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Defining of the structural robustness

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Abstract. Different methods of quantitative assessment of structural robustness has been proposed and widely discussed in recent years. This paper starts out with an outline of the requirements and discussion of the conventional and risk based methods and measures of structural robustness. The probability and consequence analysis related to the assessment of robustness usually contains the statistical, fuzzy and fuzzy-statistical information on the basic variables and parameters. The new fuzzy-probabilistic index of robustness is presented in order to consider all types of available information about different hazards and consequences which influence robustness of a structure. The proposed framework for imprecise risk assessment by means of the frequency-consequences acceptance diagram and quantification of the robustness is illustrated through a numerical example.

Key words: structural robustness, risk assessment, fuzzy information, tolerable risk.

1. Introduction

Robustness can have many various meanings in different fields of science and technology including mathematical modeling, software development, statistical or probabilistic investigation, interpretation, designing and assessing of systems, products and procedures. Generally, robustness is the property of a considered system which enables it to survive unforeseen or extraordinary exposures or circumstances that would otherwise cause them to fail or to loss of function.

Two remarks taken from the mathematical research project "Robust mathematical modeling" show limitation of the current approaches to analysis and assessment of robustness [1]:

- There is no such thing, in a real life, as a precise problem; the objectives are usually uncertain, the laws are vague and data are missing. If you take a general problem and make it precise, you always make it precise in a wrong way.
- If you bring a precise answer, it seems to indicate that the problem was exactly this one, which is not the case. The precision of the answer is a wrong indication of the precision of the question. There is now a dishonest dissimulation of the true nature of the problem.

The paper focuses on the presentation of the quantitative assessment methods of the structural robustness. It starts out with outline of definitions and requirements related to robustness of structures which are subjected to accidental actions, unexpected events, deterioration, design and construction errors, etc. Conventional deterministic, probability based and risk based measures of structural robustness are then presented and discussed. As a risk assessment in cases where the available knowledge about hazards and consequences of damage and failure is imprecise, uncertain or vague a new measure for structural robustness based on the concept of the fuzzy probabilistic assessment of a risk is proposed. The frequency-consequences diagram for acceptance of an imprecise risk is also presented. The proposed framework for the risk assessment and quantification of the robustness is illustrated through an exemplification.

2. Definitions of the robustness

There are many definitions of robustness and none of them is universally accepted. Most of them are related to insensitivity of process or system to external disturbances. Generally, being robust means that a system can handle variability and remain effective. Typically the general scientific interpretation of robustness can broadly be defined as the manner in which systems is affected by hazardous or extreme events, varying procedures or circumstances. Moreover, it is important to note that many of the input parameters necessary for a robustness assessment contain uncertainties which also need to be taken into account during analysis. In order to measure and rank the degree of robustness of a specific system, certain elements must first be clarified [2]:

1. The system must be clearly defined.
2. The intended functions/objectives of a system have to be identified.
3. The perturbations (e.g. hazards, endogenous an exogenous circumstances, deviations from design assumptions, etc.) which affect a system must be identified.
4. The overall consequences of individual perturbations have to be analyzed with regard to the mentioned before functions or objectives.

Several selected definitions of robustness from engineering and control theory could be listed as follows [3]:

- the ability of a system to maintain function even with changes in internal structure or external environment;
- the degree to which a system is insensitive to effects that are not considered in the design;
- insensitivity against small deviations in the assumptions;

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- the ability of a system to react appropriately to abnormal circumstances;
- the consequences of structural failure are not disproportional to the effect causing the failure;
- a robust solution is an optimization problem; is one which has the best performance under its worst case (max-min rule).

All the aforementioned interpretations of robustness and its assessment can be applied within engineering but an explicit definition specific to building structures is still lacking. For the purposes of clarification, robustness as a property within structural engineering systems will be referred to as structural robustness.

Robustness of structural systems is as yet not explicitly defined nor is there a clearly defined method for incorporating robustness in design/construction. It is often claimed that low scatter in performance reflects high robustness and vice-versa. For example Taguchi's definition of a robust design is: "a product whose performance is minimally sensitive to factors causing variability (at the lowest possible cost)" [4]. In fact variability reflects rather quality, not robustness. Similarly, complexity not only allows us to establish a new and useful in engineering practice definition of robustness, but it also makes it possible to actually measure it, providing a single number which reflects "the global state of health" of a system. Moreover, the following dilemmas exist: accurate models necessitate complex analysis while conversely, simple models lack applicability to the useful definition of robustness. Sometimes the structural robustness is simply defined as the ability of a structural system to survive unforeseen/extraordinary exposures or circumstances that would otherwise cause it to fail. The structure must have enough residual capacity during and after the event to maintain at least some of its intended function intact.

Many of modern building codes specify that the consequences of structural failure should not be disproportional to the effect causing the failure. The level of robustness of a structure should be analyzed in terms of the causes and consequences of failure; i.e. the consequences of structural damages should not be disproportional to the original cause. The Eurocode EN 1990 [5] provides the general principles for achieving structural robustness:

"(3)P In the case of fire, the structural resistance shall be adequate for the required period of time.

(4)P A structure shall be designed and executed in such a way that it will not be damaged by events such as: explosion, impact, and the consequences of human errors, to an extent disproportional to the original cause."

The Eurocode EN 1991-1-2 [6] deals with fire and the EN 1991-1-7 [7] deals mainly with impact and gas explosion. However, Eurocodes do not provide the explicit definition of robustness. It should be mentioned that there is a difference between robustness of the structure and its resistance to accidental loads. The partial safety method, used in to date design codes, defines reliability and safety with reference to each element, disregarding the global behavior of the structure and the possibility of progressive collapse. As a result, it is necessary to get a better understanding of the structural behavior following localized failure and resistance to accidental loads. The correctly designed and executed structure usually includes significant robustness and is able to sustain unexpected accidental loads, but the structure designed to sustain definite accidental load may not be enough robust.

3. Robustness requirements

Basic requirements related to construction works (for example these given in European Construction Product Directive) state that they shall be designed and built in such a way that the loadings acting during their construction and use will not lead to:

- collapse of the whole or part of the structure;
- deformations to and inadmissible degree;
- damage to other parts or installed equipment as result of deformation of the structure;
- damage by events to an extent disproportionate to the original cause.

The last requirement is directly related to robustness of a structure. In practice, structures may have also additional stakeholders requirements related to people affected by its construction, use and failure as well as to property affected by economic, social, environmental and business consequences. These requirements are usually considered during planning, designing, construction and permission applications and they are related to the professionals, that is to the designer, builder and manager who should take into account the following:

- Principles and measures necessary for design and evaluation of structural robustness, such as:
  - identification and specification of accidental design situations and actions;
  - verification of overall stability and stiffness of a structure;
  - verification of vulnerability of structural elements and details;
  - indirect design of alternative load paths by means of internal and perimeter, horizontal and vertical ties.
- Direct and indirect consequences due to insufficient structural robustness, such as:
  - consequences related to human safety, such as fatalities, injuries, psychological harms;
  - economic consequences including damage to the structure and surrounding properties, damage to content, loss of income, loss of customers, etc.;
  - ecological consequences, such as environmental damage and effect on wildlife;
  - social and political consequences including increase of public fears, loss of reputation, loss of political support.
- Basic principles of risk identification and assessment, such as:

– identification of considered structural system, among other things: types of structural elements (brittle or ductile), types of system (series, parallel or hybrid, with correlated or uncorrelated elements);
– identification of hazards and hazard scenarios that are the abnormal conditions which are assumed to occur during construction and lifetime of a structure;
– analysis of the conditional probabilities for local damage and global failure, as well as probabilities of hazardous events;
– quantitative analysis of direct and indirect consequences of damage or failure;
– criteria for acceptable risk and risk treatment;
– risk optimization (if considered) including identification of objective function, restrictions and risk based criteria of optimization.

4. Conventional quantitative measures of structural robustness

4.1. Probability based and deterministic measures. One of the first proposals towards a quantitative assessment of structural robustness presented by Frangopol and Curly [8] defines robustness in terms of the reliability index β:

$$\beta_r = \frac{\beta_i}{\beta_i - \beta_d}, \tag{1}$$

where β_r is the redundancy index, β_i is the reliability index of the intact structure and β_d is the reliability index of the damaged structure. Theoretically the value of β_r – index varies from zero to infinity.

Vulnerability index V indicating the increase in the probability of failure resulting from structural damage has been proposed as the measure of structural robustness by Lind [9]:

$$V = \frac{P(r_d, S)}{P(r_0, S)}, \tag{2}$$

where $P(\ )$ represents the probability of failure, r_0 is the resistance of the intact structure, r_d is the resistance of the damaged structure and S is the effect of actions.

A simple measure of structural robustness and redundancy defined in the ISO Standard 19902 [10] and used in the offshore industry, is a reserve strength ratio (RSR):

$$RSR = \frac{R_c}{S_c}, \tag{3}$$

where R_c is the characteristic value of the base shear capacity of an offshore platform and S_c is the design load corresponding to ultimate collapse.

Many other simple deterministic measures of robustness have been proposed and can be proposed, for example on the determinant of stiffness matrix of an intact structure and a structure without removal elements, degree of redundancy, etc.

4.2. Risk based measures. A definition of a robustness based on risk measures has been suggested by Ellingwood [11] and then a framework for quantitative assessment of system robustness based on risk analysis has been presented by Baker et al. [12]. They proposed the following index of robustness (I_{Rob}):

$$I_{Rob} = \frac{\sum\limits_i R_{Dir_i}}{\sum\limits_i R_{Dir_i} + \sum\limits_i R_{Ind_i}}, \tag{4}$$

which measures the fraction of total system risk resulting from direct consequences of system's damage and where R_{Dir_i} is the direct risk associated with the initial damage due to the i-action and R_{Ind_i} is the indirect risk associated with the subsequent system failure due to the i-action. The I_{Rob} – index takes values between zero and one; $I_{Rob} = 1$ if the system is completely robust and there is no risk due to indirect consequences, and $I_{Rob} = 0$ if all risk is due to indirect consequences.

Risk may be referred to as a measure of the danger or hazard that undesired events represents for people, economy and environment, and is defined as a combination (usually a product) of the probability of occurrence and the consequence of a specified hazardous or undesired event [13, 14]. For a set of hazardous design situation H_i the total risk R can be calculated as follows [15]:

$$R = \sum_{i=1}^{n_H} p(H_i) \sum_{j=1}^{n_D} \sum_{k=1}^{n_S} p(D_j \,|H_i) p(S_k \,|D_j) C(S_k), \tag{5}$$

where the structure is subjected to n_H different hazards that may damage the structure inn_D different ways and the performance of the damage structure can be discretised into n_S adverse states S_k with corresponding consequences $C(S_k)$, and $p(H_i)$ is the probability of occurrence of the *i*-th hazard H_i, $p(D_j\,|H_i)$ is the conditional probability of the *j*-th damage state of the structure given in the *i*-th hazard and $p(S_k\,|D_j)$ is the conditional probability of the *k*-th adverse overall structural performance S given in the *i*-th damage state.

The two major categories of hazard involved in the building process can be distinguished on the basis of their nature, namely natural hazards and man-made hazards. Natural hazards resulting from variations of structural materials and products properties, actions applied to a structure, geometrical data, which are modeled as stochastic variables with corresponding statistical distributions and parameters. Man-made hazards include uncertainties due to unintentional (human errors) and intentional (ill will and terrorist attacks) departure from the accepted practice and verified procedures new materials and types of structures, innovations in design and construction, new technologies and methods of execution, new or modified models for structural analysis and dimensioning, fires, explosions and other severe events caused by man who are not involved in the building process but also the pressure on the designers due to the shortage of time, money, the political climate, etc.

Negative consequences are defined as a possible outcome of desired or undesired events that may be expressed quantitatively or qualitatively in terms of personal injury, death, monetary loss, environmental and social damage. Two significant types of consequences, both immediate and arise after a certain time, associated with the exposure to a structural system may be distinguished:

- direct consequences induced by damage to the individual constituents of the system,
- indirect consequences beyond the direct consequences, induced by changes of the system.

The risk R_D associated with direct consequences due to exposure events may be assessed as follows [12, 16, 17]:

$$R_{Dir} = \sum_{k=1}^{n_{EX}} \sum_{l}^{n_{CD}} p(C_l \,|EX_k) c_D(C_l) p(EX_k), \tag{6}$$

where n_{EX} is a number of exposure events, n_{CD} is a number of possible different states of all constituents of the element C_l, $p(C_l \,|EX_k)$ is the conditional probability of the l-th damage state of the element C_l on the exposure event EX_k with probabilistic characterization $p(EX_k)$ and $c_D(C_l)$ is the direct consequence associated with the l-th of n_{CD} possible state of damage of all constituents of the element C_l.

The risk R_{ID} due to all indirect consequences of exposure events may be calculated using the formula:

$$\begin{aligned} R_{Ind} = &\sum_{k=1}^{n_{EX}} \sum_{l=1}^{n_{CD}} \sum_{m=1}^{n_{ST}} c_{ID}(S_m, c_D(C_l)) \\ &\cdot p(S_m \,|C_l \,, EX_k) p(C_l \,|EX_k) \; p(EX_k), \end{aligned} \tag{7}$$

where n_{ST} is a number of possible different structure states S_m associated with indirect consequences $c_{ID}(S_m, c_D(C_l))$ and $p(S_m \,|C_l, EX_k)$ is the conditional probability of indirect consequences on a given state of the constituents C_l and the exposure EX_k.

Successive steps of the structural robustness assessment according to the discussed risk-based approach are shown in Fig. 1.

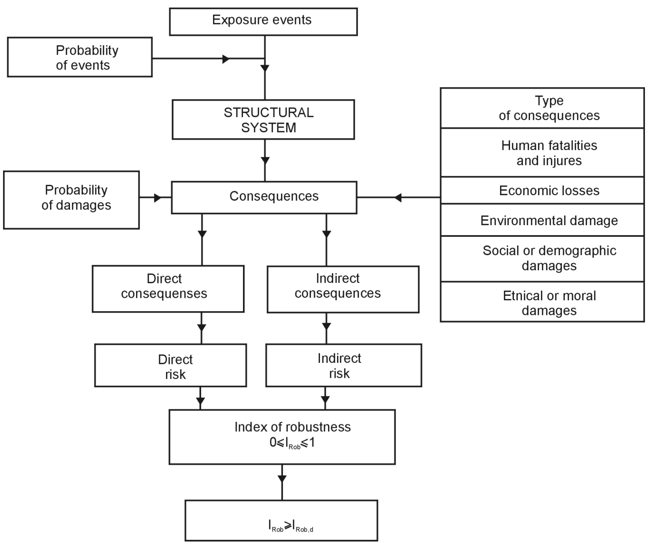

Fig. 1. Stages of the structural robustness assessment according to the probabilistic risk-based approach

5. Fuzzy-probabilistic measure of structural robustness

Information and data necessary for the evaluation of hazards occurrence probabilities of different consequences induced by damage or failure of a structure are usually highly uncertain. Generally, two types of various uncertainties can be observed in the performance of a structure; random and fuzzy. Thus three types of parameters can be used for description and analysis of structural robustness; deterministic, random and fuzzy. Using the concept of fuzzy numbers and fuzzy statistics and a scheme of approximate reasoning the subjective and qualitative information related to input variables, calculation methods, manufacturing processes, professional knowledge and intuition can be taken into account in verification and design for structural robustness.

A fuzzy set notion is a kind of generalization of a crisp set notion, proposed by L.A. Zadeh in 1965 [18]. A fuzzy set F can be described by the membership function $\mu_F(x) : X \Rightarrow [0, 1]$ defined over a universe of discourse X. Another useful notion is a fuzzy number G, described as a fuzzy set of the real line R, where $\mu_G(x) : R \Rightarrow$ [0,1]. Simplified representation of a fuzzy number $G = (m_G, \alpha, \beta)$ is very useful in practical applications of the fuzzy set theory, and its membership function can be described by the mean value m_G, a left-sided *Le* and a right-sided *Re* functions, in the following form:

$$\mu_G(x) = \text{Le}\left(\frac{m_G - x}{\alpha}\right) \quad \text{for} \;\; x \leq m_G, \;\; \alpha > 0, \tag{8}$$

$$\mu_G(x) = \text{Re}\left(\frac{x - m_G}{\beta}\right) \quad \text{for} \;\; x \geq m_G, \;\; \beta > 0, \tag{9}$$

$$\mu_G(x) = 0 \;\; \text{for} \;\; m_G - \alpha > x > m_G + \beta, \tag{10}$$

where α and β are left-sided and right-sided range of a membership function around m_G .

There are several methods which can be used to estimate a membership function of a fuzzy set or fuzzy number using results of fuzzy statistical experiments and appropriate procedures or assuming standard types of membership functions (see Fig. 2), if the initial information concerning a considered variable is available [19]. The determination of membership function can be either manual or automatic when it is "fine-tuned" based on an initial guess using Genetic Algorithm and Artificial Neural Network. The representative arithmetic operations on fuzzy numbers can be formulated on the basis of the extension principle [19, 20].

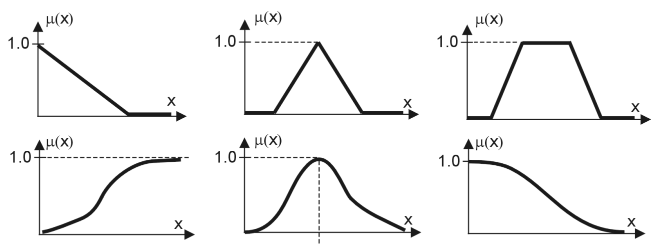

Fig. 2. Commonly used membership functions

Qualitative information or uncertain data can be formally treated by linguistic or fuzzy variables. Values of linguistic variables are named by linguistic terms, and are defined to distinguish them from numerical variables with imprecisely determined values. More or less vague relationships between some fixed numbers of variables, can be formally treated as fuzzy sets and are called fuzzy relations: e.g. "great", "medium", "small". A scheme of approximate reasoning enables the treatment of vague or qualitative information via fuzzy values of variables and via fuzzy relations between their values. L.A. Zadeh has introduced four types of rules for approximate reasoning pertaining to: modification, composition, quantification and qualification, respectively [21].

To evaluate different variables in probabilistic design and assessment procedures of structures, when our knowledge about basic variables and models is scare and imprecise, is often very hard. Our beliefs about the probability of considered events can be expressed as fuzzy numbers. The probability of a fuzzy event A that a continuous random variable X takes values within the set$A \subset R$ can be expressed as follows:

$$P(A) = \int_A f(x)dx = \int_R \mu_A(x_i) f(x)dx, \tag{11}$$

where $f(x)$ is the probability density function of a random variable X. Thus the probability of a fuzzy event according to (5) is a real number taking values in the range [0, 1] and the membership function $\mu_A(x_i) = 1$ when a sample point makes up the event A, $\mu_A(x_i) = [0, 1]$ when a sample point makes up to some degree the event A, and $\mu_A(x_i) = 0$ when a sample point does not make up the event A. There are also other definitions of the probability of fuzzy events [22].

Each measure of the structural safety and robustness can be regarded as a fuzzy-probabilistic because in practice it usually depends on various fuzzy or/and qualitative variables. D. Blockley has defined a safety measure N, which is the negative logarithm of the failure probability $N = -log_{10} P_f$ and he has evaluated the influence of human errors on that safety measure by means of an approximate reasoning procedure [23]. The discussion about applications of fuzzy-probabilistic safety measures in safety analysis and design of building structures can be also found in [24–26].

For example the probability P'_f of occurrence of the i-th hazard H_i in the formula (5), corrected due to quantitative and fuzzy information can be evaluated using the entropy method:

$$\begin{aligned} &\sum_{i=1}^{n} P'_{fi} \ln P'_{fi} = H + K \\ &= -\sum_{i=1}^{n} P_{fi} \ln P_{fi} - \Omega \sum_{i=1}^{n} \mu_{Gi}(x_i) \ln \mu_{Gi}(x_i), \end{aligned} \tag{12}$$

where H, K are the entropy of the quantitative and fuzzy information, P_f is the probability depend on quantitative variables, μ_G is the membership function of fuzzy variables, and Ω is the normalization coefficient. Conditional probabilities necessary for evaluation the direct and indirect risk can be calculated similarly.

The proposed fuzzy index of robustness ($\widetilde{I}_{Rob}$) can defined as follows:

$$\widetilde{I}_{Rob} = \frac{\sum_i \widetilde{R}_{Dir_i}}{\sum_i \widetilde{R}_{Dir_i} + \sum_i \widetilde{R}_{Ind_i}}, \tag{13}$$

where $\widetilde{R}_{Dir_i}$ is the direct fuzzy risk associated with the initial damage due to the i-action and $\widetilde{R}_{Ind_i}$ is the indirect fuzzy risk associated with the subsequent system failure due to the i-action.

6. Tolerable risk

In the predominant opinion of civil engineering researchers, a measure of tolerable risk should be based on human and economic values and expressed in the socio-economic terms. Assuming that the construction and use of building structures is an ordinary risky economic activity, it is essential to determine the reasonable investment into structural safety in order to ensure the societal lifesaving level. Social indicators that express some aspects of the life quality, inter alia: Human Development Index of the United Nations Development Program (HDI), Life Quality Index (LQI), Societal Value of Statistical Life (SVSL), are examples of complex social indicators [27, 28]. They can be used as the quantitative criteria for optimization the tolerable risk in structural engineering. However, they usually incorporate questionable or unreliable data related to the quality of life: cost of averting to fatality, life expectancy, personal well-being, time for rest, healthy ecological environment, cultural heritage, etc.

Generally, methods of risk acceptance can be divided into two categories:

- implicit methods of the comparative character which make use of quantitative risk criteria from similar structures and scenarios for other cases and sectors of industry;
- explicit methods, based on direct evaluation of risk acceptance.

The frequency-consequence diagrams (F-C diagrams) are commonly used to express the risk in terms of probability and consequences of undesired events. In the ISO Standard 2394 [29] relative costs of safety measures and consequences of failure have been defined by means of linguistic variables: high, moderate or low costs and small, some, medium or great consequences. These variables may be defined as fuzzy numbers with standard membership functions, for instance triangle $\mu_X = (m_X, \alpha_X, \beta_X)$. In Fig. 3 the F-C diagram corresponding to the target reliability levels recommended in EN 1990 [5] and proposed fuzzy measures of failure consequences is presented.

Linguistic variables can be used to express qualitative information and subjective opinions about variables, models and uncertainties which are taken into account in the evaluation of tolerable risk. Each qualitative opinion L can be described by two linguistic variables L_1 and L_2 which express the opinion in size and in weight, respectively. Using the Mamdani approach [20], first the influence of each qualitative opinion

L_i and next the influence of all opinions on allowable risk can be determined in the form of fuzzy partial relations F_i (the fuzzy Cartesian products of fuzzy sets L_{i1} and L_{i2}) and the complex fuzzy relation F (the union of fuzzy sets F_i) as follows:

$$F_i = L_{i1} \cap L_{i2}, \quad \mu_{Fi}(x_j, x_k) = \min[\mu_{Li1}(x_j), \mu_{Li2}(x_k)], \tag{14}$$

$$F = \cup F_i, \quad \mu_F(x_j, x_k) = \max\{\min[\mu_{Li1}(x_j, x_k), \mu_{Li2}(x_j, x_k)]\}, \tag{15}$$

where $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, m$, $k = 1, 2, \ldots, r$, $\mu_F(\ldots)$ membership functions of fuzzy linguistic variables, $\cap$ is the intersection and $\cup$ is the union of fuzzy sets. For known fuzzy relationship F composition of each linguistic variable L_i and F can be calculated using the following formula:

$$L'_i = L_i \circ F, \quad \mu_{Li'} = \max\{\min[\mu_{Li}(x_i), \mu_F(x_j, x_k)]\}. \tag{16}$$

The composition which shows the highest level of support, i.e. max$\mu_{Li'}$, can be identified as the measure of the influence of all qualitative opinions L_i , and the membership function of a fuzzy safety measure can be modified and deffuzified. As the general theory determining the deffuzification value of any fuzzy set is actually lacking, several practical procedures are proposed in the literature, e.g.: a single maximum point of a membership function, center-of-gravity procedure, the weighted sum procedure, etc. The most conservative and simple procedure consists in the choice of a safety measure value corresponding with the minimum value of the membership function.

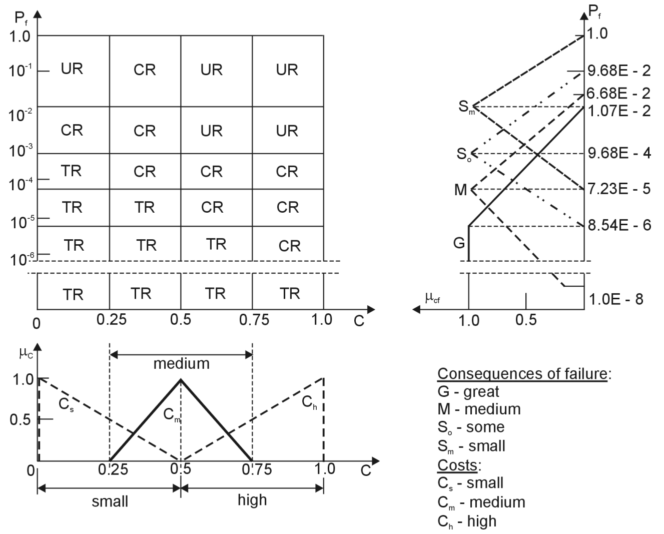

Fig. 3. Frequency–consequence diagram. UR–unacceptable risk, CR–controlled risk, TR–tolerable risk. Membership functions of linguistic variables μ_c and μ_{cf}

7. Exemplification

An existing open-area support structure for warehouse crane (86 m long), consists of $n = 2 \times 15 = 30$ prefabricated reinforced concrete beams simply supported on columns fixed in isolated footings. The structure is aged about 22 years. Due to exposure conditions (i.e. large variation of moisture and temperature as well as freezing and thawing) deterioration such as rebar corrosion, cracking and spalling or delamination of concrete cover and the premature loss of functionality or collapse of beams can occur. These exposure conditions is considered to be an event with the potential to cause damage to the considered structure.

• The preliminary inspection carried out by the author has shown the appearance of moisture and rust stains, few steep cracks and poor quality of concrete. Considered beams have been designed using the limit states method and a partial factors format. The design values load bearing capacity for bending is $M_d = 224$ kNm. Corresponding value of the notional failure probability of one beam is equal to $P_{i,f} = 5 \times 10^{-6}$, $i = 1, 2, ..., n = 30$.

• The structural system consists of 30 beams and forms the series system. For series system with perfectly correlated elements $\rho_{ij} = 1$, the initial probability of failure calculated using the β-index theory is $P'_f = P_{i,f} = 5 \times 10^{-6}$ while for series system with uncorrelated elements $\rho_{ij} = 0$, $P_f = 1 - (1 - P_{i,f})^n = 1,5 \times 10^{-4}$. Elements of the considered series system are assumed to be equally correlated and the value of correlation coefficient was assessed as equal to $\rho_{ij} = 0.60$, and the evaluated probability of failure of the whole system equals $P'_f \approx 2.1 \times 10^{-5}$. As the correlation between resistance of different beams are highly uncertain, the probability P'_f is described by the fuzzy number $\widetilde{P}'_f$ with a membership function $\mu_{P'} = (m_P, \alpha, \beta) \Rightarrow \mu_{P'} = (2.1 \times 10^{-5}, 2.1 \times 10^{-6}, 2.1 \times 10^{-4})$.

• Assuming a simplified formulas based on the Fick's law of diffusion and assuming that the increase in the volume of the crack is equal to the volume of the corrosion products produces when the diameter of the reinforcement bar is reduced, the following random variables: corrosion initiation time t_{cor}, cracking initiation time t_{cr},, spalling initiation time t_{spl} and the time to collapse t_{col} were calculated [30–33]. The Monte Carlo Method was used to perform the probabilistic calculations of probability distribution functions of considered random variables. Results of these calculations are presented in Fig. 4.

As information about considered variables is incomplete and models used in probabilistic calculations are highly uncertain, resulting mean values have been fuzzified taking into account imprecise information and subjective opinions regarding these values and described by fuzzy numbers with membership functions as follows: $\widetilde{t}_{cor} : \mu_{cor} = (17.2, 15.119.3)$, $\widetilde{t}_{crc} : \mu_{crc} = (27.5, 24.2, 30.8)$, $\widetilde{t}_{spl} : \mu_{spl} = (43.6, 38.9, 48.3)$, $\widetilde{t}_{col} : \mu_{col} = (79.3, 70.1, 88.5)$.

• Due to progress of corrosion the initial probability of the system decreases with time as follows: $\widetilde{P}'_f(t_{crc}) : \mu_{P'crc} = (3.1 \times 10^{-5}, 3.1 \times 10^{-6}, 3.1 \times 10^{-4})$, $\widetilde{P}'_f(t_{spl}) : \mu_{P'spl} = (5 \times 10^{-4}, 5 \times 10^{-5}, 5 \times 10^{-3})$, $\widetilde{P}'_f(t_{col}) : \mu_{P'col} = (2 \times 10^{-2}, 2 \times 10^{-3}, 2 \times 10^{-1})$.

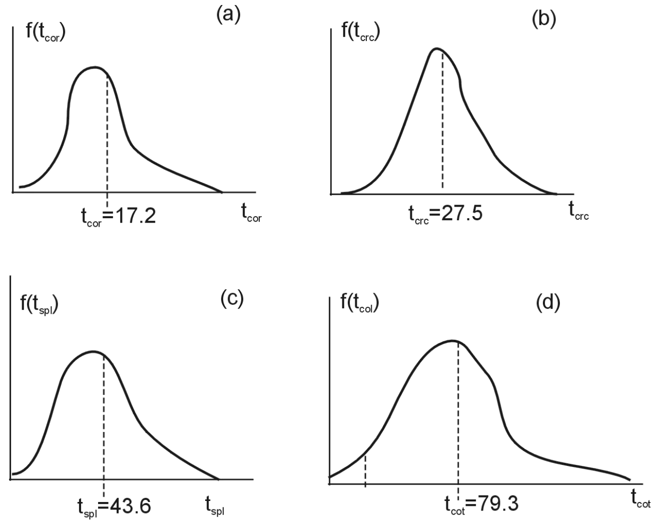

Fig. 4. Probability density functions of random variables t_i (years): (a) corrosion initiation time t_{cor}, (b) cracking initiation time t_{crc}, (c) spalling initiation time t_{spl}, (d) time to collapse t_{col}

• From the F-N diagram shown in Fig. 1. it can be noticed that the risk connected with failure of the structure can change considerably with time. For "high" and "moderate" consequences of failure and $t \geq t_{spl}$ the risk grows to the unacceptable (UN) or critical (CR) level.

• A considered catastrophic event is the collapse of at least one of prefabricated beam due to damages caused by corrosion of reinforcing steel bars and concrete. Direct consequences (c_D) of this event include deconstruction, repair or reconstruction costs (beam, supporting structure and crane), administrative and maintenance costs, etc. Indirect consequences (c_{ID}) include all consequences associated with the failure, beyond the direct consequences which are associated with the collapse, among others: human life loss or injuries costs, users, operational, safety precautions, administrative and planning costs. Both types of cost have been related to costs of replacing the existing structure by a new one and the both include fuzzy uncertainties which can be described by fuzzy numbers: $\widetilde{c}_D : \mu_{CD} = (1, 0, 0.8, 1.2)$ and $\widetilde{c}_{ID} : \mu_{CID} = (1.0, 0.5, 1.5)$.

• Fuzzy indexes of robustness $\widetilde{I}_{Rob}$ calculated for different periods of time, according to the formula (13), are the fuzzy numbers described by membership functions are as follows: $\widetilde{I}_{Rob}(\widetilde{t}_{cor}) : \mu_{IRob} \approx (0.87, 0.79, 0.95)$, $\widetilde{I}_{Rob}(\widetilde{t}_{crc}) : \mu_{IRob} \approx (0.78, 0.69, 0.87)$,

$\widetilde{I}_{Rob}(\widetilde{t}_{spl}) : \mu_{IRob} \approx (0.58, 0.46, 0.70)$ and $\widetilde{I}_{Rob}(\widetilde{t}_{col}) : \mu_{IRob} \approx (0.45, 0.32, 0.58)$.

• The $\widetilde{I}_{Rob}$ – index measures the additional risk to the structure due to indirect consequences of damage or failure. The above results show that the robustness of considered structure and that accuracy of its evaluation decrease significantly with time. In other words, damage tolerance of the structure after about 79 years is two times less than at the beginning and after about 43 years of operation indirect consequences and costs of failure are higher than direct consequences. It means that after 43 years of operation thorough inspection and appropriate repairs should be undertaken.

8. Conclusions

A risk in structural engineering is commonly analyzed and evaluated by means of quantitative criteria for identified possible hazard scenarios, probabilities of the undesired events and estimated costs of damages due to these events. The most reasonable and transparent way to manage risk is to quantify it and use crisp quantified criteria of risk acceptance. Unfortunately, in practical applications data necessary to calculate risk and consequences are generally uncertain, vogue or subjective and scattered over time.

A measure of tolerable risk are usually based on human and economic values and are expressed by linguistic variables. They can be described using somewhat arbitrary yet reasonably estimated membership functions. Qualitative acceptance criteria defined in terms of linguistic variables can be used for the risk analysis and evaluation, for instance in the form of the frequency-consequence diagrams. Using a concept of fuzzy quantities a combined approach to the risk assessment based on the probability and fuzzy methods can be derived.

The new measure and procedure for assessment of the fuzzy structural robustness based on the fuzzy-probabilistic risk analysis has been presented and illustrated with example calculations for a simple structural system. This allows to calculate and to compare robustness of different structures considering quantitative as well as qualitative information about a structural system, exposure events and different types of consequences.

Further investigations will be directed towards computation of robustness for real structures and should be helpful for formulating building codes which will be able to address design procedures for robustness.

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