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Modeling of pulsating flow in a collapsible wall

Introduction

According to estimates of the *World Health Organization* (WHO), 16.7 million people around the globe die of cardiovascular diseases every year [1]. One of the most widespread cardiovascular diseases is Abdominal Aortic Aneurysm (AAA). An aneurysm is an abnormal dilatation of the aorta, which may rupture if not surgically treated. Rupture of the aneurysm occurs when mechanical stress acting on the dilating inner wall exceeds its failure strength. The main clinical indicators used to assess the risk of aneurysm rupture are diameter and expansion rate of the aneurysm. When the AAA diameter reaches 55 mm and more, or the aneurysm expands above 10 mm/year, surgical treatment is recommended [2, 3]. Due to differences in the aorta diameter (1.5–2.5 cm) in different patients, this criterion is not accurate to estimate all patients. Various clinical studies show that more precise indicators are needed to predict the probabilities of aneurysm rupture.

The purpose of this study is to develop a non-invasive method to characterize the state of cardiovascular system and predict the probabilities of aneurysm rupture. In our previous work we developed and validated the Fluid Structure Interaction (FSI) model of pulsating flow in a silicone pipe which reflects blood flow in the vessel wall [4]. This technique involves relevant fluid-structure mechanics which takes into account the fluid force acting on the wall as well as the effect of the wall motion on the fluid. Due to complexity of the described problem, some simplifications, such as linear material property of silicone, were introduced.

In this work in order to create a more realistic FSI model, nonlinear properties of silicone were considered.

A uniaxial tensile test for silicone material was carried out in order to choose a proper form of strain energy density function to describe the nonlinear behavior of silicone.

Method

A three-dimensional (3D) model of a silicone tube which reflects pulsating blood vessel in the human body was constructed based on our experimental set-up [5]. Dimensions of the silicone vessel were similar to a human aorta; the internal diameter of the silicone vessel was 16 mm, with wall thickness of 2 mm. The material model of the silicone vessel was considered as isotropic and nearly incompressible at v = 0.49, $\rho = 1120$ kg/m³.

The nonlinear behavior of silicone can be described by the strainstress relationship. In order to perform a curve strain-stress fitting for a given sample, experimental uniaxial tensile test of silicone was carried out. ANSYS software providing curve fitting tools to obtain material constants for silicone rubber was used. Data from the uniaxial tensile test were rewritten into stress-strain text file format and then fed to the ANSYS software. Curve fitting procedure for hyperelastic material is difficult due to nonlinearity and material instability. The coefficients in strain energy potential can be considered as material constants with a specific physical meaning.

The curve fitting procedure and material constants for the chosen model are shown in fig. 1 and table 1, respectively.

The 3-term *Yeoh* model for strain energy density function was chosen to describe a nonlinear mechanical behavior of silicone rubber. The *Yeoh* model can be described by the following equation [6]:



Fig. 1. Material response described by Yeoh model

Tab. 1. Material constants for 3-term Yeoh model

Model	Material Constants	
Yeoh (3-term)	$c_{10} = 4.8188e+005$ [Pa] $c_{20} = -69527$ [Pa] $c_{30} = 33648$ [Pa]	

$$W = \sum_{i=1}^{N} c_{i0} \left(\overline{I}_{1} - 3 \right)^{i}, \qquad (1)$$

where:

W – strain energy potential,

 I_1 – first deviatoric strain invariants,

N, c_{i0} – material constants.

In general, the *Yeoh* model offers a physical interpretation and provides a better description of deformation when the parameters are based only on one deformation test (uniaxial test only).

To validate the FSI model of pulsating flow in a silicone pipe, the series of experiments were performed using the set-up developed earlier [5]. Water which reflects fluid part of the FSI model was considered as an incompressible, homogeneous and *Newtonian* fluid. The calculated *Reynolds* number was equal to 4000. According to this value turbulent flow was considered, and the Shear Stress Transport model for the simulation of turbulence was used, which is recommended for accurate boundary layer simulations.

The following boundary conditions were imposed: (i) waveform of profile velocity on the vessel inlet, and (ii) pressure waveform on the outlet (Fig. 2 and 3).



Fig. 3. Changes of pressure vs. time

Tab 2. Results of mesh independence test showing comparison of maximum pressure and maximum velocity values between different meshes

Total number of elements	Maximum pressure [Pa]	Difference %	Maximum velocity[m/s]	Difference %
13920	73704.8		0.758976	
27400	73860.1	0.2	0.756343	0.35
62400	73845.4	0.01	0.762883	0.85

A no-slip boundary condition was applied on the wall. The pulsated waveforms are represented by *Fourier* series based on the measurements taken from our experimental set-up [5]. Discretization of fluid domains was also necessary to perform calculations so, 27600 elements for the fluid region containing 18600 triangular prismatic elements in the central region were created and structural mesh with 9000 hexahedral elements on the boundary layer were added. Five layers with different sizes of elements were implemented to get a better solution of the near wall region. In order to reduce the time of calculation and due to symmetrical geometry of the tube only one quarter of the vessel was modeled.

To control mesh quality orthogonality angle, aspect ratio and mesh expansion factor for each element were checked and no errors were found. Mesh for the fluid region was made finer in the inlet region in order to let the flow develop naturally. The mesh independence test was carried out for 3 different types of mesh regarding their density. Independence in the mesh size was obtained for selected variables (maximum pressure and maximum velocity) within difference about 0.01–0.20% for maximum pressure and 0.35–0.85% for maximum velocity, respectively.

The differences in terms of maximum pressure and maximum velocity values between 13920-element mesh and the 27400-element mesh were around 0.2% and about 0.5% between the 27400-element mesh and a 62400-element mesh. Finally, mesh with 27400 elements was accepted for further calculations.

Results and discussion

The aim of this study was to perform the FSI analysis for a silicone tube with water flowing inside and to compare the value of wall displacement in the experiment and simulation for non-linear properties of the silicone wall. Fig. 4 depicts a comparison of experimental and simulated wall displacement for 5 cardiac cycles.



Fig. 4. Comparison of calculated and experimental changes of the pipe diameter

There is a good correlation in repetition and periodicity of the waveform between simulation and experimental results, while the amplitude of pulsations was estimated with about 20% error. Agreement between the simulation and experiment can be improved if additional deformation parameters are considered in the calculations (biaxial tension and planar shear tests). However, due to high costs of the deformation tests mentioned above the application of linear deformation model is justified [4].

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