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ANALYSIS OF THE BREAKAGE RATE FUNCTION FOR SELECTED PROCESS PARAMETERS IN QUARTZITE MILLING

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The paper presents the results of studies on quartzite milling in a ball mill. The milling was conducted in a batch system, for diversified compositions of balls. The milling product was subjected to granulometrical, morphological and strength analyses. On the basis of the developed Reid's theory and using the Austin–Gardner equation, a form of the function circumscribing the specific rate of comminution of selected size fractions was determined. The values of the breakage rate function $b_{i,j}$ for the mill's apparatus conditions were determined. The impact was investigated for a variable number of grinding media contact points on the values of specific rate S and the values of the breakage rate function $b_{i,j}$. Furthermore, the values of coefficients occurring in the equations circumscribing the specific rate of milling S and breakage parameter $b_{i,j}$ were determined.

Keywords: ball mill, specific grinding rate, grain morphology, contact points

1. INTRODUCTION

Comminution is one of the most important mechanical operations conducted in mineral engineering. The primary purpose of comminution is to obtain, for further process operations, an appropriate size of grains at the least possible energy input. According to literature sources (Tavares and King, 1998), comminution demands only approx. 4% of energy from the total energy expenditure necessary to drive the equipment, e.g. a mill. Mills have a very low mechanical efficiency. Numerous attempts undertaken by engineers are focused on increasing process efficiency and on the ways of better analysing results of milling, aimed at an increase in its efficacy.

Vectorial mathematical tools and matrices, as well as statistics, are used for better description of the breakage rate function and the raw material's grain classification function. Lynch (1977) presented in his article a description of comminution based on the mentioned mathematical apparatus. However, the relation between energy expenditure necessary for particle comminution and the basic laws governing the comminution process continues to be the subject of researchers' interest. There are many theories describing the correlation between the energy expenditure for comminution of a single grain and the effect of the division of this grain into smaller grains. The primary fundamental theories describing the energy issues are the studies carried out by Bond (1952), Brach (1962), Kick (1885), Rittinger (1867) and Sokolowski (1992). Their studies gave rise to equations of Charles who proposed the form of the function of size fractions of the comminuted material depending on energy expenditures (Kapur, 1971). The starting point for Charles' considerations was Equation (1) proposed by Walker and Shaw (1954).

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$$dE_x = -C\frac{dx}{x^m} \tag{1}$$

The expansion of Charles equation, including the Gates-Gaudin-Schuhmann particle size distribution formula (Schubert, 1968), and the energy needed to obtain grains of characteristic size y, starting from the initial grain size x_{max}, assumed the following form (Eq. 2):

$$E = \int_0^y \int_{x_{\max}}^x \left(-C\frac{dx}{x^m}\right) dP \tag{2}$$

$$dP_x = \alpha W_0 \frac{x^{\alpha-1}}{y^\alpha} dx \tag{3}$$

In Equation (3), value α of the distribution module is calculated from the figure (in logarithmic coordinate system) of the size distribution for comminution, as a percentage of the mass passing to a smaller fraction. Value α is determined by the straight line's angle of inclination in this figure. From the same figure, value may be determined. Value y corresponds to the size with which the straight line achieves the level of 100% raw material mass.

Stamboliadis, according to the mathematical analysis of equations which describe the energy needed for comminution, pointed to the fact that the demand for energy increases with a decrease in the characteristic size of the grain, and the increase rate depends directly on the exponent (1-m) (Stamboliadis 2002). In addition, he suggested that the magnitude of that exponent is not constant but varies with a change of the relative size of the grain and in a way which is connected with the morphology of the raw material (Hukki, 1961). The conclusion of that article (Stamboliadis 2002) indicates that the size distribution of a specific rock material after comminution is not strictly defined. Module y not only depends on the energy of grinding media but also its value is strictly connected with comminution mechanisms. Consequently, the function of the distribution of a given material does not have any constant values for all size ranges (Lynch, 1977).

Considering the above facts, the theory describing material comminution process in a ball mill should include the reactors theory in the description of the milling process. Therefore, assumed is the comminution process irreversibility, determined by the first order kinetic equation, within the particles sized from y to y+dy. Additionally assumed is the variability of the characteristic value of the comminution rate $S(y)$ for respective sizes of particles and the basic form of the particles size distribution function.

The differential equation circumscribing the distribution of grains in a reactor (Austin, 1999), exhibiting a continuous change in particles dimensions, assumes the following form (Eq. 4).

$$\frac{d^2P(x,t)}{dtdx} = \int_{y=x}^{x_{\max}} S(y)\frac{dB(x,y)}{dx}\frac{dP(y,t)}{dy}dy - S(x)\frac{dP(x,t)}{dx} \tag{4}$$

Assuming that the comminution process in the mill runs in the batch system and we deal with grains discretisation and $0 \le x \le y \le x_{max}$, the differential equation (Eq. 4) for consecutive size ranges and constant time cycles assumes the form (Eq. 5):

$$\text{Comminution index} = W\frac{dP(x_2,t)}{dt} = W\int_{y=x_2}^{x_1} S(y)B(x_2,y)\frac{dP(y,t)}{dy}dy \tag{5}$$

A consequence of notation (Eq. 5) is a simpler form of the equation, expressed in the following way (Reid, 1965):

$$\frac{dw_i}{dt} = -S_i w_i(t) + \sum_{\substack{j=1 \\ i>1}}^{i-1} S_j b_{i,j} w_j(t) \tag{6}$$

Assuming constancy of size ranges, Equation 6 may be presented in the following form:

$$\frac{dw_i(t)}{dt} = -S_1 w_1(t) \tag{7}$$

For product *i* (of the biggest sizes) we obtain a simpler form of the notation in a form of Equation 8:

$$w_1(t) = w_1(0)\exp(-S_1 t) \tag{8}$$

The difficulty in using Equations 5 and Equation 6 is connected with the need for experimental obtaining a high number of parameters occurring in them, i.e. carrying out a considerable number of screen analyses, so that experimental results would enable to obtain satisfactory results encumbered with low values of statistical errors. Therefore, such equations are searched which would satisfactorily, in statistical inference respect, allow to determine values S_i and $B_{i,j}$. $B_{i,j}$. is the mass fraction of broken products from the primary breakage of particle size *j* to *j* + d*j* which falls below particle size *i*. Breakage parameters function, depends directly on the process parameters. It is closely connected with the conditions in which the comminution process is carried out and may depend, inter alia, on the size of grinding media, rotational frequency of the mill, etc. The specific rate of comminution S_i, and actually its inverse, determines the disappearance time of size fraction *i*. These two parameters express how intense the comminution process is.

Of many submitted forms (Kapur, 1972) proposed the following equations circumscribing the specific rate of milling S_i (Eq. 9) and distribution parameter $B_{i,j}$ (Eq. 10).

$$S(y) = ay^{\alpha} \tag{9}$$

$$B(x,y) = \left(\frac{x}{y}\right)^{\beta} \tag{10}$$

For Equations 9 and 10 it is assumed that sampling and screen analysis are conducted with the accuracy enabling to determine the size of a single grain. Therefore, statistical verification is burdened with a considerable error. A partial solution of this problem was submitted by Austin and Bhatia (1973). They presented the following form of the function circumscribing the fraction parameter in the form of Equation 11.

$$B(x,y) = \phi(x/y)^{\gamma} + (1-\phi)(x/y)^{\beta}, \quad 0 \le x \le y \tag{11}$$

Although the presented equation was supported by experimental studies, their results were not encouraging from the practical point of view (Austin, 1999). The function circumscribing the grain composition, using Equation 11, is very sensitive to a change in process parameters. The longer the milling time, the greater this change. Additionally, milling products may change their shape while passing to lower size classes (Erasmus, 1994), which affects the accuracy of screen analyses. Furthermore, we should consider a change in the grains strength during the change in their characteristic dimension (Austin, 2004). At the same time we should take into account the type of impact of grinding media on the comminuted grains (Olejnik 2011). The complexity of comminution mechanisms resulting from the morphology of grains and its change during milling forces us to search for equations which include the multiple nature of the process and at the same time allow for a relatively precise description of the phenomena occurring during the milling.

2. OBJECTIVE

The study is aimed at an analysis of the comminution process of quartzite milled in a ball mill with a batch system. The results of the studies were to determine the impact of the change in selected process parameters (variable numerical and mass compositions of grinding media) on the form of the equation circumscribing the specific rate of milling S_i and the breakage rate function $b_{i,j}$ of the comminuted raw material.

3. PROCESS-APPARATUS PARAMETERS

Subjected to comminution in the ball mill was the Cambrian sedimentary quartzite from the Świętokrzyskie Mountains. In 95% the sedimentary quartzite consisted of closely adherent grains bound by silica. The choice of quartzite was justified by carrying out the milling on the material of uniform crystallographic structure. In this way some attempts were made to decrease as much as possible the effects of the grain composition change on the results of experiments. The basic data relating to the bulk density of the bed (fed material) are presented in Table 1. The bulk density was obtained after 10 minutes of shaking the measured sample. The material used for milling had the grain size of 5÷8 mm. Filling the mill with grinding media and the bed constituted 30 % of the total capacity of the mill.

The comminution was conducted in a dry process. The milling kinetics tests were carried out in a semi technical mill. The internal diameter of the mill's chamber reached 0.5 m, whereas its total capacity 0.112 m^3. The mill's rotational frequency was constant and amounted to 0.517 s^{-1}, which constituted 54 % of its critical rotational frequency.

Table 1. Bulk densities of quartzite, size fraction 5 – 8 mm

Bulk density, kg/m^3	Bulk density after thickening, kg/m^3	Average density of the bed, kg/m^3	Mass of the bed, kg
1236	1298	1267	45

Table 2. Numerical composition and mass of balls for series I, II, III and IV

Series	I	II	III	IV
Ball's diameter, mm	Ball's mass, kg / Number of contact points			
10	-	0	0	-
20	-	12.3 / 11176	12.5 / 11363	1/909
30	-	12.3 / 2035	12.5 / 2068	1/165
40	-	10/671	11/734	11/734
60	5/64	-	-	-
Sum	5/64	40.6 / 50611	1/352	3/986

The milling process was carried out periodically, using several sets of balls. The total mass of the balls used for milling was about 41 kg. The bed was sampled every 30 minutes, by collecting the mass of about 0.6 kg for analyses. The milling was conducted with four sets of balls numbered consecutively as I, II, III and IV. The sizes and masses of balls for respective measurement series are presented in Table 2. Additionally, a statistical number of contact points was estimated for each set of balls (Mort 2003).

4. ANALYSIS OF RESULTS

The samples were subjected to granulometric, morphological and strength analysis. The chemical composition, determined by the number of atoms of respective elements within the chemical compounds, as well as the morphology of grains were examined using the scanning microscope FEI QUANTA 200F. The analysis of the shape of grains and granulometric composition was carried out with the analyser AWK 3D made by Kamika Instruments. Furthermore, a classical screen analysis was carried out to exclude the measurement error connected with the applied research method. The results of analyses allowed to determine the granulometric composition of the milled material in particular moments of comminution. The grains' shape was determined using the classification according to Zingg (1935).

Figure 1 presents, in a logarithmic scale, recalculated numbers of atoms of respective elements forming chemical compounds of which quartzite grains are constructed. The values presented in Figure 1 were obtained using the device EDF X-MAX 50 made by OFURD INSTURUMENTS. The analysis of chemical composition was carried out on selected grains the characteristic sizes of which were within the following ranges: 0.2÷0.3, 0.4÷0.5, 0.8÷1.0, 1.25÷1.4 and 1.6÷2 mm. In the measurement method, the chemical compounds were recalculated in relation to the fractions of respective chemical elements.

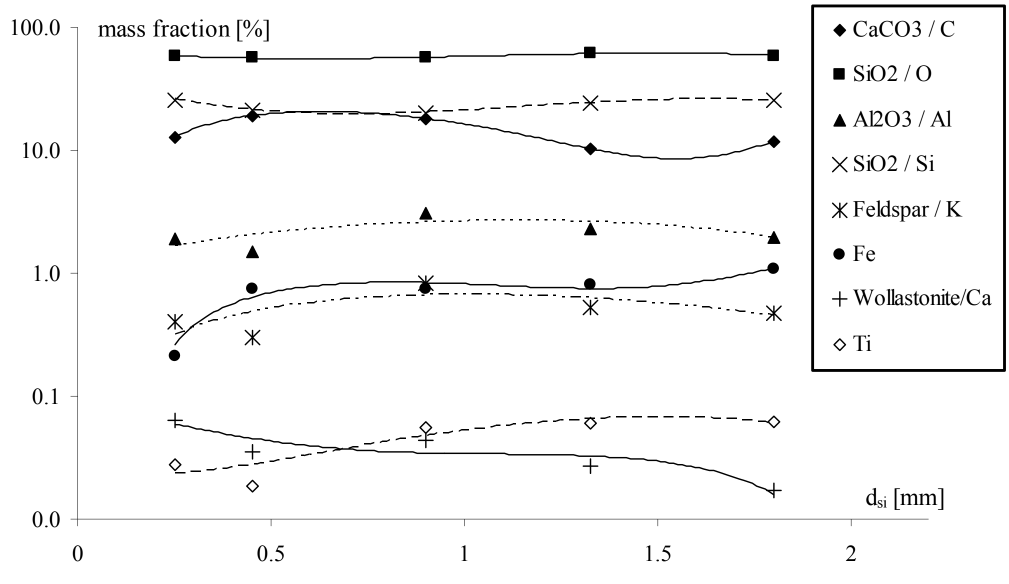


Fig. 1. The list of the number of atoms in the elements occurring in chemical compounds for various sizes of an average grain

The content of $CaCO_3$ was recalculated to the number of carbon atoms. On the basis of SiO_2 the number of oxygen atoms was calculated, whereas the content of Al_2O_3 allowed to determine the number of aluminium atoms. Potassium atoms were recalculated according to the content of feldspar. Additionally, the content of atoms of iron, titanium and calcium was determined.

With the analysis of chemical composition, the optical analysis of grains was carried out. A scanning microscope was used for this purpose. Selected grains from chosen size classes were subjected to strength tests. Single grains were subjected to compressive stresses. Crushing tests were made using INSTRON Series 3300. To standarise the results, for every size class of grains some multiple tests were carried out to exclude the measurement error. The results of strength analyses: the values of compressive destructive forces and stresses induced by them are presented in Figure 2.

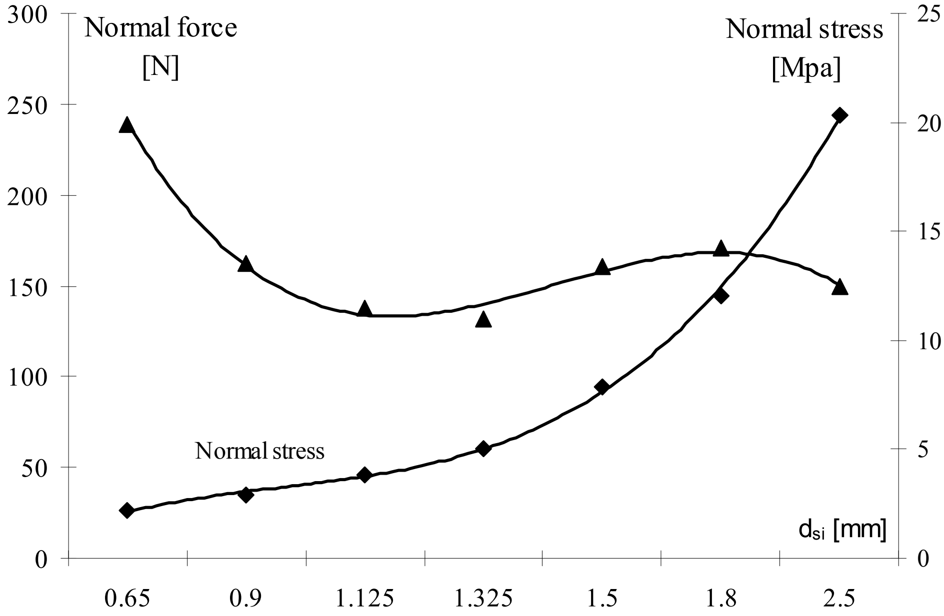


Fig. 2. Values of normal destructive forces F_N (Eq.15) and stresses σ_N (Eq.16) induced by them, for tested size ranges of grains

According to changes in granulometric composition, the comminution rates of respective size fractions were calculated, using our own computer programme. A modified Equation 6 was used for calculations, for discrete values of fractions, assuming ideal mixing of the milled material.

$$\frac{\Delta w_i(t)}{\Delta t} = -S_i w_i(t) + \sum_{j=1,i>1}^{i-1} S_j b_{i,j,t} \cdot w_j(t) \tag{12}$$

The following function was selected as an equation circumscribing the milling rate of respective size fractions:

$$S(i) = K_S \cdot d_S(i)^{ns} \cdot e^{-bs\cdot ds(i)} \tag{13}$$

Besides, the form of Equation 14 was assumed, circumscribing the dependence of coefficient $b_{i,j}$ on the average size of the grain of a given fraction:

$$b_{i,j} = \phi\left(\frac{d_i}{d_j}\right)^{\gamma} + (1-\phi)\left(\frac{d_i}{d_j}\right)^{\beta} \tag{14}$$

Table 3 presents the values of coefficients and constants occurring in Equation 13, as well as coefficients φ, γ and β occurring in Equation 14.

The strength tests of quartzite grains allowed for a mathematical formulation of the correlation between the average size of the grain d_{si} and the destructive normal load F_N and induced stresses state σ_N. The determined values were correlated by Equations 15 and Equation 16.

$$F_N(d_{si}) = -0.2797(d_{si})^3 + 3.869(d_{si})^2 - 16.246(d_{si}) + 32.641 \tag{15}$$

$$\sigma_N(d_{si}) = 1.6178(d_{si})^3 - 11.208(d_{si})^2 + 33.645(d_{si}) + 1.3186 \tag{16}$$

Noteworthy are very high correlation coefficients of Equations 15 and Equation 16, amounting to 0.988 and 0.999 respectively.

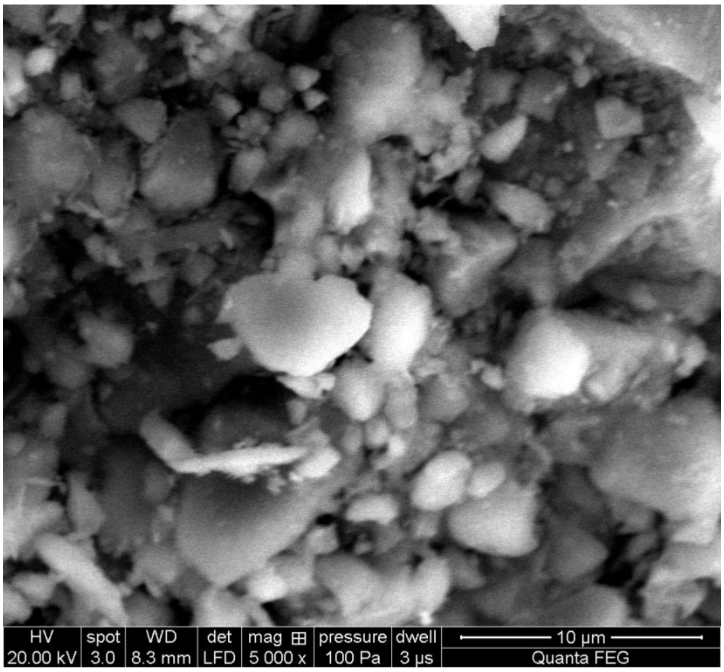
Table 3. The list of constants and coefficients occurring in Equations 13 and 14

Series	γ	β	ϕ	$K_S \cdot 10^8$	b_s	n_s	R^2
I	4.934	0.2848	0.9567	0.904	2.79	1.16	0.90
II	0.1118	4.246	0.6664	0.00249	-0.483	-3.41	0,98
III	4.627	4.635	0.7198	1.15	3.87	1.8	0,99
IV	4.284	3.2471	0.6924	1.005	3.512	1.92	0.91

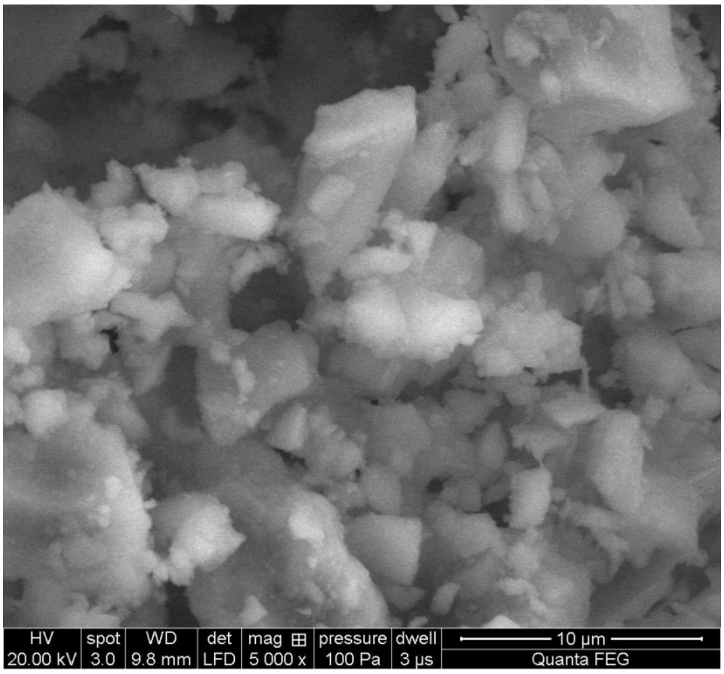
To include the impact of process parameters on the form of the breakage rate function $b_{i,j}$, a multifaceted discussion of results was carried out, analysing apart from grains morphology also their mechanical and chemical properties. As indicated in the conclusions to the studies carried out by Austin (1999) and Stamboliadis (2002), the values of parameters of function $b_{j,j}$ are affected by process parameters. Their impact is highly diversified and very often dependent on the method of the milling process, i.e. a considerable prolongation of milling introduces a greater dispersion in $b_{i,j}$ function statistics.

The results of chemical compositions analysis point to the constancy of the weight fractions of respective chemical compounds, despite a change in the grain's average size. The grains contain approximate weight fractions of hard silicon compounds, resistant to compressive stress. At the same time, the weight fraction of soft limestone binder does not change with a change in the grain's average size. The photographic analysis and analysis of chemical composition were accompanied by strength tests consisting in determination of the greatest normal forces F_N (applied to the grain's surface) at which the grain was destroyed. Furthermore, the destructive stressσ_N induced by normal forces was calculated.

We should emphasise that the results of studies indicate a change in resistance to normal destructive forces and stress they induce (Fig. 2). Grains of smaller sizes sustain less significant destructive stress. The weight fractions of chemical compounds, constituting the structural material of quartzite grains, are not changed with a change in the grain size. Therefore, a change in strength is caused by a weakened internal structure of the grain. The longer the stay in the mill a smaller average size of the grain), the higher the number of micro cracks facilitating the division into smaller particles. On the basis of an analysis of the photographs of quartzite grain surface, we can conclude that the material's grains milling products have a very similar morphological structure. Photograph 2, made for the product's grains sized from 0.2 to 0.3 mm, shows that their structure is very similar to the grains of the size class from 1.6 to 2.0 mm (Photograph 1). Both photographs show structures sized approx. 3÷5 µm and numerous grains sized less than 2 µm. For grains of the 1÷2.5 mm size range, the values of normal forces F_N amounted to approx. 160 N ± 10 N. Only for the grains sized below 1 mm the normal forces at which the grain was destroyed grew significantly, reaching the value of 240 N for grains sized 0.65 mm (Fig. 2). At the same time, there was a continuous decrease in the values of normal stress σ_N, at which the grain was destroyed, with a simultaneous decrease in the average size of the grain. However, for bigger sizes of grains, change in the values of destructive stress occurred faster than for the smallest grains. Within the grain size range above 1 mm there is a positive correlation between the change in normal forces F_N and the induced stress state σ_N. For grains sized below 1 mm the mentioned dependence was correlated negatively, i.e. the decreasing stresses σ_N were accompanied by increased force F_N, inducing this stress state. Additionally, the analysis of chemical composition indicates (Fig.1) almost constant fractions of respective chemical elements for grains from the whole examined measurement range. Therefore, a significant impact of quartzite grains chemical structure on the values of the breakage rate function $b_{i,j}$ parameters should be excluded.



Photograph 1. Quartzite grain, size class 1.6 – 2.0



Photograph 2. Quartzite grain, size class 0.2 – 0.3

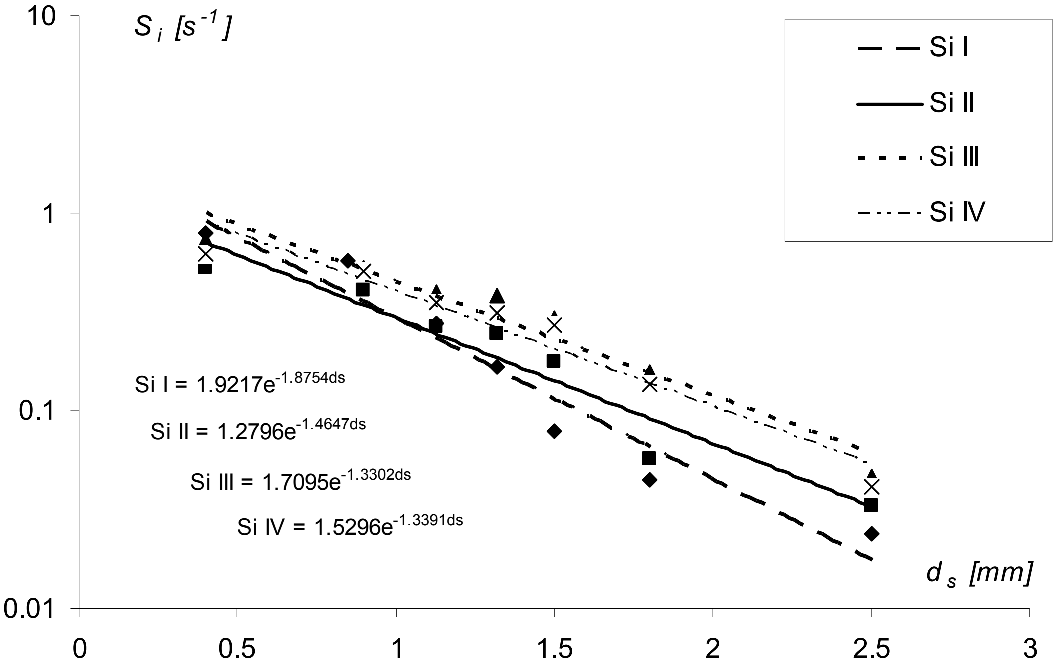


Fig. 3. Change in the value of parameter Si and correlations functions for four measurement series;
◆ – series I, ■ – series II, ▲ – series III, , **x** – series IV

The collected results are presented in Figure 2. In view of the correctness of statistical inference, the studies were carried out many times and the obtained results were subjected to a statistical analysis to determine the average standard deviation and variation coefficient. For all the samples the R^2 was above 0.88.

A more comprehensive picture of the phenomena occurring during milling was obtained by analysing the values of the breakage rate parameter (breakage rate function) $b_{i,j}$. Tables 4 - 6 present the values of the breakage rate parameter for the composition of balls I, II and III. A further analysis of the results skipped the values of the breakage rate function obtained for series IV, due to approximate values of milling rate S*i*, obtained during the milling for the composition of balls I (Fig. 3).

Table 4. List of values of the breakage rate function for the composition of balls I

Breakage rate function $b_{i,j}$, %									
i	*d*	*j* =							
		1	2	3	4	5	6	7	8
1	3	0.00							
2	2	50.19	0.00						
3	1.6	16.02	41.86	0.00					
4	1.4	10.83	27.65	51.73	0.00				
5	1.25	6.31	15.14	28.03	61.56	0.00			
6	1	5.37	8.72	14.77	31.41	89.55	0.00		
7	0.8	11.28	6.62	5.48	7.03	10.45	100.0	0.00	
8	0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table 5. List of values of the breakage rate function for the composition of balls II

Breakage rate function $b_{i,j}$, %									
i	*d*	*j* =							
		1	2	3	4	5	6	7	8
1	3	0.00							
2	2	14.60	0.00						
3	1.6	5.13	17.50	0.00					
4	1.4	4.04	12.04	24.87	0.00				
5	1.25	3.25	7.37	14.18	27.16	0.00			
6	1	6.90	8.02	10.94	17.69	37.16	0.00		
7	0.8	66.08	55.07	50.01	55.15	62.84	100.0	0.00	
8	0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table 6. List of values of the breakage rate function for the composition of balls III

Breakage rate function $b_{i,j}$, %									
i	*d*	*j* =							
		1	2	3	4	5	6	7	8
1	3	0.00							
2	2	57.00	0.00						
3	1.6	18.79	43.69	0.00					
4	1.4	12.86	29.91	53.12	0.00				
5	1.25	7.31	17.00	30.20	64.41	0.00			
6	1	3.95	9.18	16.30	34.76	97.66	0.00		
7	0.8	0.09	0.22	0.39	0.83	2.34	100.0	0.00	
8	0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

The highest values of comminution rate S_i were obtained throughout the whole size range of the grains for series II. For the mentioned composition of balls a very high value of the breakage rate function $b_{i,j}$ was obtained for the grains from approx. 0.8 mm size range. The calculation value for $b_{7,1}$ amounts to 66.08 % and is maintained at a high level - above 50% for consecutive size classes *j*. The obtained value shows a significant mass feeding of grain fractions of this particular size by the milling products from consecutive size classes *j*. The values of the breakage rate function for the smallest size classes were obtained at a highly diversified composition of the balls. We should notice that the milling conducted for series II obtained the most advantageous breakage rate parameters, despite the use of the lowest number of balls of big diameters. Therefore, we should surmise that the decisive mechanism which determined the milling process was not the impact effect of grinding media but the frictional effect. Series II was characterised by the highest theoretical number of contact points (Tab. 2), at a relatively low mass of grinding media.

For series II (Tab. 5), a decisive mechanism of comminution was the frictional effect between the grinding media and raw material's grains. We should also pay attention to an increase in normal destructive forces (Fig. 2) for the smallest size fractions of grains, which would confirm the thesis that for such grains a decisive part during comminution is played by the frictional effects. The obtained correlation between normal destructive forces and grain sizes accounts for high values of comminution rates during the milling accomplished by means of the composition of balls I (Tab.4). Admittedly for this series, lower values of the breakage rate function $b_{i,j}$ were obtained as compared to series II, but for the size class $i=2$ and $i=3$, the value of the breakage rate function $b_{i,j}$ reached respectively: $b_{2,1} = 50.19$ and $b_{3,1} = 16.02$. The obtained values for classes 2 and 3 indicate they were fed with mesh fraction from class 1. Therefore we can conclude that for the biggest grinding media and resultant least number of contact points, the basic mechanism of comminution is their impact effect. This fact is important for quartzite because of an increased normal force which is conditioning the destruction of grains from the smallest size classes (Fig. 2).

Series III was characterised by the highest values of the breakage rate function $b_{i,j}$ for indirect values i (Tab.6). Values $b_{2,1}$ and $b_{3,1}$ are for this series respectively higher, as compared to the measurement series I and amount to 57.00 and 18.79 respectively, although for the grains of class i=7 the breakage rate function values are by one order of magnitude lower, as compared to series I and II. Therefore we should surmise that for series III the impact mechanisms account mainly for the formation of grains the size of which enables passing by three or four size classes downwards. However for the smallest grains the effects of frictional forces are insufficient to feed the smallest size fractions with numerous grains.

A change in the value of the breakage rate function for the discussed process parameters may lead to searching for the most optimum conditions of milling, assuming as the criterion obtaining the highest rates of milling S_i. We may obtain appropriate values S_i through an appropriate selection of grinding media's composition. However, the obtained results should be confirmed for other raw materials which exhibit less uniform morphology.

5. CONCLUSIONS

Taking into account the material presented in the study, certain general conclusions may be drawn:

- A change in the number of grinding media's contact points significantly affects a change in the value of the breakage rate function $b_{i,j}$.
- An appropriate selection of the size of grinding media causes a significant improvement of the conditions of quartzite milling.
- It is possible to obtain approximate values of specific rate S_i of quartzite grain comminution for diversified compositions of balls.

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SYMBOLS

$b_{i,j}$	mass of the fraction of the comminuted raw material from the initial fraction of index j, which appears in the mass of the raw material of index i; ($b_{i,j} = B_{i,j} - B_{i+1,j}$), kg
$B(x, y)$	mass of the comminuted product's fraction from the initial fraction of milling of grains sized from y to y + dy, which in result of comminution passed to the size fraction ranging from 0 to x, kg
bs	parameter of correlation Equation (13), -
C	constant in Equation (1) higher than 0, -
dE_x	the incremental specific energy required to reduce by dx, the size of particle of size x, J/kg
dP	defined by Equation (3), kg
d_s	mean particle size, m

F_N	destructive normal load, N
K_s	parameter of correlation Equation (13), -
m	constants in Equation (1) higher than 0, -
ns	parameter of correlation Equation (13), -
$P(x, t)$	mass of grains fraction in the mill, bigger or equal to grains sized x, after the comminution time t; kg
S_i	specific rate of comminution of the material of index i, s^{-1}
$S(y)$	specific rate of comminution of grains sized from y to y+dy; s^{-1}
t	time of grinding, s
W	mass of the milled material, kg
W_0	the total mass of the milled material, kg
$w_i(t)$	mass of the raw material's fraction of index i, -
x	grain size, m
x_1, x_2	are the upper and lower size fractions of range 1, kg
y	module of the size of comminuted raw material (max. size for the GGS distribution), m

Greek symbols

α	module of the distribution of comminuted raw material, -
β	experimentally determined coefficients in Equation (9 and 10), -
γ	experimentally determined coefficient in Equation (14), -
φ	experimentally determined coefficients in Equation (11) and (14), from approximation in the region where the size distribution in the interval is approximately linear, -
σ_N	breaking stress, MPa

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