RESEARCH
IN LOGISTICS
& PRODUCTION

A HYBRID ANT COLONY FOR MULTIRESPONSE MIXED-INTEGER PROBLEMS

Sunil Kushwaha*, and Indrajit Mukherjee**

* Department of Mathematics, Indian Institute of Technology Bombay,
Email : hiskushwaha@gmail.com
** Shailesh J. Mehta School of Management, Indian Institute of Technology Bombay,
Email: indrajitmukherjee@iitb.ac.in

Abstract In this paper, a hybrid ant colony optimization (ACO) is used to solve a multiple response optimization problem with mixed-integer (MI) search space. The work reported in this paper may be clasified into three part. The first part discusses on relevant litratures and the methodology to solve multiple response optimization problem. The second part provide details on the working principal, parameter tuning of a hybrid ACO proposed for mixed integer state space. In the hybrid ACO, genetic algorithm (GA) is used for intensification of the search strategy. Standard single response (objective) test functions are selected to verify the suitability of hybrid ACO. The third part of this research work illustrates the application of the hybrid ACO in a multiple response optimization (MRO) problem. Statistical experimentation, partial least square regression, 'maximin' desirability function, and hybrid ACO is used to solve the MRO problem. The results confirm the suitability of the hybid ACO for a typical MI MRO problem

Paper type: Research paper

Published online: 29 October 2012
Vol. 2, No. 4, pp. 317-327

ISSN 2083-4942 (Print)
ISSN 2083-4950 (Online)

Keywords: *ant colony optimization(ACO), desirability functions, genetic algorithm(GA), mixed-integer (MI) problem, multiple response optimization(MRO)*

1. INTRODUCTION

A common problem generally encountered in process industry involves optimization of multiple quality characteristics (responses) desired by the customers. The problem can be stated as determining the settings of in-process conditions (e.g. machine settings) that simultaneously optimize all the responses in presence of noise conditions (e.g. raw material composition, environmental conditions). Simultaneous optimization of multiple quality characteristics is generally referred to as 'multiple response optimization' (MRO) (Khuri, 1996, p. 377). In practice, MRO problem typically involves responses that are conflicting (multiobjective) in nature. In other words, optimizing a particular response will result in sacrificing the optimality of another response. Thus, there is always a trade-off between the conflicting responses.

In this context, a graphical response surface approach, so called overlaying contour plot (Lind, Goldin, and Hickman, 1960, p. 62) is suggested to handle multiple response problems. However, the number of in-process (independent) variables is practically limited to less than three due to two-dimensional solution approach. Harington (1965, p. 494) first proposed a desirability function-based approach to convert a higher dimensional MRO problem to a single objective optimization problem using suitable transformation functions.

In his approach, the relevant desirability function is used to transform each predicted response variables to a corresponding scale-free desirability value (say, d_j), which lies between zero and one ($0 \leq d_j \leq 1$). The value of d_j increases as the "desirability" of the corresponding response increases. Subsequently, all the desirability values of predicted responses are transformed to a single measure, so-called 'composite desirability' or 'overall desirability index (D)'. Derringer and Suich (1980, p. 214) extended the concept of Harrington (1965, p. 494), and suggested a more generalized transformation scheme for predicted responses to desirability values. Their individual desirability transformation function varied according to its desired target of the response, viz., nominal-the-best (NTB), larger-the-better (LTB), and smaller-the-better (STB).

The individual desirability function for STB type of responses is given below.

$$d(\hat{y}_j(\mathbf{X})) = \begin{cases} 1 & \text{if } \hat{y}_j(\mathbf{X}) \leq y_j^{\min} \\ \left[\dfrac{y_j^{\max} - \hat{y}_j(\mathbf{X})}{y_j^{\max} - y_j^{\min}} \right]^{t_1} & \text{if } y_j^{\min} < \hat{y}_j(\mathbf{X}) < y_j^{\max} \\ 0 & \text{if } \hat{y}_j(\mathbf{X}) \geq y_j^{\max} \end{cases} \qquad (1)$$

where $d(\hat{y}_j(\mathbf{X}))$ is the desirability function of the j^{th} predicted response. $y_j^{\min}$ and $y_j^{\max}$ are lower and upper bound of j^{th} response, respectively. t_1 is the exponential parameters that determine the desirability function form and shape.

Kim and Lin (2000, p. 311) proposed a ‘maximin’ composite desirability index function as an optimization criterion for correlated responses. Optimization technique integrated with this criterion attempts to maximize the minimum of all individual desirability values. The ‘maximin’ desirability index (D) is expressed as

$$\mathrm{D} = Maximize\left(Minimum\{d_1, d_2, . \;\; . \;\; . \;\; ., d_r\}\right) \;\; \forall\, j = 1(1)r \qquad (2)$$

There are two broad categories of optimization search techniques which are generally recommended for MRO problems (Mukherjee, 2007, p. 103). First category is the conventional nonlinear search techniques. The second category is unconven-tional metaheuristic search strategies. GA and ACO belong to this second category. Real coded GA (Haupt and Haupt, 2004) work on a population concept rather than single individual iterative search approach. Deep et al. (Deep, Singh, Kansal, and Mohan, 2009, p. 505) extended the concept of real-coded GA to solve typical mixed integer optimization problems. Deep et al. (Deep, Singh, Kansal, and Mohan, 2009, p. 505) proposed a MI-LXPM algorithm, which is an extension of real-coded genetic algorithm proposed by Deep and Thakur (2007, p. 211). MI-LXPM is shown capable of solving integer and mixed integer nonlinear optimization problems. In MI-LXPM, Laplace crossover and power mutation are modified and extended for integer-type decision (independent) variables. In addition, a special truncation procedure for satisfaction of integer restriction on decision variables and a ‘parameter free’ penalty approach for constraint handling are also proposed.

ACO for discrete optimization problem is proposed by Dorigo (1992). Socha and Dorigo (2008, p. 1155) proposed a variant of continuous ant colony optimization (CACO) strategy, so-called ant colony optimization in real space (ACO_R). ACO_R uses gaussian kernel probability density function, which is the weighted sum of several one-dimensional gaussian functions. By this strategy, a solution archive table structure is used to store the information of independent variable(s), and its corresponding fitness value. The concept of ACO_R is extended to mixed integer state space, and so-called ACO_{MV} (Liao, 2010/2011).

ACO_{MV} relaxes discrete variables to continuous variable for generating new solutions. In this paper, an intensification and diversification scheme of ACO is used, which is based on GA and intrinsic parameter functions, to solve a MRO problem with mixed integer search space.

2. HYBRID ACO STRATEGY

The starting point before developing the idea of hybrid ACO strategy is working with ACO_R (Bera and Mukherjee, 2012, p. 312) to solve single response continuous search space optimization problem. In ACO_R, the pheromone information is stored as a solution archive. A solution archive is a collection of *k* best solutions at given time point. In ACO_R (Socha and Dorigo, 2008, p. 1155; Bera and Mukherjee, 2012, p. 312), a solution is probabilistically constructed by generating Gaussian kernel pdf function, $G^i(x)$, for each dimension (i, where $i = 1,.......,n$) of the state space. The PDF G^i is constructed using only the i^th coordinates of all the k solutions from the archive. The Gaussian kernel PDF $G^i(x)$ is parameterized with three vectors of parameters, viz. ω is the vector of weights associated with the individual Gaussian functions, μ^i is vector of means, and σ^i is the vector of standard deviations. The cardinality of all these vectors is equal to the number of Gaussian functions that constitute the Gaussian kernel. Various parameters are expressed as

$$\omega_l = \frac{1}{qk\sqrt{2\pi}} e^{-\frac{(l-1)^2}{2q^2k^2}} \tag{3}$$

$$\boldsymbol{\mu}^i = \{\mu_1^i,.......,\mu_k^i\} = \{s_1^i,........,s_k^i\} \tag{4}$$

$$\sigma_l^i = \xi \sum_{e=1}^{k} \frac{\left|s_e^i - s_l^i\right|}{k-1} \tag{5}$$

where 'q' and 'ξ' are heuristic parameters. When q is small, best-ranked solutions are strongly preferred, and when q is large, the probability becomes more uniform. The parameter ξ (and ξ >0) is same for all dimensions and has an effect similar to pheromone evaporation rate in basic ACO strategy. Higher the value of ξ , the lower the convergence speed of the algorithm**.** For convenience, a parameter k is used, and $|\omega| = |\mu^i| = |\sigma^i| = k$. The solutions (s_l) are stored according to their ranks or quality [depending on the value of $f(s_l)$]. The weight (ω) of each gaussian function is calculated using their rank value. Pheromone updates are done by adding the set of newly generated solutions to the existing solution archive, and subsequently removing the same number of worst solutions.

To improve the performance of ACO strategy for continuous and mixed integer state space, a diversification scheme is used in this paper. It works only when the solution stagnates to a specific value for a specified number of iteration. In such situation, the best solution searched so far is retained, and new $k-1$ (k is archive size and se-lected as 100 in this study) random points are generate. To diversify the search, a low convergence rate (ξ) and a high q value is initially selected. Subse-quently, value of

ξ is increase and q is decreased upto a predefined number of iter-ation, based on the following scaled sinusoidal functions (as given in Eqn. 6 and Eqn. 7).

$$\xi_{new} = 0.05 + 0.4308 \cdot \left(0.9285 + \frac{x}{\sqrt{1+x^2}} \right), -2.5 \le x \le 2.5 \tag{6}$$

$$q_{new} = 0.05 - 0.4308 \cdot \left(0.9285 + \frac{x}{\sqrt{1+x^2}} \right), -2.5 \le x \le 2.5 \tag{7}$$

The optimal value of ξ is selected as 0.85. The starting value of ξ is taken as 0.05 and increased still 0.85 for 10000 iterations of ACO with equal step size. The start value of x is selected as -2.5, and subsequently increased by calculated step size for 10000 iteration till it reaches 0.85. Similar logic is used to decrease the value of q from 0.9 to 0.1 in 10000 step size (Khuswaha and Mukherjee, 2012).

Although diversification helps to avoid local optima, a local search will intensify and improve the heuristic. Instigated by this philosophy, ACO_{MV} integrated with MI-LXPM GA is used to solve mixed integer problems in this paper. GA considers the best solution achieved by ACO (using 2 ants in an iteration) and stored in archive as the initial start point. An adaptive local search neighbourhood region is generated to create the initial population size. The maximum norm ($\| . \|_\infty$) of the vector that separates a randomly selected solution and the best solution from the ACO archive is used to define the neighbourhood region. GA is terminated if 10000 iterations are reached or relative change in fitness functional value is less than 10^{-12}. Other parameters of GA are selected based on Taguchi's design of experiment trials on standard test functions.

2.1. Tuning the GA parameters

Proper selection of parameters for GA in the hybrid ACO can plays a very important role on convergence of the algorithm to optimal solution as well as on the speed of convergence. In this research work, Taguchi L_{16} orthogonal array design (Mukherjee, 2007, p. 39) is used to tune the GA parameters. The goal of the simulation experiment is to determine a suitable combination of GA parameters that can result in faster convergence to global optimal for test cases. The factors and their levels selected is given in Table 1.

Table 1 Factors and Levels

	Level-1	**Level-2**	**Level-3**	**Level-4**
Crossover rate	0.2	0.4	0.6	0.8
Migration rate	0.2	0.4	0.6	0.8
Mutation rate	0.004	0.02	0.1	0.5
Population size	25	50	100	200

30 independent simulation run (replicates) for each of the 7 continuous test functions, viz. spheroid, hyper ellipsoid, difference of powers, Ackley, Rastrigin, Rosenbrock, Griewank (Molga and Smutnicki, 2005) were used for analysis. 10 dimensions continuous optimization problem with 4 parameters at 4 level is selected in the Taguchi's L_{16} experimental setup. To analyze the Taguchi's design, different type of input test functions are introduced as noise for any row of experimentation matrix. As each of the chosen test function are 'smaller-the-better(STB)' type, appropiate Signal-to-Noise (S/N) ratios was selected. The analysis of variance (ANOVA) tables and response tables for S/N ratio and mean on linear models using Taguchi's experiment is provided in Tables 2 to Table 5.

Table 2 Analysis of variance for S/N ratios

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Crossover	3	289.74	289.74	96.58	19.69	0.018
Migration	3	16.59	16.59	5.531	1.13	0.462
Mutation	3	23.59	23.59	7.863	1.6	0.354
Population	3	810.46	810.46	270.153	55.08	0.004
Residual Error	3	14.71	14.71	4.905		
Total	15	1155.10				

Table 3 Response table for S/N ratios

Level	Crossover	Migration	Mutation	Population
1	1.349	0.746	0.763	1.910
2	0.907	0.925	0.843	0.862
3	0.520	0.924	0.937	0.379
4	0.556	0.737	0.788	0.180
Delta	0.828	0.187	0.174	1.730
Rank	2	3	4	1

Table 4 Analysis of variance for Means

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Crossover	3	1.783	1.783	0.594	10.62	0.042
Migration	3	0.133	0.133	0.044	0.79	0.573
Mutation	3	0.071	0.071	0.023	0.43	0.750
Population	3	7.173	7.173	2.391	42.7	0.006
Residual Error	3	0.167	0.167	0.056		
Total	15	9.329				

Table 5 Response table for Means

Level	Crossover	Migration	Mutation	Population
1	1.349	0.746	0.763	1.910
2	0.907	0.925	0.843	0.862
3	0.520	0.924	0.937	0.379
4	0.556	0.737	0.788	0.180
Delta	0.828	0.187	0.174	1.730
Rank	2	3	4	1

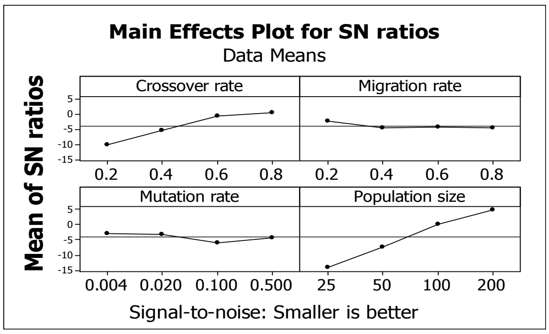

Fig. 1 Main effect plots for S/N ratios

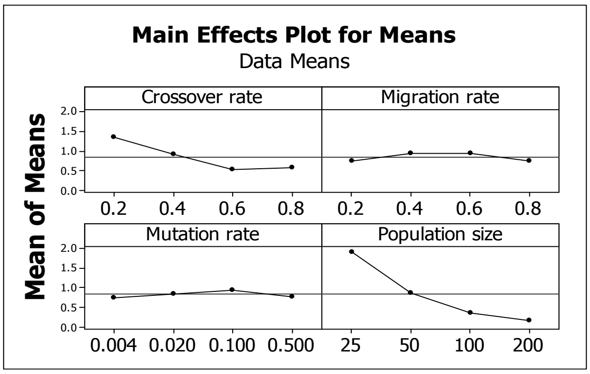

Fig. 2 Main effect plots for Means

The main effect plots for means and S/N ratio is shown in Fig. 1 and Fig. 2, respectively. As the goal is to maxize the S/N ratios and minimize means, seeing all the tables and figures above, the best parameter setting for GA was selected as (i) crossover rate as 0.8, (ii) migration rate as 0.2, (iii) mutation rate as 0.004, and (iv) population size as 200.These final setting was also selected in the mixed integer and multiple response optimization problems.

2.2. Results and Discussion

All the computational runs reported are performed on specific desktop personal computer (PC) using matlab software. The PC configuration is2.80 GHz intel dual core processor with 1.98 GB RAM. The results of 30 computational run for each test function in mixed integer problems, using ACO_{MV} with only diversification scheme, and ACO_{MV} integrated with MI-LXPM and diversification scheme is provided in Table 6 and Table 7, respectively. Success in a run is achieved in any particular run for a test case, if the difference between global optimum and solution obtained by a metaheuristic is less than 10^{-4}.

Comparing the results of Table 6 and Table7, it is evident that integrating GA with ACO improves the solution quality and also the success rate. Instigated by the above results, a multiple response optimization case is selected for investigating the performance of hybrid ACO strategy.

Table 6 ACO_{MV} with diversification scheme in mixed integer state space

Test Function	**Success**	**Mean of Solution**	**Standard Deviation of Solution**	**Average Number of Function Evaluation**	**Run Time (Secs)**
Sphere	30	_	_	2380	4.168
Hyper ellipsoid	30	_	_	2516	7.524
Sum of Different Power	30	_	_	2062	3.278
Ackley	30	_	_	5266	9.458
Rastrigin	29	0.033	0.182	32211	51.645
Rosenbrock	5	24.00	40.05	194574	340.608
Griewank	26	0.001	0.002	231669	382.721

Table 7 ACO_{MV} integrated with diversification scheme and MI-LXPM GA in mixed integer state space

Test Function	**Success**	**Mean of Solution**	**Standard Deviation of Solution**	**Average Number of Function Evaluation**	**Run Time (Secs)**
Sphere	30	_	_	23274	7.31
Hyper ellipsoid	30	_	_	30030	20.74
Sum of Different Power	30	_	_	17705	3.01
Ackley	30	_	_	59271	6.90
Rastrigin	30	_	_	232277	50.38
Rosenbrock	30	_	_	2821425	916.37
Griewank	30	_	_	4357317	1231.00

3. A MULTIPLE RESPONSE OPTIMIZATION CASE

A full factorial design of experiment data on a wear and friction tester machine is used for analysis. In the machine, the tester operates with a pin positioned perpendicular to a flat circular hardened steel disc. 72 experiments were conducted with 3 independent variables (two are ordinal discrete and one is continuous variable). The variables are materials (aluminum and brass, x_1), load (4 unit levels, x_2), and linear velocity (3 unit levels, x_3). The responses are coefficient of friction under sliding contact (y_1), frictional force (y_2), wear rate (y_3), and surface temperature (y_4). The objective of experiment is to determine the best set-tings of control variables that minimize all the response values. A partial least square regression model (Rosipal and Kramer, 2006, p. 34) is used to develop the response surface. SAS statistical software is used to estimate the regression coefficient. The overall multiple response problem is formulated as,

$$\begin{aligned} &\min f(y_i) = \beta_0^i + \beta_1^i x_1 + \beta_2^i x_2 + \beta_3^i x_3 \\ &s.t. \quad x_1 \in \{1,0\},\ x_2 \in \{3,4,5,6,7\},\ 1 \le x_3 \le 2 \end{aligned} \tag{8}$$

Table 8 Best setting using Hybrid ACO

Optimization Technique	Metal	Load (Kgf)	Linear velocity (m/s)	Maximum Composite Desirability
Hybrid ACO	Brass	4	1.2931	0.4275

Desirability functions (with t_1 as 1) are used to reduce the dimensionality and convert the problem to a single objective optimization case. 30 successful computational run achieved the same best setting as given in Table 8. The target values of all responses are set to zeros (STB type), and the overall desirability value (D) achieved is 0.4275.

4. CONCLUSION

This paper illustrates the suitability of a hybrid ACO strategy for MRO problems with mixed integer search space. GA-based local search was integrated in the hybrid ACO to improve the solution quality. The search strategy was verified using single and multiple response optimization problems. Fine tuning of the intrinsic parameter of GA is based on Taguchi's experimental concept of robust design. However, scope exists to test and improve the hybrid strategy in complex real life industrial mixed integer MRO problems.

REFERENCES

Bera, S. and Mukherjee, I. (2012) “An Ellipsoidal Distance-based Search Strategy of Ants for Nonlinear Single and Multiple Response Optimization Problems”, [in:] European Journal of Operations Research, vol. 223, pp. 321-332, 2012.

Deep, K., Singh, K. P., Kansal, M. L., & Mohan, C., (2009), “A real coded genetic algorithm for solving integer and mixed integer optimization problems”. [in:] Applied Mathematics and Computation, Vol. 212, pp. 505–518.

Deep, K., & Thakur, M. (2007), “A new mutation operator for real coded genetic algorithms”, [in:] Applied Mathematics and Computation, Vol. 193, pp. 211–230.

Derringer, G. , Suich, R., (1980), "Simultaneous optimization of several response variables”, [in:] Journal of Quality Technology, Vol. 12, No.4, pp. 214-219

Dorigo, M.(1992), Optimization, Learning and Natural Algorithms (in Italian). Ph.D. thesis, ipartimento di Elettronica, Politecnico di Milano, Italy.

Harrington, E. C. Jr., (1965), "The desirability function”. [in:] Industrial Quality Control, Vol. 21, No.10, pp. 494–498.

Haupt, R. L., Haupt, S. E., (2004), Practical genetic algorithms. 2nd ed. New York: John Wiley & Sons, Inc.

Khuri, A. I., (1996), Ghosh, S., & Rao, C. R. (Eds.), "Multiresponse surface methodology”, Handbook of Statistics: Design and Analysis of Experiments, 13, pp. 377–406, North Holland: Elsevier Science.

Kim, K., and Lin, D., (2000), "Simultaneous optimization of mechanical properties of steel by maximizing exponential desirability functions”. [in:] Applied Statistics: Journal of the Royal Statistical Society, Series C, Vol. 49, No. 3, pp. 311-325.

Kushwaha, S. and Mukherjee, I. (2012) A Hybrid GA-based Ant Colony Strategy for Continuous Correlated Multiple Response Optimization Problem, [in:] IEEE Colloquium on Humanities, Science & Engineering Research, Malaysia 2012, In Press.

Liao, T., (2010/2011), Improved ant colony optimization algorithms for continuous and mixed discrete-continuous optimization problems. Supervisors: Dorigo, M. and Stützle, T: [in:] Report d’avancement de recherche Année , IRDIA.

Lind, E. E., Goldin, J., & Hickman J. B., (1960), "Fitting yield and cost response surfaces”, [in:] Chemical Engineering Progress, Vol. 56, No.11, pp. 62–68.

Molga, M., & Smutnicki, C., (2005), “Test functions for optimization needs”, available [in:] http://www.zsd.ict.pwr.wroc.pl/files/docs/functions.pdf.

Mukherjee, I., (2007), Modeling and Optimization of Abrasive Metal Cutting Processes. PhD Thesis, Indian Institute of Technology (IIT) Kharagpur, India.

Rosipal, R., & Krämer, N. (2006), Overview and Recent Advances in Partial Least Squares. In: Saunders, C. et al. (Eds.), LNCS, Vol. 3940, pp. 34–51.

Socha, K., Dorigo, M., (2008), “Ant colony optimization for continuous domains”. [in:] European Journal of Operations Research, Vol.185, pp. 1155-1173.

BIOGRAPHICAL NOTES

Mr Sunil Kushwaha is a statistician in the Cytel Statistical Software and Services, since June 2012. He did his masters in Applied Statistics and Informatics (ASI) from Department of Mathematics, IIT Bombay. His research focuses on metaheuristics optimization, multiple response optimization, and statistical computing using nonparametric methods.

Dr Indrajit Mukherjee is an assistant professor in Shailesh J. Mehta School of Management IIT Bombay, India. He teaches operations management and quality management. His research interests are multiple response optimization, quality engineering, quality management, applied statistics and operations research. His paper appear in numerous journals including european journal of operations research, applied soft computing, computers and industrial engineering.