1+\lambda^2+2\lambda\cos\beta\right)^{\frac{1}{2}}-c}{z\left(1+\lambda^2+2\lambda\cos\beta\right)^{\frac{1}{2}}-c\left(1+\delta'\right)} \tag{7}$$

and the analogue for the external gear is

$$\xi_a = \frac{z\left(1+\lambda^2+2\lambda\cos\beta\right)^{\frac{1}{2}}-c}{c\delta'} \tag{8}$$

where

$$c = \frac{r_c}{e} \tag{9}$$

On the profile point, where the directions of sliding and relative velocities are in agreement, specific sliding is positive, and where they are not in agreement, it is negative.

Based on formulae (7) and (8), it can be concluded that the values of specific sliding become endlessly great when the values of relative velocities are equal zero. These points are singular for the distribution of the specific sliding of the meshing profiles.

At first, we analysed the conditions when $\xi_t \to \infty$, respectively, when $v_{rt} = 0$. Starting with formula 7 and making it equal to zero is obtained the following:

$$c = \frac{z\left(1+\lambda^2+2\lambda\cos\beta\right)^{\frac{3}{2}}}{z+\lambda^2+\lambda(z+1)\cos\beta} \tag{10}$$

The obtained equation has shown that ξ_t is not defined when the value of the equidistant radius is equal to the curve radius of the base epitrochoid **[L. 2]**. As the value of the equidistant radius is chosen to be less then minimum value of the epitrochoid curve radius, it appears that the singularity for ξ_t is eliminated. However, to avoid the extreme great values of specific sliding, it is recommended that the choice of the equidistant radius value is considerable less then limited.

On the profile of external gear exists the point with endlessly great specific sliding. The position of this singular point is defined with critical angle β_0 by that the common normal of the meshing profiles in the contact point and is in agreement with the contact line on the pitch circles **[L. 2]**. At this point the change of sign of relative velocity is approached, and $v_{ra} = 0$, and, based on relations (4) and (5), the critical value of angle can be determined, in reference to the following equation:

$$\beta_0 = \arccos\left(-\frac{1}{\lambda}\right) \tag{11}$$

and it corresponds to the last point of the active part of the profile of the meshing envelope.

CONDITIONS FOR THE DETERMINATION OF UNIFORM TEETH WEAR

One of the main indicators of the influence of the geometrical and kinematical parameters on the size of the sliding and the intensity of teeth profile wear is the profile of specific sliding [8]. The aim of the optimal construction of the gerotor pump is that, with the corresponding choice of the geometrical gearing parameters, we can realise the uniform teeth wear of the meshing profiles in the gears meshing process. Because it is necessary to realise the equality of the specific sliding in the points with the greatest sliding velocity, respectively, in the points that are farthest from the pitch point. Using formulae 7 and 8 reveals that, for the given values of the teeth number *z* and trochoid coefficient λ, the uniform wear of the teeth profiles can be realised when the condition of equality fulfilled for the relative velocities in regard to when the chosen value of the coefficient of the equidistant radius is equal as shown in the following equation:

$$c = \frac{z\left(1+\lambda^2+2\lambda\cos\beta\right)^{\frac{3}{2}}}{\lambda^2-1+2z\left(1+\lambda\cos\beta\right)} \tag{12}$$

It is necessary to remark that the singular points ($\beta = \pm\beta_0$) are taken for consideration.

This condition can be written for the points with the greatest sliding velocity, and they are the points on the top of the trochoidal profile, $(\beta = 0)$, in the form:

$$c = \frac{z(1+\lambda)^2}{2z+\lambda-1} \tag{13}$$

Geometrical interpretation of the obtained formula is given in the **Fig. 4** (diagram designed with 1), when the values of the teeth number *z* and coefficient λ are varied.

It can be observed that, in the area of recommended values of the trochoid coefficient for pumps, $\lambda < 2$, the teeth number does not have the essential influence on the condition for the uniform wear, and it is especially expressed by the greater teeth number.

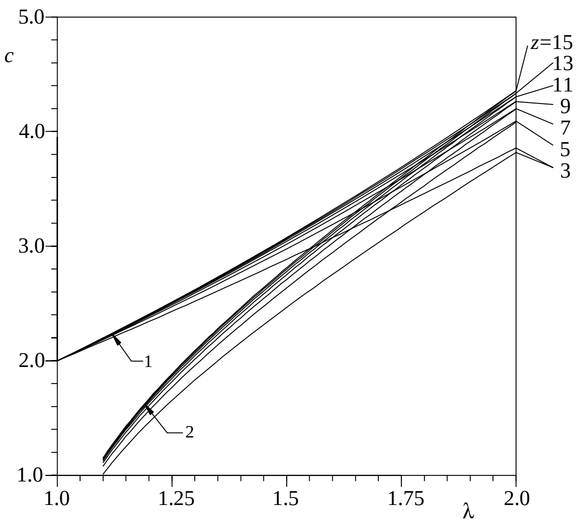

Fig. 4. Diagrams for choice of the equidistant radius from the condition for the uniform wear of the meshing profiles: 1 – in the point with the greatest sliding velocity, 2 – in the point with the greatest curvature of the trochoidal profile

Rys. 4. Diagramy do wyznaczania równoodległych promieni z warunku równomiernego zużycia współpracujących profili: 1 – w punkcie z największą prędkością poślizgu, 2 – w punkcie z największą krzywizną profilu trochoidalnego

On the similar way, the condition on equality of the relative velocities in the point with the least curve radius of trochoid can be written. Starting from the formula for the determination of the angle that corresponds to the point with the least curve radius on the internal gear profile, given in reference **[L. 2]**, and the condition defined with formula (12) is as follows:

$$c = \frac{z\left[3(z-1)\left(\lambda^2-1\right)\right]^{\frac{3}{2}}}{(z+1)^{\frac{1}{2}}\left(1-3z+2z^2\right)\left(\lambda^2-1\right)} \tag{14}$$

Geometrical interpretation of the obtained formula is given in the **Fig. 4** (diagram designed with 2).

It means that, with the choice of the corresponding coefficient of the equidistant radius, we can realise the equality of the specific sliding in the corresponding points of the trochoidal gear pair with the internal gearing, and it is illustrated in **Figure 5**. Except for this, it is also possible to decrease the extremely great values of the specific sliding in the area of the point with the greatest curve of the trochoidal profile.

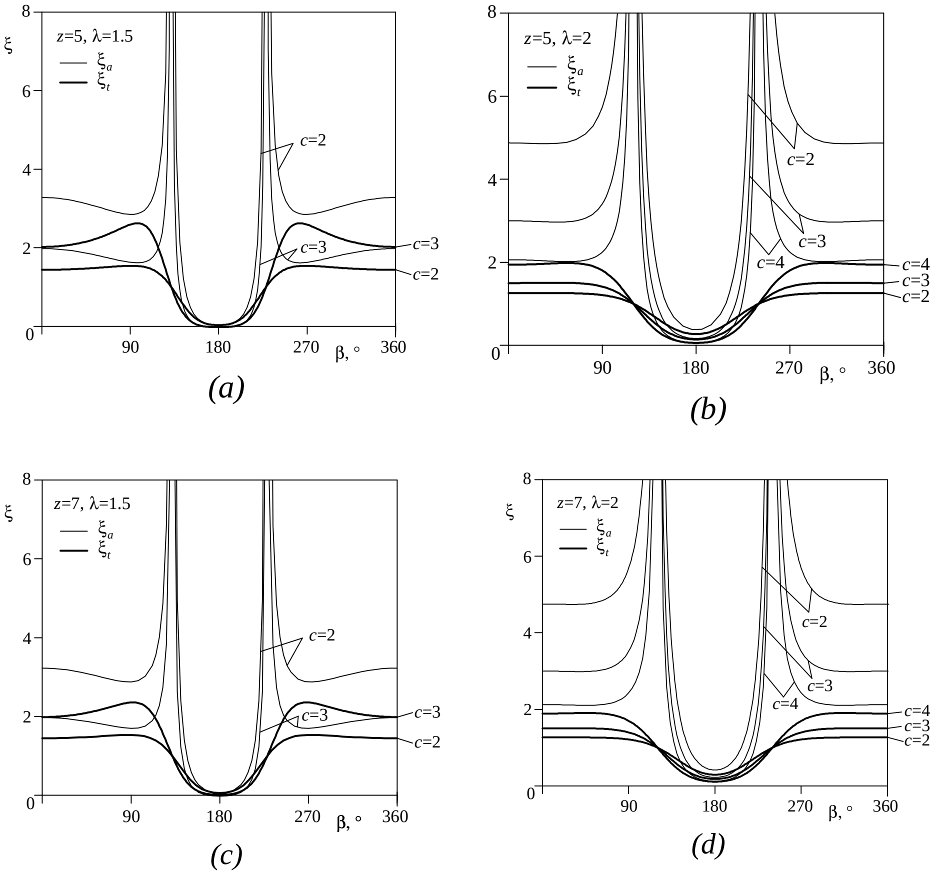

Fig. 5. Comparison of the absolute value of the specific sliding of the profile contact point at the trochoidal gear pairs for the different values of the teeth number by different values of the coefficient c and for different values of the trochoid coefficient λ: (a) z = 5, λ = 1.5; (b) z = 5, λ = 2; (c) z = 7, λ = 1.5 i (d) z = 7, λ = 2

Rys. 5. Porównanie wartości bezwzględnych poślizgu względnego w punkcie styku zazębienia trochoidalnego dla różnych wartości liczb zębów z różnymi wartościami współczynnika c i różnymi wartościami współczynnika trochoidy λ: (a) z = 5, λ = 1,5; (b) z = 5, λ = 2; (c) z = 7, λ = 1,5 i (d) z = 7, λ = 2

Based on **Figure 5** (a) and (c), we can be concluded that the kinematical aspect supplied the uniform wear of the meshing teeth profiles of gear in the profile points that are the greatest from the pitch point, because the values of the specific sliding in these points are equal.

CONCLUSION

On the basis of the obtained formulae, we can be conclude that the values of the specific sliding become endlessly great when the values of the relative velocities are equal zero. These points are singular for the distribution of the specific sliding of the meshing profiles. On the profile of the internal gear, the specific sliding does not have singular points, so that, on the profile of the external gear, exists the point with the endlessly great specific sliding.

In this paper is shown that with the corresponding choice of the geometrical parameters can realise the equality of the specific sliding in the points with the greatest sliding velocity, from the kinematical aspect, they ensured the uniform teeth wear of the meshing gears in the process of the meshing profiles.

REFERENCES

1. Ansdale R.F., Lockley D.J.: The Wankel RC Engine, Iliffe Books Ltd., London, 1970.
2. Ivanović L.: Identification of the Optimal Shape of Trochoid Gear Profile of Rotational Pump Elements, PhD dissertation, The Faculty of Mechanical Engineering in Kragujevac, Kragujevac, 2007 (in Serbian).
3. Ivanović L., Josifović D.: Specific Sliding of Trochoidal Gearing Profile in the Gerotor Pumps, FME Transactions, Vol. 34, No 3, pp. 121–127, 2006.
4. Josifović D., Ivanović L.: Kinematical analysis of trochoidal gearing by IC engines lubricating pumps, Monograph, Mobility and Vehicles Mechanics, pp. 119–132, Kragujevac, 2005.
5. Litvin F.L.: Theory of gear meshing, Nauka, Moscow, 1968 (in Russian).
6. Maiti R., Sinha G.L.: Kinematics of active contact in modified epitrochoid generated rotary piston machines, Mechanism and Machine Theory, Vol. 23, No 1, 39–45, 1988.
7. Manco G., Manco S., Rundo M., Nervegna N.: Computerized generation of novel gearings for internal combustion engines lubricating pumps, The International Journal of Fluid Power, vol. 1, pp. 49–58, March 2000.
8. Shanikov V.M.: Planetary transmitter with the outcentroidal meshing, Mashgiz, Moscow, 1948 (in Russian).

9. Vecchiato D., Demenego A., Argyris J., Litvin F.: Geometry of a cycloidal pump, Computer methods in applied mechanics and engineering, pp. 2309–2330, 2001.

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Streszczenie

W referacie przedstawiono model teoretyczny zmodyfikowanego zazębienia trochoidalnego, które jest stosowane w pompach systemu Gerotor. Pompy Gerotor należą do maszyn wirujących, których kinematyka oparta jest na zasadzie mechanizmu planetarnego z zazębieniem wewnętrznym. Za pomocą modeli geometrycznych i kinematycznych przeprowadzono szczegółową analizę poślizgu względnego w punktach styku profili; podano też zależności do jego określenia. Poślizg względny jest jednym z najbardziej istotnych czynników spośród parametrów geometrycznych decydujących o warunkach tarcia i zużyciu kontaktujących się powierzchni. Zasadniczym zadaniem w konstrukcji pomp Gerotor jest właściwy dobór parametrów zazębienia dla uzyskania równomiernego zużycia w punkcie z największą prędkością poślizgu, tj. w punkcie najbardziej oddalonym od bieguna zazębienia. Metodologia przedstawiona w referacie umożliwia określenie warunków dla uzyskania równomiernego zużycia z punktu widzenia kinematyki.