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APPLICATION OF SEMI-MARKOV'S PROCESS FOR CONTROL OF AVAILABILITY OF EXECUTIVE SUBSYSTEM WITH A THRESHOLD STRUCTURE

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Abstract

The article presents the method for control of availability of executive subsystem with the threshold structure, consisting of elementary subsystems of the type <driver-vehicle>. The whole of the consideration was presented using the example of a chosen authentic system of the operation and maintenance of transport means. The basic objective of the operation of transport systems is fulfilling the transportation needs as a result of the carrying out of transport on particular routes. The effectiveness of the operation of transport system depends on the possibility of correct carrying out of the assigned transport task. Among many significant criteria used for assessment of a transport system operation there is one of special importance, that is availability of the executive subsystem to perform the assigned tasks. Availability of the executive system to perform the assigned tasks depends on availability of particular technical objects (transport means) used in the system of transportation and their number. The presented method consists of determining and comparison of the values of the required and actual availability of the executive subsystem. The article deals with the way of determining the required availability of an individual technical object (means of transport) as well as the required number of technical objects used in the system, in order for the obtained value of the availability of the executive subsystem to exceed the value of the required availability determined by the parameters of transport task. The availability of the individual technical object was defined on the basis of a constructed mathematical model of operation and maintenance process realized in the tested transport system. Exemplary results have been presented on the basis of experimental data, obtained from the tests carried out in a real operation and maintenance system of transport means.

Keywords: *transport system, availability, operation and maintenance process, threshold structure*

1. Introduction

In general, the transport system consists of two main subsystems: logistics and executive. In the logistics subsystem, the processes carried out are supposed to assure task-based efficiency of the used technical objects. The goal of the executive subsystem is to perform the assigned transport tasks over set routes, with a given frequency, according to a set schedule. A proper accomplishment of the transport task is possible only if the required number of technical objects (transport means) is prepared to perform the assigned task in a given time.

The effectiveness of the operation of transport system depends on the possibility of correct carrying out of the assigned transport task. In the executive subsystem, the direct carrying out of the transport task is performed by elementary subsystems such as operator – means of transport, linked by an appropriate structure. One of the factors strongly influencing the possibility of correct carrying out of the transport task is the availability of the executive subsystem to carry out the task. The executive subsystem availability for the carrying out of the assigned transport task depends on many factors, such as:

- availability of the technical objects (means of transport) used in the system,
- number of technical objects used in the system,
- the structure with which the technical objects are linked,
- reliability and receptivity to servicing and repair of technical objects used in the system,
- availability and efficiency of the logistics subsystem.

On the basis of the above statement, it can be said that control of the executive subsystem availability can be carried out both by selecting technical objects with the required availability and matching the required number of the objects which are to be used in the system in order to ensure proper accomplishment of the transport tasks.

In this work there has been discussed a method for determination of the required availability of a single object as well as the required number of technical objects indispensable to perform appropriately the assigned transport task, on the basis of the transportation system availability assessment. The method consists of determining and comparison of the values of the required and actual availability of the executive subsystem for the carrying out of the assigned transport task and choosing the required availability of technical objects and the required number of objects used in a transport system, so that obtained value of actual availability of the executive subsystem is higher than the value of required availability, determined by the parameters of the transport task. The value of actual availability of the executive subsystem is defined for a given availability of a single technical object as well as a given structure which links the individual technical objects. The actual availability of the technical objects (transport means) was defined based on a mathematically built model of the operation and maintenance process carried out in the tested transport system.

2. The method for control of availability of executive subsystem with a threshold structure

In order to assure the correct realization of the assigned transport task it is necessary for the value of the availability of the executive subsystem G^{PW} to be at least equal to the value of required availability G_w^{PW} :

$$G^{PW} \geq G_w^{PW} . \tag{1}$$

In the tested transport system the individual technical objects (means of transport) are coupled the threshold structure of the k_w of N type, where k_w marks the required number of technical objects available for the carrying out of the assigned transport task, while N the number of all technical objects used in the system. Then the availability of the executive subsystem with a threshold structures marked by the formula [2, 6]:

$$G^{PW} = \sum_{i=k_w}^{N} \binom{N}{i} \cdot \left(G^{OT}\right)^i \cdot \left(1 - G^{OT}\right)^{N-i} . \tag{2}$$

Taking into consideration the fact that the required minimum number of objects available for the particular transport task is constant $k_w = const.$, the availability of the executive subsystem G^{PW} depends on the number N of technical objects used in the system as well as the availability of a single technical object G^{OT} .

Availability of a single technical object defined on the basis of the semi-Markovian model of operation and maintenance process is determined as the sum of limit probabilities p_i^* of remaining at states S_i of proces $X(t)$, belonging to the availability states set S_G [2, 5]:

$$G^{OT} = \sum_i p_i^*, \quad dla \quad S_i \in S_G \ . \tag{3}$$

The required availability of the executive subsystem for the realization of the assigned transport task is determined depending on the required minimum number of available technical objects k_w, defined in the description of the assigned transport task as well as the number of all technical objects used in the transport system N, according to the following formula:

$$G_w^{PW} = \frac{T_w^{PW}}{T_w^{PW} + U_w^{PW}} = \frac{k_w \cdot \tau_z}{k_w \cdot \tau_z + n_w \cdot \tau_z} = \frac{k_w}{N}, \tag{4}$$

where for the given assigned transport task:

T_w^{PW} – required time of availability of the executive subsystem,
U_w^{PW} – required time of non-availability time of the executive subsystem,
k_w – the required minimum number of available technical objects,
n_w – the required maximum number of non-available technical objects,
N – the number of all technical objects used in the transport system,
τ_z – required time for the realization of the assigned transport task.

In the presented method the evaluation of executive subsystem, is done on the basis of the comparison of its value and the value of the required availability which the executive subsystem should have for the assigned task to be carried out correctly. Then for the evaluation of the availability of the executive subsystem for the realization of the assigned task the following criteria were adopted:

– facilitating the definition of the required number of technical objects used in the transport system N_w for a given value of the availability of a single technical object G^{OT} :

$$N = N_w \Leftrightarrow \sum_{i=k_w}^{N} \binom{N}{i} \cdot \left[G^{OT}\right]^i \cdot \left[1 - G^{OT}\right]^{N-i} \geq \frac{k_w}{N}, \tag{5}$$

– facilitating the definition of a required value of availability of a single technical object G_w^{OT} for a given number of technical objects used in the transport system N:

$$G^{OT} = G_w^{OT} \Leftrightarrow \sum_{i=k_w}^{N} \binom{N}{i} \cdot \left[G^{OT}\right]^i \cdot \left[1 - G^{OT}\right]^{N-i} \geq \frac{k_w}{N}. \tag{6}$$

3. Semi-Markovian model operation and maintenance process of transport means

In result of carried out analysis of assumptions and restrictions, semi-Markov's process $X(t)$ was assumed to be the model operation and maintenance process of technical objects. Using semi-

Markov's process for the operation and maintenance process mathematical modeling, the following assumptions have been accepted:

- semi-Markov's process *X(t)* reflects the modeled real process properly enough from the point of view of the tests,
- the modeled process of operation and maintenance has a finite number of states S_i, i=1,2,….,16,
- random process *X(t)* which is a mathematical model of operation and maintenance process is a homogenous process,
- in time t=0 the process is in state S_1 (S_1 is the initial state).

The model of operation and maintenance process was created on the basis of the analysis of state space as well as operational events pertaining to technical objects (municipal buses) used in the analyzed real transport system. Due to the identification of the analyzed transport system and the multi-state process of technical object operation utilized in it, crucial operation states of the process as well as possible transfers between the defined states were designated. Based on this, a graph was created, depicting the changes of operation and maintenance process states, shown in Fig. 1.

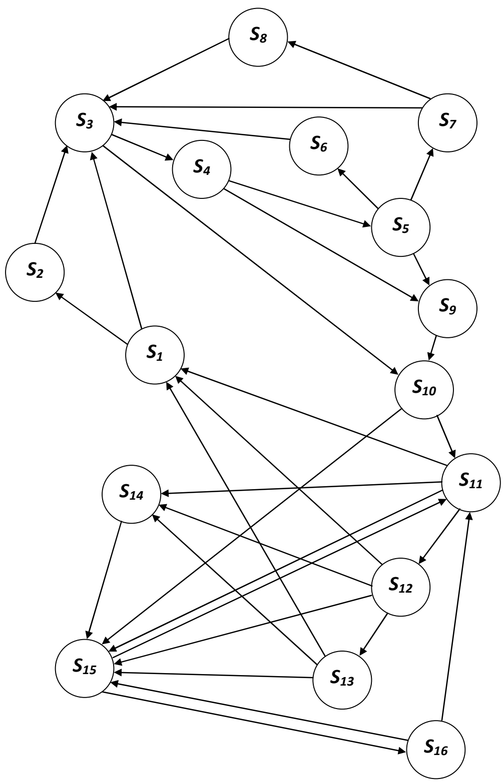

Fig. 1. Directed graph representing the operation and maintenance process of transport means
S_1 – awaiting the carrying out of the task at the bus depot parking space, S_2 – repair at the bus depot parking space without losing a ride, S_3 – carrying out of the transport task, S_4 – waiting for the decision of the traffic controller after occurrence of the vehicle damage, S_5 – diagnosing by the emergency service unit, S_6 – repair by technical support unit without losing a ride, S_7 – repair by the emergency service with losing a ride, S_8 – awaiting the start of task realization after technical support repair, S_9 – emergency exit, S_{10} – waiting for action of the maintenance subsystem, S_{11} – refueling, S_{12} – maintenance check on the operation day, S_{13} – realization of periodical servicing, S_{14} – prior to repair diagnosing in the serviceability assurance subsystem, S_{15} – repair in the serviceability assurance subsystem, S_{16} – diagnosing after the repair in the serviceability assurance subsystem

The homogenous semi-Markovian process is unequivocally defined when initial distribution and its kernel are given [1, 2, 3]. Form our assumptions and based on the directed graph shown in figure 1, the initial distribution $p_i(0)$, i = 1,2,…,16 takes up the following form:

$$p_i(0) = P\{X(0) = i\} = \begin{cases} 1 & when \quad i = 1 \\ \\ 0 & when \quad i \neq 1 \end{cases}, \tag{7}$$

whereas the kernel of process *Q(t)* takes up the form:

$$Q(t) = \begin{bmatrix}
0 & Q_{1,2}(t) & Q_{1,3}(t) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & Q_{2,3}(t) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & Q_{3,4}(t) & 0 & 0 & 0 & 0 & 0 & Q_{3,10}(t) & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & Q_{4,5}(t) & 0 & 0 & 0 & Q_{4,9}(t) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & Q_{5,6}(t) & Q_{5,7}(t) & 0 & Q_{5,9}(t) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & Q_{6,3}(t) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & Q_{7,3}(t) & 0 & 0 & 0 & 0 & Q_{7,8}(t) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & Q_{8,3}(t) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q_{9,10}(t) & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q_{10,11}(t) & 0 & 0 & 0 & Q_{10,15}(t) & 0 \\
Q_{11,1}(t) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q_{11,12}(t) & 0 & Q_{11,14}(t) & Q_{11,15}(t) & 0 \\
Q_{12,1}(t) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q_{12,13}(t) & Q_{12,14}(t) & Q_{12,15}(t) & 0 \\
Q_{13,1}(t) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q_{13,14}(t) & Q_{13,15}(t) & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q_{14,15}(t) & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q_{15,11}(t) & 0 & 0 & 0 & 0 & Q_{15,16}(t) \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q_{16,11}(t) & 0 & 0 & 0 & Q_{16,15}(t) & 0
\end{bmatrix}, \tag{8}$$

where:

$$Q_{ij}(t) = P\{X(\tau_{n+1}) = j, \tau_{n+1} - \tau_n \leq t \mid X(\tau_n) = i\}, \quad i, j = 1,2,...,16, \tag{9}$$

means that the state of semi-Markovian process and the period of its duration depends solely on the previous state, and does not depend on earlier states and periods of their duration, where τ_1, τ_2, …, τ_n, are arbitrary moments in time, so that $\tau_1 < \tau_2 < \ldots < \tau_n$, as well as

$$Q_{ij}(t) = p_{ij} \cdot F_{ij}(t), \tag{10}$$

where:

p_{ij} – means that the conditional probability of transfer from state S_i to state S_j, as well as

$$F_{ij}(t) = P\{\tau_{n+1} - \tau_n \leq t \mid X(\tau_n) = i, X(\tau_{n+1}) = j\}, \quad i, j = 1,2,...,16, \tag{11}$$

is a distribution function of random variable Θ_{ij} signifying period of duration of state S_i, under the condition that the next state will be state S_j.

Limit probability p_i^* of staying in states of process were assigned on the basis of limit theorem for semi-Markovian processes [2]:

If hidden Markov chain in semi-Markovian process with finite state S set and continuous type kernel contains one class of positive returning states such that for each state $i \in S, f_{ij} = 1$ *and positive expected values* $E(\Theta_i), i \in S$ *are finite, limit probabilities:*

$$p_i^* = \lim_{t \to \infty} p_i(t) = \frac{\pi_i \cdot E(\Theta_i)}{\sum_{i \in S} \pi_i \cdot E(\Theta_i)}, \tag{12}$$

where probabilities $\pi_i, i \in S$ *constitute a stationary distribution of a hidden Markov chain, which fulfills the simultaneous linear equations:*

$$\sum_{i \in S} \pi_i \cdot p_{ij} = \pi_j, \quad j \in S, \quad \sum_{i \in S} \pi_i = 1. \tag{13}$$

4. Results

Based on the source data obtained from operational research carried out in an existing municipal bus transport system, with the use of the MATHEMATICA software, the limit probability p_i^* of staying in states of semi-Markov process were determined. Results are given in Tab. 1.

Tab. 1. Values of probability p_i^* and border semi-Markov process distribution

p_1^* = 0,30282	p_2^* = 0,00042	p_3^* = 0,54035	p_4^* = 0,00052
p_5^* = 0,00204	p_6^* = 0,00039	p_7^* = 0,00117	p_8^* = 0,00085
p_9^* = 0,00163	p_{10}^* = 0,09614	p_{11}^* = 0,00614	p_{12}^* = 0,00518
p_{13}^* = 0,00281	p_{14}^* = 0,00149	p_{15}^* = 0,03668	p_{16}^* = 0,00137

In order to define availability of technical objects (means of transport) based on the semi-Markovian model of operation and maintenance process, the states of the process of technical object should be divided into availability states S_G and non-availability states S_{NG} of the object for the carrying out of the assigned task. The states of availability of the technical objects are such states, when the object, including the operator remains in the operational system, is efficient and supplied, or will be repaired and/or supplied in a period of time shorter than the period of the time reserve assigned for the task. Non-availability states are such states, when the object or the operator remain outside of the operational system (efficient or not) as well as when an inefficient and/or unsupplied object remains within the operational system. In the presented model, the following technical object availability states were enummerated:

state S_1 – awaiting the carrying out of the task at the bus depot parking space,
state S_2 – repair at the bus depot parking space without losing a ride,
state S_3 – carrying out of the transport task,
state S_6 – repair by technical support unit without losing a ride,
state S_8 – awaiting the start of task realization after technical support repair.

Then, the availability of technical objects of the transport system were determined:

$$G^{OT} = p_1^* + p_2^* + p_3^* + p_6^* + p_8^* = 0{,}84483 . \tag{14}$$

Then for the values of parameters describing assigned to the executive subsystem transport task (k_w = 159), the values of actual availability G^{PW}, required availability of the executive subsystem G_w^{PW} and the graphs $N_w = f(G_w^{PW})$, $N_w = f(G^{OT})$, $G_w^{OT} = f(G_w^{PW})$, $G_w^{OT} = f(N)$ were determined. Results are presented in Fig. 2 to 5.

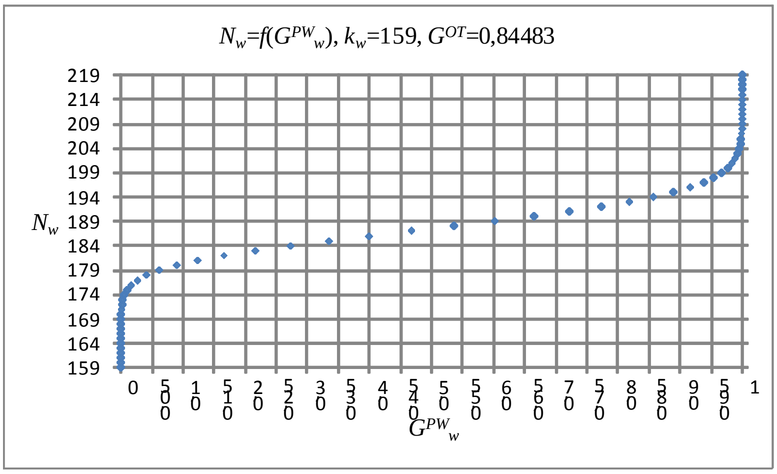

Fig. 2. Required number of the technical objects used in the system of transport N_w in the function of required availability of the executive subsystem G^{PW}_w, for given value k_w and G^{OT}

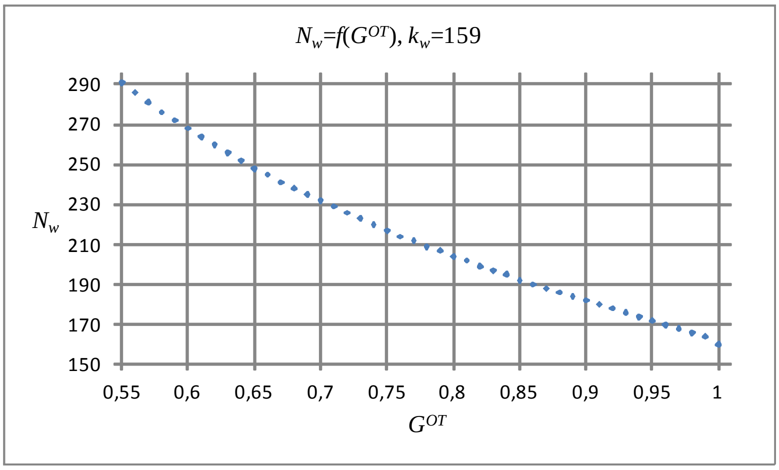

Fig. 3. Required number of the technical objects used in the system of transport N_w in the function of availability of the technical objects G^{OT}, for given value k_w

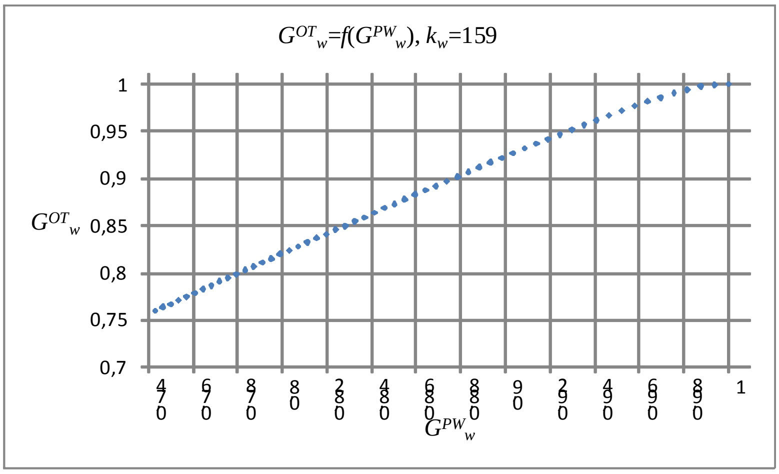

Fig. 4. Required availability of technical object G^{OT}_w in the function of required availability of the executive subsystem G^{PW}_w, for given value k_w

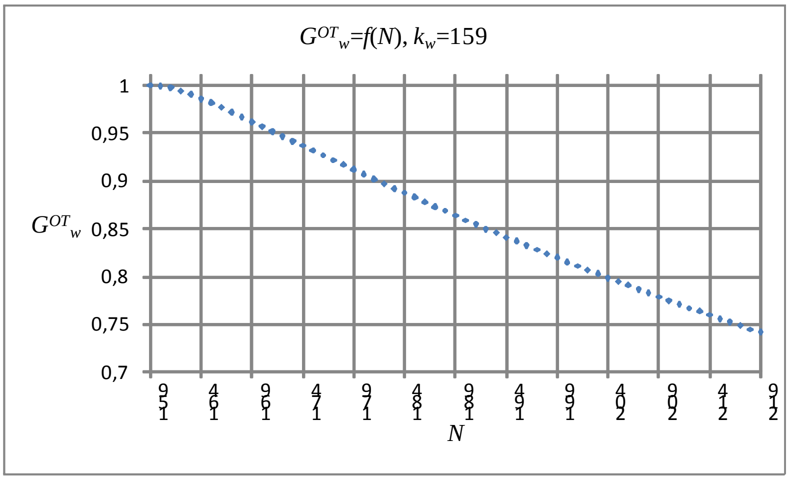

Fig. 5. Required availability of technical object G^{OT}_w in the function of number of the technical objects used in the system of transport N, for given value k_w

5. Conclusions

The presented method enables the selection of the required availability G_w^{OT} and the required number N_w of technical objects coupled the threshold structure so as to ensure proper realization of the tasks assigned to a transport system.

The method can be used both for designing or upgrading the transport system (selection of the required values G_w^{OT} and N_w) and for the assessment and selection of variants for transport tasks that can be implemented (for determined values G^{OT} and N).

On the basis of the graphs drawn, for given number of technical objects, that should to realize the assigned transport tasks, eg $k_w = 159$, it is possible to determine:

a) required number N_w of technical objects used in the system:
   - depending on the required availability of executive subsystem G_w^{PW} (Fig. 2),
   - depending on the actual availability of technical objects G^{OT} (Fig. 3);

b) required availability G_w^{OT} of technical objects used in the system:
   - depending on the required availability of executive subsystem G_w^{PW} (Fig. 4),
   - depending on the number of all technical objects used in the transport system N (Fig. 5).

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