



## APPLICATION OF MARKOV PROCESS FOR MODELING CHANGES OF TRANSPORT MEANS OPERATION STATES

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### *Abstract*

*The article deals with selected problems connected with modeling, prognosis and control of the operation process of a certain class of technical objects used in a complex operation system. The research object is an urban bus transportation system. The authors present assumptions be used for developing a model of the process carried out within the research object. A model of the operation process (Markov process) of an urban transportation bus has been presented. On the basis of identification of the research object and the operation process carried out in it, there has been built a graph of the transport means operational states and possible transitions between these states have been determined. The mean 'life time' of the system in which the modeled operation process is carried out has been established. The whole study has been illustrated by a computational example. Values of the model parameters have been estimated on the basis of results of the initial tests performed in areal transport means operation system.*

**Keywords:** *maintenance process, modelling, Markov process, urban public transport, lifetime of system*

### **1. Introduction**

Supporting people, responsible for making decisions on the control of complex operation processes, can involve prognosis of the operation system and assessment of the impact of selected decision variants on the operation process course [3].

Vehicles involved in the operation process go through many operational states which make up a space of  $S$  states. A random process with a finite number of states  $S$  and a set of parameters  $R^+$  is a natural model of the operation process [7]. An assumption has been accepted that a homogenous Markov model is the one which describes the real process of the vehicle operational states changes [1,2,3,5,6,8]. In practical application, it is necessary to verify if there occur any reasons to reject the assumptions resulting from the related mathematical apparatus.

The purpose of this elaboration is to present the possibilities of using Markov model of the technical object operation process for an analysis of the operation system behavior after changing the model initial parameters. A Change in the model initial parameters values can simulate the impact of different factors on the system behavior and characteristics of the vehicle operation process (duration times of the operation states, probabilities of the state change, numbers of entrances to states). In order to illustrate the discussion, a simplified computational example, concerning the initial prognosis of the system "life time" with the assumption that the objects withdrawn from operation have been replaced by new ones, has been demonstrated. For this

purpose, the so called absorption state has been specified in the model of the transport means operation. In such a case the vehicle “lifetime” is the expected value of a random variable, standing for the time period, after which process  $X(t)$ , being a model of the transport means operation process, will be found in the absorption state.

In the analyzed system of operation, determination of the “life time” of the system, viewed as the length of time interval after which the system stand-by is exhausted (stand-by vehicles), and the number of vehicles being in the state of serviceability is too small to perform fully the accepted range of transport tasks.

## **2. Research Object**

The research object is a complex system of transport means operation (technical objects) including the process of technical objects operation. The process of operation is referred to as all the processes concerning technical objects in the phase of operation, thus determining advisability of their application and efficiency of the related system.

An urban bus transportation system used in a big urban agglomeration is an example of a complex system of technical objects operation which has been used for illustration of the study presented in this work.

The main goal of the examined system operation is safe and efficient performance of transport tasks in a defined quantity and over an assigned territory. Transport tasks are directly performed by elementary socio-technical subsystems of the type O-TO (operator –technical object). Elements of these subsystems, that is, the driver (operator) and a vehicle (technical object), are coupled in the by a series structure in the sense of reliability. Transport tasks are performed with the use of buses. Further, in this work, an assumption is accepted that in the set of operated buses there can be distinguished systems of objects, homogenous from the point of view of the analysis purpose.

The main processes carried out in the research object include, apart from transport means operation processes, processes connected with ensuring serviceability of the vehicles.

In the analyzed system there can be distinguished a subsystem responsible for providing the vehicles with serviceability which includes: [3,8]:

- servicing station with stands for current repairs – in which the damaged vehicles are provided with serviceability,
- subsystem of, the so called, teams for maintenance of traffic during performance of transport tasks- it provides damaged vehicles with serviceability while performing transport tasks ( definite range of carried out processes).

In order to guarantee efficient performance of transport tasks, in the accepted quantity and over the assigned territory, it is necessary to have  $m$  fit for use vehicles. Ensuring the continuity of tasks performance in case of the vehicle damage is possible through maintenance of  $k$  stand-by vehicles. In the considered object there are used  $n=m+k$  number of buses. They perform transport tasks during a 24 hour-cycle.

## **3. Simplified model of the vehicle operation**

For further studies it has been assumed that there can be distinguished the technical object operation states that are significant from the point of view of the research goal. In a given time the object can be only in one of the analyzed operational states.

A stochastic model describing changes of the vehicle operational states has been accepted to be its operation process model.

On the basis of an analysis of the space of the transport means operational states it is possible to distinguish states which are significant in terms of these systems operation efficiency analysis. The size of the distinguished state spaces depend, among others, on the goals of the carried out

analyses and studies. Due to the character of the research, five operational states of a city bus transportation system have been analyzed. However, the presented approach to the model description enables its use for objects with different, than the analyzed, space of states. In works [3,4,7,8,9,10,11] models of an urban bus transportation operation process are analyzed for more complex spaces of states.

In result of identification of the research object and the related operation process, the following operation states of vehicle have been distinguished:

- $S_1$  – performance of transport tasks,
- $S_2$  – performance of, the so called, servicing processes, on the day of operation,
- $S_3$  – servicing the vehicle at the station,
- $S_4$  – withdrawal of the vehicle from operation (the so called liquidation of the vehicle, in the analyzed example in result of damage whose removal is economically unjustified, e.g. being the effect of a road collision),
- $S_5$  – standby of the vehicle at the premises of the depot ( according to the schedule of transport tasks).

It has been assumed that duration time of state  $S_4$  approaches infinity (the vehicle is crossed out from the vehicle register and is not replaced by a different one).

On the basis of an analysis of the operation process realized within the research object, there have been determined transitions between the vehicle operation states – fig. 1.

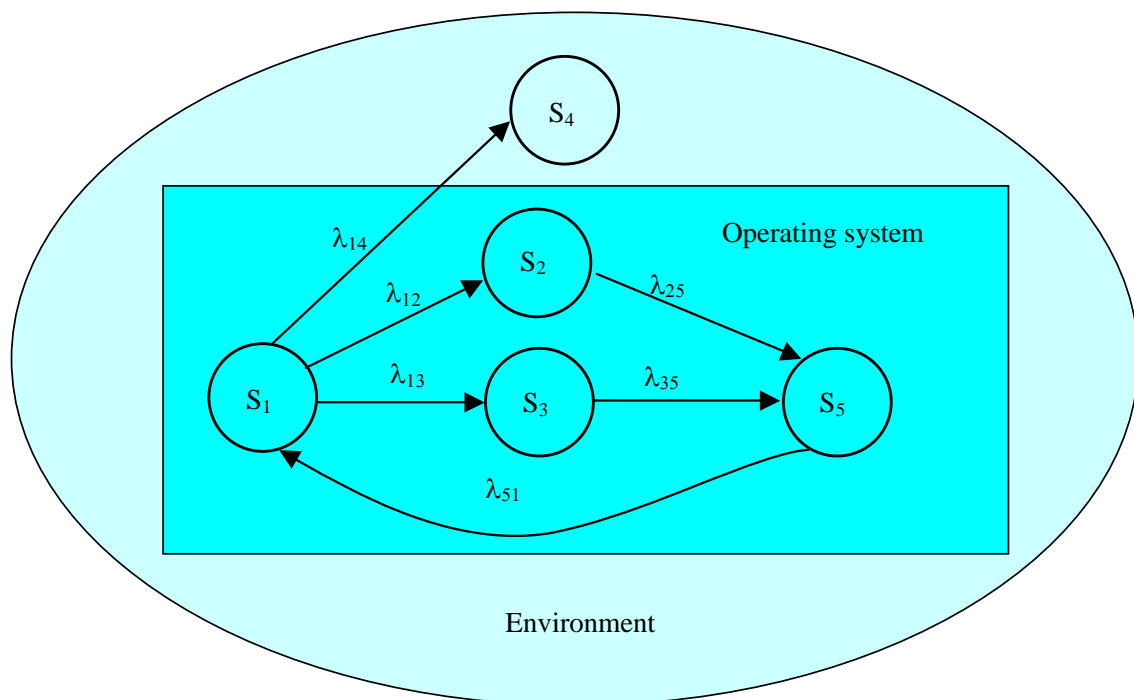


Fig 1. Directed graph of operational states

A homogenous Markov process  $X(t)$  has been accepted to be a model of the vehicle operation process. The analyzed process  $X(t)$  has an infinite phase space  $S = \{ S_1, S_2, S_3, S_4, S_5 \}$ . If  $X(t) = S_i$ ,  $i = 1, 2, 3, 4, 5$ , then the vehicle is in state  $S_i$ , in time  $t$ . For the analyzed model of the vehicle operation process, state  $S_4$  is an absorption one.

Let  $P_i(t) = P\{X(t) = S_i\}$  denote probability that process  $X(t)$  is in state  $S_i$ ,  $i=1,2,3,4,5$ . State  $S_1$ , is assumed to be the initial one which means that the initial distribution is in the form:

$$\begin{aligned} P\{X(0)=S_1\}&=1, \\ P\{X(0)=S_i\}&=0, \text{ dla } i=2,3,4,5. \end{aligned}$$

Intensity of transitions from the process state to state has been accounted for in matrix  $\Lambda$ :

$$\Lambda = \begin{bmatrix} -(\lambda_{12} + \lambda_{13} + \lambda_{14}) & \lambda_{12} & \lambda_{13} & \lambda_{14} & 0 \\ 0 & -\lambda_{25} & 0 & 0 & \lambda_{25} \\ 0 & 0 & -\lambda_{35} & 0 & \lambda_{35} \\ 0 & 0 & 0 & 0 & 0 \\ \lambda_{51} & 0 & 0 & 0 & -\lambda_{51} \end{bmatrix}. \quad (1)$$

For the purpose of simplification notation  $\lambda = \lambda_{12} + \lambda_{13} + \lambda_{14}$  has been introduced.

Matrix of intensity of transitions  $\Lambda$  allows to build the differential equations layout as form [2]:

$$\begin{aligned} P'_1(t) &= -\lambda P_1(t) + \lambda_{51} P_5(t), \\ P'_2(t) &= \lambda_{12} P_1(t) - \lambda_{25} P_2(t), \\ P'_3(t) &= \lambda_{13} P_1(t) - \lambda_{35} P_3(t), \\ P'_4(t) &= \lambda_{14} P_1(t), \\ P'_5(t) &= \lambda_{25} P_2(t) + \lambda_{35} P_3(t) - \lambda_{51} P_5(t). \end{aligned} \quad (2)$$

System of differential equations (2), in a matrix notation, assumes the form:

$$\begin{bmatrix} P'_1(t) \\ P'_2(t) \\ P'_3(t) \\ P'_4(t) \\ P'_5(t) \end{bmatrix} = \begin{bmatrix} -\lambda & 0 & 0 & 0 & \lambda_{51} \\ \lambda_{12} & -\lambda_{25} & 0 & 0 & 0 \\ \lambda_{13} & 0 & -\lambda_{35} & 0 & 0 \\ \lambda_{14} & 0 & 0 & 0 & 0 \\ 0 & \lambda_{25} & \lambda_{35} & 0 & -\lambda_{51} \end{bmatrix} \begin{bmatrix} P_1(t) \\ P_2(t) \\ P_3(t) \\ P_4(t) \\ P_5(t) \end{bmatrix}. \quad (3)$$

In order to provide the above system of equations with a unique solution it is necessary to accept the initial conditions. On the basis of the initial distribution of  $X(t)$  process, it can be noted:

$$\begin{aligned} P_1(0) &= 1, \\ P_2(0) &= 0, \\ P_3(0) &= 0, \\ P_4(0) &= 0, \\ P_5(0) &= 0. \end{aligned} \quad (4)$$

It is convenient to solve the systems of linear differential equations by means of Laplace transform which with accounting for initial condition (4) has the form:

$$\begin{aligned} s \tilde{P}_1(s) - 1 &= -\lambda \tilde{P}_1(s) + \lambda_{51} \tilde{P}_5(s), \\ s \tilde{P}_2(s) &= \lambda_{12} \tilde{P}_1(s) - \lambda_{25} \tilde{P}_2(s), \\ s \tilde{P}_3(s) &= \lambda_{13} \tilde{P}_1(s) - \lambda_{35} \tilde{P}_3(s), \\ s \tilde{P}_4(s) &= \lambda_{14} \tilde{P}_1(s), \\ s \tilde{P}_5(s) &= \lambda_{25} \tilde{P}_2(s) + \lambda_{35} \tilde{P}_3(s) - \lambda_{51} \tilde{P}_5(s). \end{aligned} \quad (5)$$

Taking into consideration notation (1) system of equations (5) can be written as follows:

$$P'(t) = \Lambda^T P(t), \quad (6)$$

where:

$$P'(t) = [P'_1(t), P'_2(t), P'_3(t), P'_4(t), P'_5(t)]^T,$$

$$P(t) = [P_1(t), P_2(t), P_3(t), P_4(t), P_5(t)]^T.$$

The matrix notation of linear system of equations for transforms has the following form:

$$\begin{bmatrix} -\lambda + s & 0 & 0 & 0 & \lambda_{51} \\ \lambda_{12} & -\lambda_{25} + s & 0 & 0 & 0 \\ \lambda_{13} & 0 & -\lambda_{35} + s & 0 & 0 \\ \lambda_{14} & 0 & 0 & s & 0 \\ 0 & \lambda_{25} & \lambda_{35} & 0 & -\lambda_{51} + s \end{bmatrix} \begin{bmatrix} \tilde{P}_1(s) \\ \tilde{P}_2(s) \\ \tilde{P}_3(s) \\ \tilde{P}_4(s) \\ \tilde{P}_5(s) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}. \quad (7)$$

It is convenient to solve equation system (7) by Cramer's method. The prime determinant of the equation system is as follows:

$$W(s) = \begin{vmatrix} s + \lambda & 0 & 0 & 0 & -\lambda_{51} \\ -\lambda_{12} & s + \lambda_{25} & 0 & 0 & 0 \\ -\lambda_{13} & & s + \lambda_{35} & 0 & 0 \\ -\lambda_{14} & 0 & 0 & s & 0 \\ 0 & -\lambda_{25} & -\lambda_{35} & 0 & s + \lambda_{51} \end{vmatrix}. \quad (8)$$

After performance of successive developments and computing of 3-degree determinant, we receive:

$$W(s) = s(s+\lambda) (s+\lambda_{51}) (s+\lambda_{25}) (s+\lambda_{35}) - s\lambda_{51}[\lambda_{12}\lambda_{25} (s+\lambda_{35}) + \lambda_{13}\lambda_{35} (s+\lambda_{25})]. \quad (9)$$

Similarly specified were determinants:

$$\begin{aligned} W_1(s) &= s(s+\lambda_{51}) (s+\lambda_{25}) (s+\lambda_{35}), \\ W_2(s) &= s\lambda_{12}(s+\lambda_{35}) (s+\lambda_{51}), \\ W_3(s) &= s\lambda_{13}(s+\lambda_{25}) (s+\lambda_{51}), \\ W_4(s) &= \lambda_{14}(s+\lambda_{25}) (s+\lambda_{35}) (s+\lambda_{51}), \\ W_5(s) &= s[\lambda_{12}\lambda_{25} (s+\lambda_{35}) + \lambda_{13}\lambda_{35} (s+\lambda_{25})]. \end{aligned} \quad (9a)$$

In order to determine the mean "life time" of the system it is necessary to determine the expected value of the sum of times during which the vehicle is in non absorbing states. Without determination of return transforms which enable determination of probabilities  $P_i(t)$ , where  $i=1,2,3,4,5$  it is possible to establish the expected values of the vehicle being in particular states. It is known [1] that:

$$E(T_1+T_2+T_3+T_5) = \lim_{s \rightarrow 0^+} \left[ \frac{1}{s} - \tilde{P}_4(s) \right], \quad (10)$$

where:

$T_i$  - denotes time of being in state  $i$ ,  $i=1,2,3,5$ ,  
and:

$$\tilde{P}_4(s) = \frac{W_4(s)}{W(s)} = \frac{\lambda_{14}(s + \lambda_{25})(s + \lambda_{35})(s + \lambda_{51})}{s[(s + \lambda)(s + \lambda_{51})(s + \lambda_{25})(s + \lambda_{35}) - \lambda_{51}[\lambda_{12}\lambda_{25}(s + \lambda_{35}) + \lambda_{13}\lambda_{35}(s + \lambda_{25})]]}. \quad (11)$$

Let  $W(s) = sM(s)$ , then:

$$\frac{1}{s} - \tilde{P}_4(s) = \frac{1}{s} - \frac{W_4(s)}{sM(s)} = \frac{M(s) - W_4(s)}{sM(s)}. \quad (12)$$

After computing values  $M(s)$  and  $W_4(s)$  we receive:

$$M(0) = \lambda_{51}\lambda_{14}\lambda_{25}\lambda_{35}, \quad (13)$$

$$W_4(0) = \lambda_{51}\lambda_{14}\lambda_{25}\lambda_{35}. \quad (14)$$

Boundary

$$\lim_{s \rightarrow 0^+} \frac{M(s) - W_4(s)}{sM(s)} \quad (15)$$

is of the type  $[0/0]$ , where  $M(s)$  is a polynomial of fourth degree in relation to  $s$ , and  $W_4(s)$  is a polynomial of third degree.

Coefficient near  $s$  for  $M(s)$  is equal to:

$$\lambda\lambda_{51}\lambda_{25} + \lambda\lambda_{51}\lambda_{35} + (\lambda + \lambda_{51})\lambda_{25}\lambda_{35} - \lambda_{51}\lambda_{12}\lambda_{25} - \lambda_{51}\lambda_{13}\lambda_{35}, \quad (16)$$

and for  $W_4(s)$ :

$$\lambda_{14}(\lambda_{25}\lambda_{35} + \lambda_{51}\lambda_{25} + \lambda_{51}\lambda_{35}). \quad (17)$$

In relation to the above, we receive:

$$\lim_{s \rightarrow 0^+} \left[ \frac{1}{s} - \tilde{P}_4(s) \right] = \frac{\lambda_{25}\lambda_{35}(\lambda + \lambda_{51} - \lambda_{14}) + \lambda_{51}\lambda_{13}\lambda_{25} + \lambda_{51}\lambda_{12}\lambda_{35}}{\lambda_{51}\lambda_{14}\lambda_{25}\lambda_{35}}. \quad (18)$$

The expected value of system ET life time can be determined from dependence (18).

In table 1 there are values of the model parameters, estimated on the basis of initial results of tests performed in the analyzed system. In fig. 2 the dependence between the mean "life time" and the intensity value  $\lambda_{14}$  has been presented.

Tab. 1. Estimated values of the model parameters

$\lambda_{12}$	$\lambda_{13}$	$\lambda_{14}$	$\lambda_{25}$	$\lambda_{35}$	$\lambda_{51}$
0,055351	0,002768	0,000017	1,500000	0,344828	0,163043

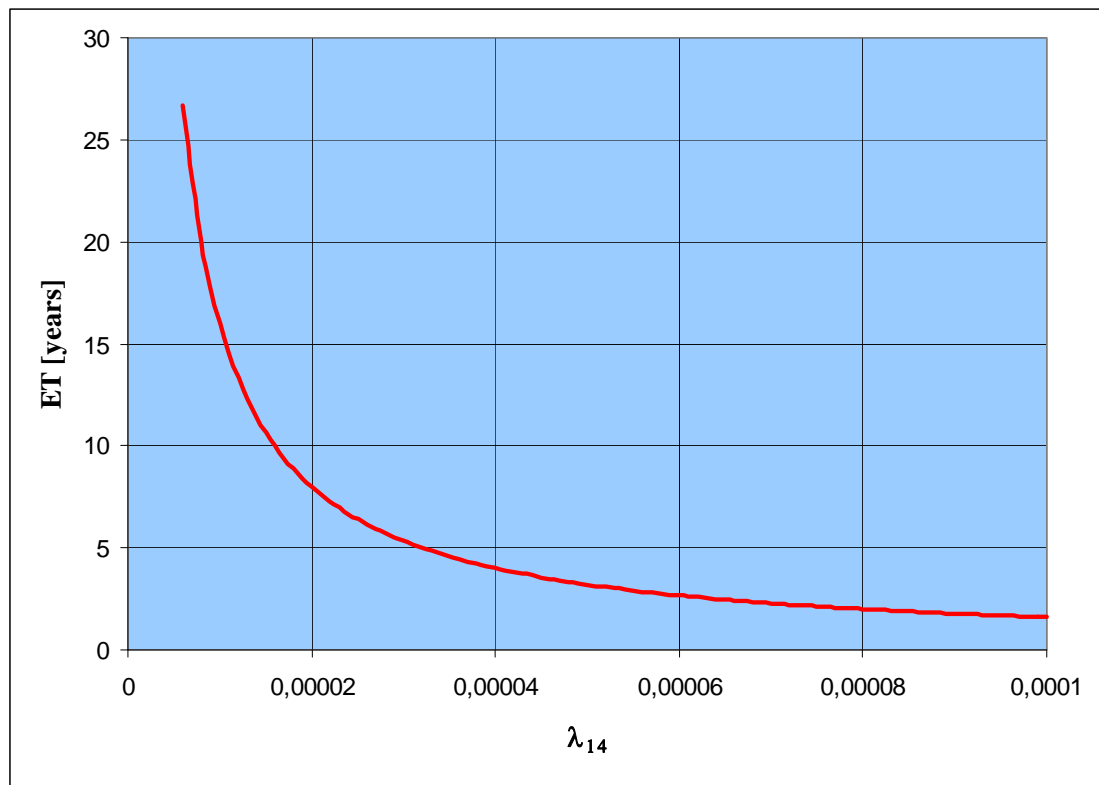


Fig. 2. Mean time of the system "life" in dependence on the intensity value  $\lambda_{14}$

#### 4. Summary

One of the purposes of the research presented in this work is to show the possibility of using selected stochastic processes for mathematical modeling of the vehicle operation system and process.

The assessment and prognosis of efficiency in complex systems of transport means operation is connected with mathematical modeling of the technical objects operation processes. It involves a necessity of providing a formalized description of the processes connected with the transport means operational states changes. Changes of operational states are caused by changes in the vehicle technical states and the processes of their operation and utilization. These processes are of random character and depend on each other. Their mathematical models will naturally be simplified. The analysis of these models tests results, made for the value of their parameters, determined on the basis of experimental tests performed in a real transportation system, makes it possible to formulate conclusions and assessments of both qualitative and quantitative (in a limited scope) character. Practical conclusions resulting from these models tests should be formulated carefully.

The discussed computational example is characterized by a significant simplification degree. Estimation of the values of the model parameters, simulating a change of control impacts of the operation process is an important problem (analysis of different decision variants). Assessment of the influence of the number of stand-by vehicles is difficult in the analyzed example due to the change of transitions intensity values occurring along with the stand-by exhaustion.

Solution of differential equation system (5) and introduction of a non-zero value  $\lambda_{51}$  of (purchase of new vehicles) matrix (1) enables an analysis of a wider range of issues concerning the analyzed system.

The presented approach to building models of the vehicle operational states changes and application of Markov for modeling the process of an urban transportation buses operation can be

relatively easily applied for an analysis of, different than the considered, characteristics of the process and for other operation systems.

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