Prosimy cytować jako: Inż. Ap. Chem. 2011, 50, 2, 15-16

Józef FLIZIKOWSKI

e-mail: fliz@utp.edu.pl

Department of Production Technology and Engineering, Faculty of Mechanical Engineering, University of Technology and Life Sciences, Bydgoszcz

Self-regulation of intelligent grinding system

Interrelations at grain deformations

Self-regulation is based on changes of speed of interrelations, balancing of potentials, reactions to load, transport and stirring movement, and relates to states of stresses, changes and consequences of the grinding process.

The aim of the work is to find a model of plastic strain for free torsion at quasi-cutting of grain/granules in respect of previously given principles of the intelligent grinding system and performance parameters with minimum energy – aims of its operation.

In an analysis of the self-regulation of stresses and strains, a system of dependencies (eq. (12) in [12]) may be presented using the following matrixes:

$$\begin{bmatrix} \sigma_x - \sigma_{śr}, & \tau_{xy}, & \tau_{xz} \\ & \sigma_y - \sigma_{śr}, & \tau_{yz} \\ & & \sigma_z - \sigma_{śr}, \end{bmatrix} = 2G \begin{bmatrix} \varepsilon_x - \varepsilon_{śr}, & \frac{1}{2}\gamma_{xy}, & \frac{1}{2}\gamma_{xz} \\ & \varepsilon_y - \varepsilon_{śr}, & \frac{1}{2}\gamma_{yz} \\ & & \varepsilon_z - \varepsilon_{śr} \end{bmatrix} \quad (1)$$

where left and right terms of the equations (12), respectively proportional, take an identical position.

For example, term $(\sigma_y - \sigma_{śr})$ from the second line and the second column of the left matrix corresponds in the right matrix to term also located in the second line and the second column – $(\varepsilon_y - \varepsilon_{śr})$ and this term is known to be proportional to the first one. It was established that the left matrix that consists of components of the stress influencing the change of shape of the ground grains is called the stress deviator, whereas the right matrix – the strain deviator [1, 2, 4].

Therefore, the generalized interrelation between comminution-related stress and strain may be symbolically described as follows:

$$D_n = 2GD_o \quad (2)$$

i.e.: the stress deviator is directly proportional to the strain deviator.

Expressions (1a) and (2) are called self-regulation at the change of shape of the ground grains (granules) [1, 3, 4].

Invoking the principle of the volume change and using the concepts concerning volumetric stress tensor [1, 3], relationship (11) in [12] may be presented as follows:

$$A_n = E_o A_o \quad (3)$$

i.e. the volumetric stress tensor is directly proportional to the volumetric strain tensor.

A different formulation is also possible: first invariants of stress and strain tensor are proportional to each other, i.e. $\sigma^T = E_o \varepsilon^T$

Coefficient of cubic elasticity (volumetric modulus of elasticity) is:

$$E_o = \frac{E}{1 - 2\mu} \quad (4)$$

at $\mu \to 0,5$ approaches infinity.

Out of four constants E, μ, G, E_o obviously only two are independent. Considering the existence of the described two parameters of the strain principles and moduli E_o and G included in the same, it is logical to consider modules E_o and G, physical features of the ground grains, to be the basic ones, i.e. independent.

Volumetric modulus of elasticity E_o describes the resistance of the material to a volume change without an interrelated change of shape (hydrostatic compression). The shear modulus G (also known as rigidity modulus), on the contrary, describes the desistance of the material to the change of its shape, which is not accompanied by an interrelated volume change.

Regulation of comminution loads

It is assumed that the cross-section of the ground grain is an ellipse with semi-axes a and b. Because for in the cross-section profile, where $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, the function of stresses should equal zero, the following form of the function may be assumed:

$$F = A\left(\frac{x^2}{a^2} + \frac{y^2}{b^2} - 1\right) \quad (5)$$

Coefficient A is calculated on the basis of the condition that inside a profile, we should obtain the following: $\nabla^2 F = -2$. When placing function (5) in differential equation $\nabla^2 F = \nabla^2 \psi - \nabla^2\left(\frac{x^2 + y^2}{2}\right) = -2$ the following is obtained [1]:

$$A = -\frac{a^2 b^2}{a^2 + b^2} \quad (6)$$

and as a consequence, considering the torsion angle for ends of grains/granules ϑ, the following is obtained:

$$M_s = \frac{G\vartheta\pi a^3 b^3}{l(a^2 + b^2)} \quad (7)$$

$$\tau_{xz} = -\frac{2M_s}{\pi a b^3} y, \quad \tau_{yz} = \frac{2M_s}{\pi a^2 b} x \quad (8)$$

The total steady stress equals:

$$\tau = \sqrt{\tau_{xz}^2 + \tau_{yz}^2} = \frac{2M_s}{\pi ab}\sqrt{\frac{x^2}{a^4} + \frac{y^2}{b^4}}$$

and it reaches the highest value at the end of the axis of grain (granules) (i.e. for $a>b$ at $y = \pm b$):

$$\tau_{\max} = \frac{2M_s}{\pi a b^2} \quad (9)$$

Because elliptical section area is $\Omega = \pi ab$ and the polar moment of inertia for this section:

$$J_o = J_x + J_y = \frac{\pi a b^3}{4} + \frac{\pi a^3 b}{4} = \frac{\pi ab}{4}(a^2 + b^2)$$

expression (7) may also be presented as follows:

$$M_s = G\frac{\vartheta}{l}\frac{\Omega^4}{\pi^2 J_o} \quad (10)$$

This formula, generally correct only for the elliptical section, is widely-used in the grinding practice [4, 9, 11] for the calculation of the twisting moment for a given torsion angle (or vice versa), for any compact cross-section of non-concave profile considering relevant values Ω and J_o in (10).

Using *Prandtl* stress function $F = \psi - \frac{x^2 + y^2}{2}$, for elliptical section (5) and relationship (6) as (a) for $\psi = \varphi$ the following expression was obtained:

$$\varphi = \frac{1}{2(a^2 + b^2)}\left[2a^2 b^2 - (a^2 - b^2)(x_2 - y^2)\right] \quad (b)$$

Prosimy cytować jako: Inż. Ap. Chem. 2011, 50, 2, 15-16

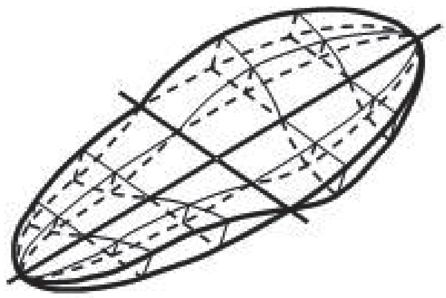

Fig.1. Deformation of elliptical section of micro grain (granules) when twisted [1, 9]

Based on expressions that bind interrelated functions φ and ψ the following is obtained:

$$\varphi = \int \frac{\partial \psi}{\partial y} \partial x + f_1(y)$$

or

$$\varphi = \int \frac{\partial \varphi}{\partial x} \partial y + f_2(x)$$

When function (b) is differentiated in respect of x and y the following is obtained:

$$\frac{\partial \psi}{\partial x} = -\frac{a^2 - b^2}{a^2 + b^2} x$$

$$\frac{\partial \psi}{\partial y} = -\frac{a^2 - b^2}{a^2 + b^2} y$$

therefore

$$\varphi = \frac{a^2 - b^2}{a^2 + b^2} y \int dx + f_1(y) \tag{c}$$

$$\varphi = \frac{a^2 - b^2}{a^2 + b^2} x \int dy + f_2(x) \tag{d}$$

Comparison of (c) and (d) gives $f_1(y) = f_2(x) = C$, so

$$\varphi = \frac{a^2 - b^2}{a^2 + b^2} xy + C \tag{e}$$

The formula of warp/deformation of points in the cross-section takes the following form now:

$$w = \frac{\vartheta}{l}\left(\frac{a^2 - b^2}{a^2 + b^2} xy + C\right)$$

Because the comminution deformation, of one of the points of the cross-section, may be assumed (depending on the limit kinematic conditions that do not restrict strains during the torsion of grains), if it is assumed that the centre of twist ($x = y = 0$) is not altered ($w = 0$), then $C = 0$.

The above analysis gives the final formula of self-regulating movement along z axis, i.e. of the deformation of the section of the ground micro grain, as follows:

$$w = \frac{\vartheta}{l} \frac{a^2 - b^2}{a^2 + b^2} xy \tag{11}$$

In the case of the circular section ($a = b$) based on formula (11) $w = 0$, i.e. self-regulation of such section does not result in deformation of the section.

Using dependence (7) an expression for the deformation of the section is presented in a different form, namely:

$$w = \frac{M_s(a^2 - b^2)}{G\pi a^3 b^3} xy \tag{12}$$

Formula (12) indicates that the lateral elliptical section of the grain (granules), when twisted, is subject to the comminution deformation, assuming the shape of a hyperbolic paraboloid.

Summary

If the algebraic sum of the grain/granule volume contained between the deformed section area and its initial plane is called the comminution volume of the section, then this volume equals zero. The volume of the comminution deformation equals zero for the torsion of the cross-section of any shape. The last conclusion concerning self-regulation of comminution strains and stresses may also be presented in the following analytical form:

$$\int_F w dF = 0$$

The analysis concerned micro grains with the elliptical section held, as per [6] at the left end and subject to the twisting moment at the right end. It was assumed that the moment is a result of only tangential forces distributed in the section as per formulas (8). In a given case, a non-free torsion occurs and therefore the model of a deformation of the section (12) previously described for a free torsion is not acceptable for a part of the ground grains/granules located close to the left end [12].

REFERENCES

[1] *N. I. Biezuchow:* Teoria sprężystości i plastyczności. PWN Warszawa 1957.

[2] *Z. Drzymała (Red.):* Badania i podstawy konstrukcji młynów specjalnych. PWN Warszawa 1992.

[3] *J. Flizikowski*: Rozprawa o konstrukcji. ITEE, Radom 2002.

[4] *J. Flizikowski:* Projekt implementacji inteligentnego systemu wspomagania konstrukcji młynów specjalnych, MNiI, ATR-AGH, Warszawa, Bydgoszcz, Kraków 2005.

[5] *J. Flizikowski:* Plastic devices for Energy Engineering. Nanoenergy, Tapis Rouge, Paryż 2008.

[6] *W. Grellmann, S. Seidler:* Deformation and fracture behavior of polymers. Springer-Verlag, Berlin 2001.

[7] *I. Rokach:* Numeryczno-analityczne modelowanie prób udarowych. Monografie nr 50, Wyd. Pol. Świętokrzyskiej, Kielce 2005.

[8] *J. Sidor:* Badania, modele i metody projektowania młynów wibracyjnych. Wyd. AGH Rozprawy/Monografie nr 150, Kraków 2006.

[9] *G. Szala:* Analiza energochłonności technologicznego ścinania ziaren zbóż. Grant MNiI/UTP Warszawa, Bydgoszcz 2007.

[10] *J. Yan, J. S. Strenkowski:* Journal of Materials Processing Technology 174, 102, (2006).

[11] *J. Zawada (Red.):* Wprowadzenie do mechaniki maszynowych procesów kruszenia. Wyd. ITE, Radom, Warszawa 2005.

[12] *J. Flizikowski:* Inż. Ap. Chem. **50**, nr 2, 13 (2011).

The research financed by the funds of the National Centre for Research and Development in 2010/2013 as a development project.