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A methodology for the assessment of historical structures based on finite element models

Metodologia oceny historycznych konstrukcji w oparciu o modelowanie metodą elementów skończonych

Keywords: Historical structures, Finite element, Masonry, Stone, Brick, Arch, Vault, Flying buttress

Słowa kluczowe: historyczne konstrukcje, Finite Element, kamieniarka, kamień, cegła, łuk, sklepienie, przypora

1. INTRODUCTION

The most common methodology applied currently to verify the safety of masonry historical structures is the graphic statics under the limit analysis [1]. It is based on a comparative analysis between the equilibrium situation of the structure and its limit situation considering the stability. The result of this analysis is the geometrical factor of safety. It is an indicator of the safety of the structure considering its stability when the loads are applied and fixed supports are considered (without foundation settlements, spread of the abutments, etc.).

The equilibrium situation of the structure, applying the current methodology, is determined considering the equilibrium equations, but not the compatibility and the material constitutive equations. Therefore, the calculated result only depends on the geometry of the structure, the applied loads and the consider reactions at the supports. The solution of the equilibrium equations system is obtained applying the graphic statics [2], determining the funicular polygon that equilibrate the loads acting on the structure and, from it, it is deduced the thrust line. It is assumed the following hypothesis [3]:

- The masonry blocks are rigid elements and it is assumed that the material failure in compression is not possible.
- The masonry tensile strength is null. It is not considered the cohesion due to the mortar located between blocks.
- The sliding between blocks is not possible.

The thrust line, obtained from the structural analysis, allows knowing the minimum geometry of the structure necessary to be in situation of equilibrium and, comparing the minimum geometry with the real geometry it is determined the geometrical factor of safety. It is usually considered that the masonry historical structures are safe when their geometrical factor of safety is equal or higher than 2, but this value varies according to the construction style. In case of masonry bridges, both the graphic statics under the criterion of the limit analysis and finite element models that simulates the bridges behavior [4] are currently applied. In this last case, the stone blocks, fills and pavement of the bridges are modeled and over them it is applied the loads to calculate the bridge response to verify their safety considering the stability.

In this work a methodology based on the finite element method to analyze historical structures is presented, considering the following hypotheses:

- There is no tensile strength at the joints between blocks.
- Shear strength of the joints is limited by the shear strength of the mortar and the static frictional coefficient in the interface block-mortar-block.
- Brittle facture occurs in tension as the elastic limit in tension is reached.
- Plastic deformation and crushing appear as the elastic limit in compression is reached.

Applying this methodology, the following aspects can be verified:

- Situation of equilibrium of the structure.
- Stresses in the different elements of the masonry.
- Possible sliding between blocks.
- Capacity of the structure to resist foundation settlements.

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- Influence of the degradation of some blocks considering the durability, the decreasing of their density and their elastic modulus.
- Consequence of the installation of reinforcements (actives or passives) in the structure and the stresses in them.
- Effect of the cracks seal with non shrinkage mortar.
- Soil bearing capacity under the foundations.
- Conditions in which the structure collapses and the collapse progress.

2. GENERAL EXPOSITION

The proposed methodology to analyze and evaluate masonry historical structures of monumental buildings can be summarized in the following steps:

Study of the building "on site" to obtain the necessary information: all the dimensions and levels of the structure, its location coordinates, high from the level sea and wind exposure, the material properties and the soil bearing capacity.

Study of the loads acting on the structure. Considering the current standards, the actions [5] and their combinations [6], which depend on the limit state to verify, can be obtained. The simultaneity factors to apply are defined in the current standards and the safety partial factors to consider are shown in table 1.

Table 1. Safety partial factors

Persistent or transitory situation			Accidental situation		
Stability of the structure	$\gamma_{G,stb}$ = 0,90	$\gamma_{Q,dst}$ = 1,80	Stability of the structure	$\gamma_{G,stb}$ = 0,90	$\gamma_{Q,dst}$ = 1,20
Slidding between blocks	$\gamma_{G,stb}$ = 0,90	$\gamma_{Q,dst}$ = 1,50	Slidding between blocks	$\gamma_{G,stb}$ = 0,90	$\gamma_{Q,dst}$ = 1,20
Masonry resistance	γ_{G} = 1,35	γ_{Q} = 1,50	Masonry resistance	γ_{G} = 1,00	γ_{Q} = 1,00
Reinforcement resistance	γ_{G} = 1,35	γ_{Q} = 1,50	Reinforcement resistance	γ_{G} = 1,00	γ_{Q} = 1,00
Soil bearing capacity	γ_{G} = 1,00	γ_{Q} = 1,00	Soil bearing capacity	γ_{G} = 1,00	γ_{Q} = 1,00

Creation of the geometric model. The scaled masonry structure is graphically represented using a CAD application. The geometrical model consists in an assembly of pieces in contact and each piece can represent a single block of the structure or a group of blocks.

Preparation of the model to analyze the structure. All the pieces of the geometric are meshed at this step. It is also assigned to the assembled pieces the boundary conditions (loads and imposed displacements), the contact behaviour between pieces and the material properties (density, Young's modulus, Poisson's ratio, etc.).

Calculation of the model applying the finite element method. The method formulation [7] allows to avoid the imposed restrictions in the graphic statics.

Evaluation of the results. According to figure 1, after the analysis, if there is an equilibrium solution the safety of the structure can be determined verifying several parameters. On the other hand, if do not exist such solution, it means that the structure will collapse under the imposed boundary conditions on the model and then, the collapse evolution can be determined applying a progressive analysis.

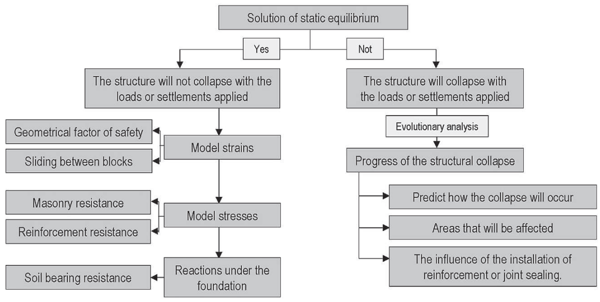

Fig. 1. Verifications that can be checked applying the proposed methodology

3. METHODOLOGY DEVELOPMENT

3.1. Creation of the geometric model

The pieces of the geometric model should be created using a CAD application and considering two possible criteria. Each piece can represent a single block of the masonry structure or, on the other hand, a group of blocks. The correct criteria to create pieces that represent a group of blocks can be summarized in the following points:

- The blocks with common joints where all the contact surfaces are in compression can be represented in the geometric model by just one piece. The objective is to find groups of blocks with rectangular or trapezoidal contact pressure distributions between them, considering their behaviour as only one elastic solid. This situation appears when the thrust line lies inside the kernel of the blocks section. It the above condition is not satisfied, the joint between blocks will be cracked, so these blocks are associated to different pieces in the geometric model.
- The masonry blocks located at geometric discontinuities must be associated to different pieces of the geometric model, e.g. where an abutment changes its direction or its thickness, an abutment or a column supports an arch, a vault or a flying buttress, etc.
- It is recommended, in the safety side, to group the blocks of domes, semi-domes, etc. in pieces with segments shape according to the "cuts method" [1] of the graphic statics. The vertical cracks generated in the geometric model are necessary to consider the typical vertical cracks shown in the figure 2 appearing in domes due to the horizontal tensile stress in the base of this type of structures.

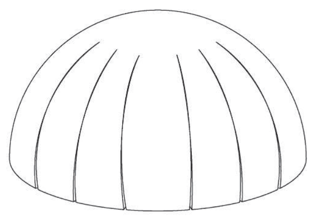

Fig. 2. Typical vertical cracks in the base of the domes

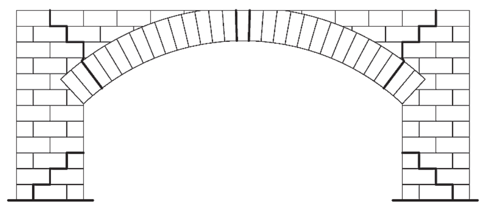

Fig. 3 Typical cracked areas in abutments that support arches

– It is also recommended to separate in different pieces the blocks located in areas where high shear loads and low axial loads of compression are expected. It is necessary to verify the possibility of sliding between the pieces of the geometrical model and, therefore, between the blocks of the masonry structure.
– Finally, it is important to create a joint at the base of the abutments of the geometric model as shown in the figure 3 (forming 45° with the horizontal plane) to take into account the possible cracking that usually appears in the bases of the abutments. When the thrust line does not go through the kernel of the section of the abutment base, a crack appears initially in the intrados of the base and later, due to the high self weigh of the abutment, the abutment is broken and a new crack appears forming 45° with the base [8].

The above criteria are recommended to simplify the geometrical model in order to achieve reliable results with the minimum computational cost. However, it is not compulsory to simplify the model but, in that case, the computational cost will increase a lot and the results will be the same.

3.2. Preparation of the model

Once the geometric model has been created, it is necessary to assign the boundary conditions to the pieces, that is to say, the loads that act on the structure, the supports below the foundations, the imposed displacements that simulate the effect of settlements, etc. It is also necessary to mesh and to assign the material properties to each piece and finally, to define the contact behaviour between pieces.

The loads and their combinations are indicated in the current standards applying the safety partial factors shown in the table 1 and, on the other hand, the settlements that should be considered in the foundation to verify the safety of the masonry structure are:
– A maximum absolute settlement: δ_{vert} = 25 mm.
– A maximum relative settlement: δ_{vert} = L/2000, where L is the distance between the considered points to obtain the máximum relative settlement between them.
– The real settlements observed in the structure during the study of the building "on site".

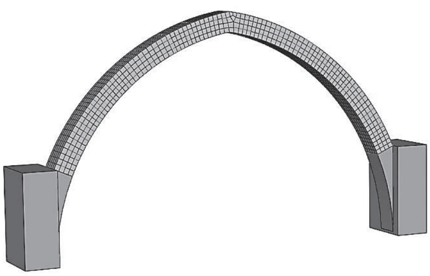

Fig. 4. Equilateral arch meshed

The recommended elements to mesh the pieces, as shown in the figure 4, are quadratic bricks of 20 nodes or quadratic tetrahedra of 10 nodes where the geometry is complicated.

The material properties that should be assigned to the meshed pieces should be obtained considering parameters determined from structural tests "on site" or specialized bibliography. These necessary material properties are:
– Masonry density (ρ).
– Young's modulus of the masonry (E_m), calculated as:

$$E_m = E_b \cdot \frac{1+\alpha}{1+\alpha\cdot\beta\cdot(1+\varphi)} \tag{1}$$

where the E_b is the Young modulus of the masonry blocks, α is the ratio of the average thickness of the mortar bed joints and the average high of the blocks, β is the ratio of the Young's modulus of the blocks and the Young's modulus of the mortar and φ is a parameter that takes account the masonry creep which can be considered null analizing historical structures. The values of the α and β parameters can be obtained applying the following ecuations:

$$\alpha = \frac{h_m}{h_b} \qquad \beta = \frac{E_b}{E_m} \tag{2}$$

– Poisson's coefficient of the masonry (ν).
– Design value of the masonry compressive strength ($f_{m,d}$). It is the limit from which the non linear behaviour of the material starts and can be obtained applying the Ohler formula [9]:

$$f_{m,k} = \frac{a\cdot f_{b,k}}{1+10\cdot b\cdot\alpha} \qquad f_{m,d} = \frac{f_{n,k}}{\gamma_R} \tag{3}$$

where γ_R = 4 is the partial factor for the material properties, $f_{b,k}$ is the characteristic value of the blocks compressive strength and the 'a' and 'b' non-dimensional parameters can be obtained from the table 2 in function of α.

Table 2. Parameters of the Ohler formula

	a	b
∞ ≤ 0,02	1.00	2.22
0,02 < ∞ < 0,05	0.81	0.96
∞ ≥ 0,15	0.66	0.66

– Design axial tensile strength of the blocks ($f_{b,td}$). It is the limit from which the cracking of the material starts and can be calculated as:

$$f_{b,tk} = 0.05\cdot f_{b,k} \qquad f_{p,td} = \frac{f_{p,tk}}{\gamma_R} \tag{4}$$

where γ_R = 4 is the partial safety factor for the material properties, $f_{b,k}$ is the characteristic value of the blocks compressive strength and $f_{b,tk}$ is the characteristic axial tensile strength of the blocks.

The contact properties that must be assigned to the contact surfaces should be as follows:
– Normal behaviour: it is disabled the possibility of interference between the pieces, that is to say, each piece can transfers thrusts to next pieces and cannot penetrate in them. It is also allowed the separation of the contact surfaces after the contact.
– Tangential behaviour: it is allowed the sliding between pieces if the maximum friction load is exceeded at their contact surfaces considering an static frictional coefficient.

After the preparation of the model, it is calculated applying the finite element method. It is recommended to start the analysis procedure fixing all the pieces of the model to ensure an initial mathematical convergence to an equilibrium solution (if it exists). After that, assigning contact conditions instead of the fixing conditions in successive iterations, the solution of the model considering the contact conditions recommended in the proposed methodology will be obtained.

3.3. Results analysis

The safety of the structure considering the stability is verified obtaining equilibrium solutions in all analyzed cases. The scope of these analyzed cases should be all the possible situations of the structure during its working life (e.g. extreme loads, foundation settlements, blocks deterioration, etc.).

On the other hand, historical structures have high execution tolerances and are also characterized by their self-weight as the dominant action; therefore their design geometry is very important because their safety considering the stability assuming the execution tolerances depends mainly on the geometrical design. To take into account the effect of the execution tolerances on the safety of the structure from the stability point of view, one of the following verification criteria must be used:

– The execution tolerances can be considered in the creation of the geometrical model.
– The geometrical model can be created without the execution tolerances. Therefore they should be considered calculating the geometrical factor of safety of the structure analyzing the deformed model calculated applying the self-weight of the masonry structure on the pieces and without consider settlements on the foundation, that is to say, in the situation when the centering is removed.

The geometrical factor of safety is usually the result obtained applying the graphic statics under the limit analysis criteria. Applying the proposed methodology the geometric factor of safety can be determined from the analysis of the cracks opened between the pieces in the deformed model. Considering the ratio of the compressed area of the contact surface and the whole area of the contact surface can be determined discretely the local geometrical factor of safety at any joint (e.g. at the joint "i" the $GSF_i = 3$ when the ratio = 1, $GSF_i = 2$ when the ratio = 0.75 and $GSF_i = 1$ when the ratio ≤ 0.1). This discrete procedure to obtain the local geometrical factor of safety at any joint (GSF_i) must be changed for a new criterion that defines this factor in a continuum way considering plane flexo-compression according to the figure 5.

The geometrical factor of safety of any structural element (GSF_{elem}) can be obtained applying the equation 9 considering the local geometrical factor of safety of its critical joints (GSF_i), that is to say, of the contact surfaces of its pieces where "hinging" cracks [1] are opened. Finally, the geometrical factor of safety of the whole structure is equal to the lower value of the GSF_{elem} of the all elements of the analyzed structure:

$$GSF_i = \frac{1}{2}\frac{h_i}{e_i} \qquad GSF_{elem} = \frac{\sum_{i=1}^{n} GSF_i}{n} \tag{5}$$

where n is the number of critical joints in the structural element. In the particular case where n is null because there is not critical joints, $GSF_{elem} \geq 3$ in this element.

The safety of the structure considering the masonry resistance is verified comparing the maximum value of the compressive stress field with the design value of the masonry compressive strength ($f_{m,d}$). The safety of the structure considering the reinforcement resistance is verified comparing the maximum value of tensile stress in the reinforcement with the design value of the reinforcement tensile strength specified by the manufacturer or the current standard. Finally, the soil bearing capacity is checked considering the reactions at the foundation under the current standard criteria, e.g. the CTE-SE-C [10] in Spain.

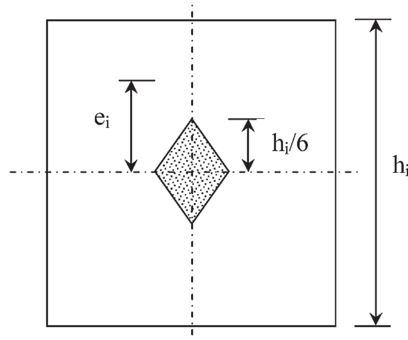

Fig. 5. Kern of a rectangular section

4. HISTORICAL MASONRY STRUCTURE ANALYSIS

4.1. Definition of the structure, creation and preparation of the model

In the figure 6 is shown the masonry structure to be analyzed applying the proposed methodology in this work. It is composed by an arch, two flying buttress and two abutments. All of them are 1 m depth. In this case, only the self-weight of the masonry acts on the structure. It has been considered a 2500 kg/m^3 of masonry density. Figure 6 shows the pieces in which the structure has been divided to create the geometrical model with a CAD application.

The following tasks were performed to prepare the model:

– Meshing the pieces with quadratic brick elements of 20 nodes and 100 mm of maximum width and assembling them how is shown in the figure 7. Abaqus program [11] was used.
– Assigning the masonry properties: Young's modulus E_m = 15000 MPa, Poisson's coefficient $\nu = 0.2$, design value of the masonry compressive strength $f_{m,d}$ = 5 MPa and the design axial tensile strength of the blocks $f_{b,td}$ = 0.25 MPa.
– Assigning also the recommended contact properties, considering a static frictional coefficient $\mu = 0.3$, to the contact surfaces.
– Applying the boundary conditions: the loads (only the self-weigh) and the supports (that depends on the analyzed case).

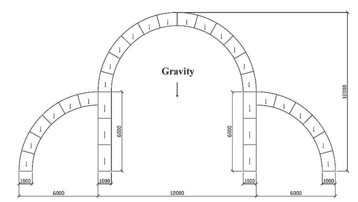

Fig. 6. Proposed structure (dimensions in mm)

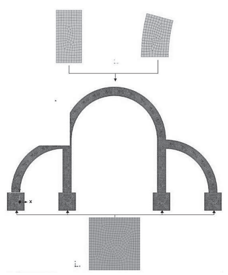

Fig. 7. Assembly of meshed pieces

4.2. Results

The deformed model in equilibrium situation applying the self-weight of the structure and without consider foundation settlements, is shown in the figure 8. It can be obtained that the GSF = 1.2 from these results, so if the proposed structure was built, it could collapse when the centering would be removed unless the structure was built perfect (without significant execution tolerances). The GSF should be ≥ 2 to ensure the stability of any historical structure taking account the instability effects due to the real imperfections. On the other hand, no sliding occurs between pieces in the analyzed situation.

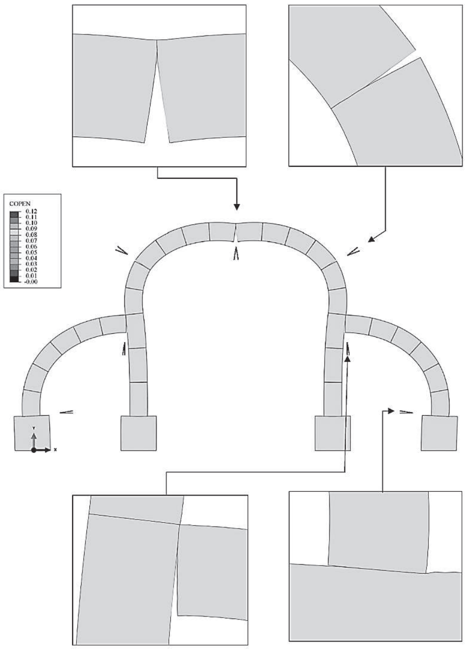

Fig. 8. Deformed model (x 2000)

The stress field of the model obtained applying the design loads and without consider foundation settlements is shown in the figure 9. The maximum compressive stress is 0.77 MPa in this case and the design value of the masonry compressive strength is 5 MPa, therefore the safety of the structure is high considering the material strength (the safety factor is 6.49).

The stress field on the deformed model obtained applying the design loads and considering a 25 mm of settlement in the left foundation, is shown in the figure 10. The maximum compressive stress is 3.14 MPa in this case, so the structural safety is reduced considering the material strength when a settlement appears (the safety factor is 1.59 in this case). Regarding the deformed model, there is a small sliding between pieces under the left flying buttress but however, its effect is not enough to collapse the structure because an equilibrium solution exists.

The model has been also calculated applying the self-weight of the structure and an arch maintenance load (without consider foundation settlements) to verify the structural safety if the arch maintenance is carried out.

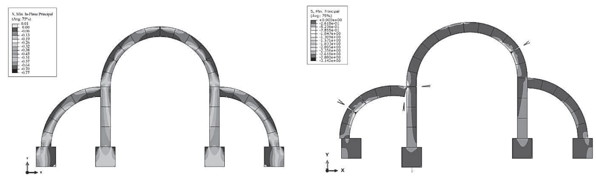

Fig. 9. Stress field in the model without Settlements

Fig. 10. Stress on the deformed model with 25 mm settlement

The maintenance load is 15 kN acting on the keystone (the keystone is composed by the two upper pieces of the model) and there is not equilibrium solution in this case, so the structure collapses. The progress of the structural collapse is shown in the figure 11, obtained from an evolutionary analysis.

Finally, the model is reinforced with a temporally steel bar to avoid the structural collapse when the arch maintenance is carried out. The position of the steel bar has been determined analyzing the figure 11 (c) which shows the collapse mechanism that should be avoided. The stress field on the deformed reinforced model obtained applying the design loads and without considering foundation settlements, are shown in the figure 12. The maximum compressive stress in the masonry is 1.29 MPa in this case, so the structure safety is high considering the material strength (the safety factor is 3.87) when the arch maintenance is carried out. Regarding the deformed model, no sliding occurs between pieces in the analyzed situation. The scope of this work does not include the verification neither the soil bearing capacity nor the tensile yield strength of reinforcement.

5. CONCLUSIONS

In spite of the existing analogies between the developed methodology in this work and the graphic statics under the

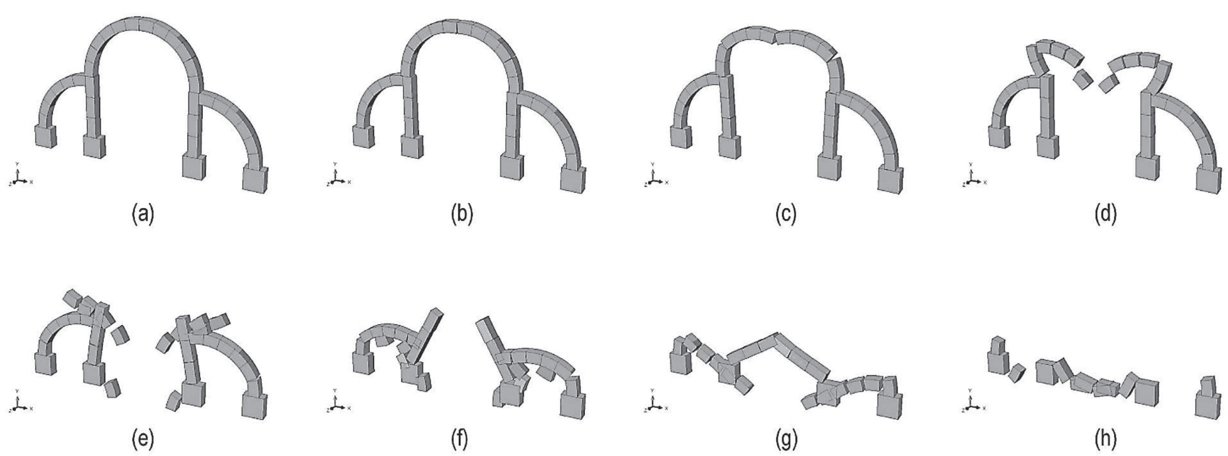

Fig. 11 Progress of the structural collapse

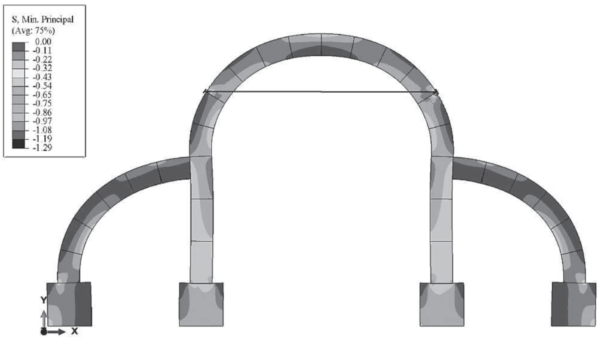

Fig. 12 Stress field on the deformed reinforced model (x 2000) due to the maintenance load

limit analysis criteria, this last methodology has several limitations when comparing with the proposed methodology. The main advantages are:

Addresses much more situations: It can consider foundation settlements, blocks deterioration, different types of material behaviour, active or passive reinforcement, joint sealing with non shrinkage grout, etc.

Besides the geometrical factor of safety, the proposed methodology allows to verify the structure safety considering the failure by sliding between blocks, the material failure and the loss of the overall stability due to foundation settlements, etc.

The collapse progress can be predicted in case that the structure loss the stability.

Tree-dimensional masonry structures can be analyzed (the graphic statics method can analyze only planar structures).

Due to the reasons exposed above, the methodology developed in this work represents a way for the assessment of historical monumental structures with high level of reliability in order to to determine their structural safety in different situations.

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Abstract

This work proposes a methodology to analyze masonry historical structures (stone or brick). They are mainly characterized by their self-weight which is the dominant action that acts on them, such us churches, cathedrals, etc. The proposed methodology is based on the analysis of finite element models that simulate the real structure behaviour. Therefore, much more information can be obtained about the structure safety than applying the usual methodology (the graphical statics considering the limit analysis). Using the proposed methodology, the structure response can be determined applying some loads on the model and considering some support conditions that simulate foundation settlements. The effect of structural reinforcement, wear of some blocks, non linear properties of the materials, etc. can be also considered. The safety level of the studied structure is evaluated with the obtained analytical results.

Streszczenie

W niniejszej pracy zaproponowano wykorzystanie metody elementów skończonych do analizy historycznych konstrukcji murowanych (kamienia lub cegły). Zasadniczą cechą tych konstrukcji jest ich ciężar własny, który stanowi w ich przypadku podstawowe obciążenie np.: kościoły, katedry itp. Zaproponowana metodologia jest oparta na analizie metodą elementów skończonych modeli, które symulują zachowanie rzeczywistej konstrukcji. Pozwala to na uzyskanie większej ilości informacji dotyczących bezpieczeństwa konstrukcji, niż przy użyciu zwykłych metod (statyka wykreślna z uwzględnieniem analizy stanów granicznych). Przy wykorzystaniu proponowanej metody, reakcja konstrukcji może być określona poprzez zwiększenie obciążenia na modelu i przebadanie warunków podparcia, które symulują osiadanie fundamentów. Efekt wzmocnienia konstrukcji, zużycie materiałów, ich nieliniowe właściwości, itp. mogą również zostać uwzględnione. Poziom bezpieczeństwa badanej konstrukcji jest oceniany na podstawie uzyskanych wyników analiz.