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DETERMINATION OF CYLINDER LINER FREE VIBRATION FREQUENCIES IN DIESEL MARINE ENGINES

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Abstract

This article presents a method to determine free vibration frequencies of cylinder liners in marine gas engines based on the power method. The cylinders were treated as thin-walled tubes getting distorted because of a long term variable load. The assumptions for the model have been made, mathematical procedure has been shown as well as a description of cylinder free vibration types. The final description of free vibrations was related to the way in which the cylinder was mounted searching for a minimum free vibration frequency.

Key words: *marine engine, cylinder liner, vibrations, frequency of resonance vibrations*

1. Introduction

Determination of free vibrations of cylinder liners is a significant task at the designing stage and an important one at Diesel engine operation. Cylinder vibrations induced as a result of interaction of the impulse force accompanying piston strokes when the crankshaft is passing the Top Dead Center (TDC), are the main cause of cavity formation in the cylinder block cooling areas [4, 5]. It is valid for medium and high speed engines whose cylinders get damaged as a result of cavitation.

2. Simplifying assumptions for the model

The ratio of the thickness of a cylinder of an engine with self-ignition to its radius fulfills the inequality $\delta/R < 0{,}1$, which was the basis for accepting a cylinder liner as a thinned-wall one that is getting distorted under variable load [1]. To determine the frequency of free vibrations, the following assumptions have been taken [6]:

- cylinder material does not have mechanical hysteresis,
- the sum of kinetic and potential energy is constant and energy dissipation does not occur,
- during cylinder vibrations only a continuous process of transition from one form of energy into another takes place,
- cylinder vibrations are generated as a result of interactions of the impulse piston force when the crankshaft is passing the Top Dead Center (TDC),

– action of forces generating cylinder vibrations are asymmetrical as the contact of the piston with the cylinder wall takes place in the plane of the crankshaft movements.

Parameters of liner distortion are influenced by the type of vibration generating forces and the properties of the liner which are determined by the possible types of vibrations and frequency values accompanying them. As a consequence, the searched for vibration type will also be asymmetrical. In such a case, radial displacements are accompanied by those on the circumference tangent to the contour of the cross-section as well as diagonal distortions along the axis of the cylinder. A calculation diagram of the cylinder as well as a thin-walled liner with the discussed asymmetrical load is shown in Figure 1 [6].

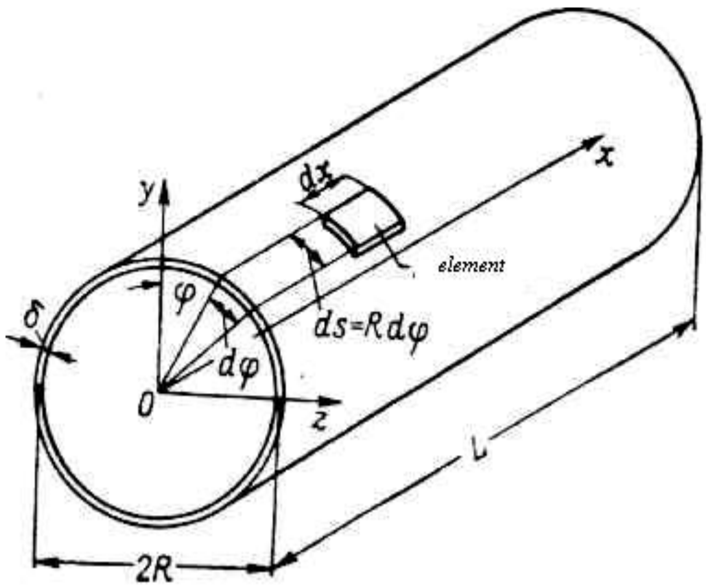

Fig. 1. A calculation diagram of a cylinder as a thin-walled liner

Determination of thin-walled cylinder liner vibration types and their corresponding frequencies was carried out using the power method suggested in paper [1]. A static equilibrium of the system was assumed there, treating mass inertia forces as a load for the external construction.

3. Mathematical description of vibrations

With such assumptions, a single element of the surface gets displaced in radial direction *w*, direction *v* and axial direction *u* . Then, in the central area of the cross-section of the cylinder, relative deformations of surface elements will take place:

along the x axis

$$\varepsilon_x = \frac{\partial u}{\partial x} \tag{1}$$

in the circumferential direction

$$\varepsilon_\varphi = \frac{w}{R} + \frac{1}{R}\frac{\partial v}{\partial \varphi} \tag{2}$$

and displacement deformations of the central area of the cross-section

$$\gamma = \frac{\partial u}{R\partial \varphi} + \frac{\partial v}{\partial x} \tag{3}$$

In relation to the cross-section of the central area of the cross-section, the following hypotheses were applied: on the rigidness of the liner in the circumferential direction and on the lack of displacements which point to $\varepsilon_\varphi = 0$ and $\gamma = 0$ deformations. Such an assumption allows to find a relationship between displacements in *w*, *v* and *u* directions which finally makes it possible to substitute all unknown forces and deformations with one, which significantly simplifies the task.

As a result of surface deformation, a simultaneous change of central area curvature will take place :
in the direction of generatrix

$$\kappa_x = -\frac{\partial^2 w}{\partial x^2} \tag{4}$$

and in the circumferential direction

$$\kappa_\varphi = -\frac{1}{R^2}\left(\frac{\partial^2 w}{\partial \varphi^2} - \frac{\partial v}{\partial \varphi}\right) \tag{5}$$

Radial displacements can be presented as a product of three functions dependent on x, φ coordinates and on time, t:

$$w(x,\varphi,t) = \psi(x)\cos n\varphi \sin \omega t \tag{6}$$

where:

ω – frequency of liner free vibrations,

$n = 2, 3, 4, \ldots$ – natural numbers characterizing the number of half-periods in the cross-section.

To determine the unknown types of vibrations $\psi(x)$ and their frequencies, the condition of the minimum potential energy was applied. Considering relations (2) and (3), circumferential and axial distortions have been presented in the form of following relationships:

$$v(x,\varphi,t) = -\frac{1}{n}\psi(x)\sin n\varphi \sin \omega t \tag{7}$$

$$u(x,\varphi,t) = -\frac{R}{n^2}\frac{d\psi(x)}{dx}\cos n\varphi \sin \omega t \tag{8}$$

The total potential energy of the system may be obtained from the following expression :

$$U = \int_0^L \Gamma dx \tag{9}$$

where: Γ – the total potential energy of the liner per length (L) unit of the cylinder

$$\begin{aligned}\Gamma &= \oint\left[\frac{1}{2}m_\varphi\kappa_\varphi + \frac{1}{2}\sigma_x\delta\varepsilon_x + \frac{1}{2}\rho\delta_\Sigma\left(\frac{\partial^2 w}{\partial t^2}w + \frac{\partial^2 v}{\partial t^2}v + \frac{\partial^2 u}{\partial t^2}u\right)\right]Rd\varphi = \\ &= \left(\frac{D_ш\left(n^2-1\right)^2}{2R^4}\psi^2 + \frac{E\delta R^2}{2n^4}\left(\frac{d^2\psi}{dx^2}\right)^2 - \frac{\omega^2\rho\delta_\Sigma}{2}\left(\psi^2\left(1+\frac{1}{n^2}\right) + \frac{R^2}{n^4}\left(\frac{d\psi}{dx}\right)^2\right)\right)\pi R\sin^2\omega t.\end{aligned} \tag{9a}$$

In (9a) the following symbols denote:

$D_{ш} = \dfrac{E\delta_{ш}^3}{12\left(1-\mu^2\right)}$ – cylinder rigidness, characterizing cylinder wall resistance to bending,

E – modulus of elasticity of the material (Young's modulus),

R – radius of the cylinder,

P – density of the cylinder material,

μ – Poisson's coefficient,

$m_\varphi = D_{ш}\left(\kappa_\varphi + \mu\kappa_x\right)$ – bending momentum in the circumferential direction,

$\sigma_x = \dfrac{E}{1-\mu^2}\dfrac{\partial u}{\partial x}$ – axial tension.

It has been assumed that external load is due to mass inertia forces whose projections on unitary area of the cross-section were defined in the following way

$$P_x = -\rho\delta_\Sigma \frac{\partial^2 u}{\partial t^2}; \qquad P_y = -\rho\delta_\Sigma \frac{\partial^2 w}{\partial t^2}; \qquad P_\varphi = -\rho\delta_\Sigma \frac{\partial^2 v}{\partial t^2}$$

Then expression $\dfrac{1}{2}\rho\delta_\Sigma\left(\dfrac{\partial^2 w}{\partial t^2}w + \dfrac{\partial^2 v}{\partial t^2}v + \dfrac{\partial^2 u}{\partial t^2}u\right)$ determines the potential of external forces, used with a reverse sign.

As in expression (9a) defining Γ the potential of external forces with a reverse sign is used, then from formula (9) the following condition emerges:

$$U = \int_0^L \Gamma dx = 0 \tag{10}$$

meaning that at radial displacement, the equity of work performed by internal and external forces is noted when the liner is in the equilibrium state.

The choice of thickness δ, δ_Σ, $\delta_{ш}$ used for calculations depends on the cylinder shape. For cylinders whose external part of the wall is made in the form of spiral grooves as $\delta_{ш}$ wall thickness in the region of protrusions should be taken and for δ wall thickness in the groove region should be assumed. Then, the total thickness of the reinforced liner may be defined as the sum of the following form:

$$\delta_\Sigma = \delta + \frac{f_{ш}}{a} \tag{11}$$

where:

a – spiral step; $f_{ш}$ – the area of the cross-section of the protrusion.

For cylinders with smooth walls and collars $\delta_{ш}$ is equal to the corresponding wall thickness defined according to the generally used relationship:

$$\delta_{ш} = d = d_1\left[\frac{\left(\eta^2 - 1\right)}{\dfrac{1+\mu}{1-\mu}\eta^2 + 1}\right]^{\frac{1}{3}} \tag{12}$$

where:

d_1 – collar thickness; $\eta = \frac{a_2}{a_1}$;

a_1 – external radius of the cylinder; a_2 – cylinder radius in the collar region.

δ refers to the wall thickness at the end of the cylinder (generally denoted as d in the applied methodology – formula 12) and the total thickness of the liner is determined taking into consideration all non-uniformities of the whole thickness of cylinder walls

$$\delta_\Sigma = \frac{1}{L}\sum_i f_{шi} \tag{13}$$

where: $f_{шi}$ – is the cross-section area of particular parts with defined wall thickness (supportive belt, dredging, scarf etc).

4. Problem solving method

Relying on the condition of the minimum potential energy of the system, radial displacements *ψ(x)*, which were used to determine all deformations, internal forces and external loads can be determined using Euler equation for the variation problem:

$$\frac{\partial \Gamma}{\partial \psi(x)} - \frac{d}{dx}\frac{\partial \Gamma}{\partial \psi'(x)} + \frac{d^2}{dx^2}\frac{\partial \Gamma}{\partial \psi''(x)} = 0 \tag{14}$$

As

$$\frac{\partial \Gamma}{\partial \psi(x)} = \left(D_ш \frac{\left(n^2-1\right)^2}{R^4} - \omega^2 \rho \delta_\Sigma \left(1+\frac{1}{n^2}\right)\right)\psi(x)\pi R \sin^2 \omega t \tag{15}$$

$$\frac{d}{dx}\frac{\partial \Gamma}{\partial \psi'(x)} = -\omega^2 \rho \delta_\Sigma \frac{R^2}{n^4}\frac{d^2\psi}{dx^2}\pi R \sin^2 \omega t \tag{16}$$

$$\frac{d^2}{dx^2}\left(\frac{\partial \Gamma}{\partial \psi''(x)}\right) = E\delta \frac{R^2}{n^4}\frac{d^4\psi}{dx^4}\pi R \sin^2 \omega t \tag{17}$$

Then from relations (14) – (17) a uniform differential equation of the fourth order was obtained:

$$\frac{d^4\psi}{dx^4} + \frac{\rho\delta_\Sigma}{E\delta}\omega^2\frac{d^2\psi}{dx^2} - \left(\omega^2 \frac{\rho\delta_\Sigma\left(n^2+1\right)n^2}{E\delta R^2} - D_ш \frac{\left(n^2-1\right)n^4}{E\delta R^6}\right)\psi(x) = 0 \tag{18}$$

whose solution is function (19):

$$\psi(x) = C_1 ch\ k_1 x + C_2 \cos ik_2 x + C_3 sh\ k_1 x + C_4 \sin ik_2 x \tag{19}$$

describing the type of vibrations along the x axis, where :

C_1, C_2, C_3, C_4 – integral constants dependent on the limiting conditions,

k_1 , k_2 – roots of the fourth order characteristic equation,

$$k^4 + ak^2 + b = 0 \qquad (19a)$$

where: $a = \frac{\rho\delta_\Sigma}{E\delta}\omega^2$; $\qquad b = D_ш \frac{(n^2-1)^2 n^4}{E\delta R^6} - \omega^2 \frac{\rho\delta_\Sigma n^2(n^2+1)}{E\delta R^2}$;

$$k_1^2 = \frac{-a + \sqrt{a^2 - 4b}}{2}; \qquad k_2^2 = \frac{-a - \sqrt{a^2 - 4b}}{2} \qquad (19b)$$

If for the actual construction of the cylinder, the following conditions are fulfilled:

$b < 0$; $\qquad |a| << |b|$ $\qquad$ and $\qquad k_1^2 \approx -k_2^2 \approx \sqrt{|b|}$

Then, it can be assumed that

$$k_1 \approx ik_2 = k\ .$$

Substituting relation (19), which describes *ψ(x)*, free frequency of the cylinder has been obtained in the following form:

$$\omega_m^2 = \frac{(k_m R)^4 + \frac{D_ш}{R^2 E\delta}(n^2-1)^2 n^4}{\frac{\rho\delta_\Sigma}{E\delta} R^2\left((k_m R)^2 + (n^2+1)n^2\right)} \qquad (20)$$

5. Summary

In contrast to the generally used methods of calculations applied for determination of free vibration of cylinders [1, 6, 7], relation (20) is a general one, valid for all limiting ways of cylinder mountings and its application has no restrictions regarding different mounting of the cylinder in the block, not only at the edges but also with intermediate support.
Taking into account limiting conditions, referring to the way in which the cylinder is mounted in the block, leads to a series of exact values k_m. Limiting conditions for the ψ (x) function constitute a system of equations versus C_1, C_2, C_3, C_4 constants, whose solution uncommonly exists when the system determinant equals zero.

In the simple cases of symmetrical mounting at the edges of the cylinder, trigonometric equations, whose known roots are expressed with the π number, can be obtained. Taking into consideration the actual mounting of the cylinder in the block, set up both at the edges as well as between them, leads to more complicated limiting conditions.
Then, it is indispensable to determine the k_m value from the condition that the determinant is equal to zero for limiting conditions, and the expression for calculating free vibrations of the cylinder is given in the from:

$$\omega_m^2 = \frac{(k_m R)^4 + \frac{D_ш}{R^2 E\delta}(n^2-1)^2 n^4}{\frac{\rho\delta_\Sigma}{E\delta} R^2\left((k_m R)^2 + (n^2+1)n^2\right)},\ \ m = 1,2,3\ldots, \qquad (21)$$

where: *m* – the number of half- periods along the x axis. The first type of vibrations m=1 is the one without knots with the minimum frequency where: ω_m. Each vibration type in the axial direction is characterized by m half-terms along the x axis and has one value in the circumferential direction, n_m, at which the frequency of vibrations will be minimal [2, 3].

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