Journal of Polish CIMAC
Gdansk University of Technology
The Faculty of Ocean Engineering and Ship Technology

MODEL OF PROFIT IMPROVEMENT IN MAINTENANCE SYSTEM

Leszek Knopik

University of Technology and Life Science
Faculty of Management Science
Fordonska st. 424, 85-790 Bydgoszcz, Poland
Email: knopikl@utp.edu.pl

Abstract

The objective of this paper is to study a profit maximization model under decreasing the number of instantaneous failures. We derive the objective function (profit from work of the technical object) under general assumptions. The method is illustrated by a two numerical examples. In the first example the times between two instantaneous failures have two point distribution, in the other two parameters Weibull distribution

Keywords: *lifetime, mixture of distribution, Weibull distribution, exponential distribution, reliability function, instantaneous failures, profit.*

1. Introduction

In the real maintenance system the reduction of the number of the failures of a technical objects causes the growth of the profit from the work of an object. Reduction of the number of the failures depends on the expenditure on the diagnostic and improvement of the organization of the repairs. However, it is known that elimination of all failures is impossible or demands high cost. In this paper, we construct the profit model under reduction of the instantaneous failures.
The lifetime distribution is very important in reliability studies. An important topic in the field of lifetime data analysis is based on the selection of the most appropriate lifetime distribution for statistical data coming from an experiment. This distribution is to describe the time of a component failure, a subsystem or a system. Some failures result from natural damages of the machines, while the other ones may be caused by an inefficient repair of the previous failures, resulting from an incorrect organization of the repairs. The analysis of the results of the operation and maintenance investigations, regarding the moments the failures occurrence proves that the set of the failures may be divided into two subsets, namely into the set of primary failures and the set of secondary failures. It follows the fact that the population of times to failure is often heterogeneous. In this case, the resulting population of lifetimes can be described by using statistical concept of a mixture of two distributions.

2. The model of the profit maximization

We consider a mixture of the lifetimes T_1 and T_2 with the densities $f_1(t)$, $f_2(t)$, the reliability functions $R_1(t)$, $R_2(t)$, the failure rate functions $\lambda_1(t)$, $\lambda_2(t)$ and the weights p and $1 - p$, where $0 < p < 1$. The mixed density function is then written as

$$f(t) = p\, f_1(t) + (1 - p)\, f_2(t)$$

and the reliability function is

$$R(t) = p\, R_1(t) + (1 - p)\, R_2(t) \qquad (1)$$

The failure rate function of mixture can be written as the mixture [1]:

$$\lambda(t) = w(t)\, \lambda_1(t) + (1 - w(t))\, \lambda_2(t),$$

where $w(t) = R_1(t) / R(t)$.
The mean value of the mixture is following:

$$ET = p\, ET_1 + (1-p)\, ET_2, \qquad (2)$$

where ET_1 and ET_2 are the mean values of T_1 and T_2. We assume that $ET_1 < ET_2$.
Reduction of the number of the instantaneous failures is realized by decreasing the value of the parameter p. We assume, that the profit from the work of the technical object is proportional to the expected value of time to failure. This profit, we describe by $g_1(t) = d\, ET$, where $d > 0$. It can be easily seen that $g_1(0) = d\, ET_1$, $g_1(1) = d\, ET_2$. By $h_2(t) = - g_2(t)$, we describe the expenditure on elimination of instantaneous failures. We assume that the function $g_2(t)$ is twice differentiable on (0, p) and the following assumptions are fulfilled:

$$g_2(t) < 0, \;\; g_2(0^+) = -\infty, \;\; g_2(p^-) = 0, \qquad (3)$$

$$g_2'(t) > 0, \; g_2'(0^+) = +\infty, \; g_2'(p^-) = 0, \qquad (4)$$

$$g_2''(t) < 0, \qquad (5)$$

The objective function expresses the profit from the work of the technical object:

$$g(t) = g_1(t) + g_2(t)$$

The first derivative of the objective function g(t) is

$$g'(t) = d\, (ET_1 - ET_2) + g_2'(t)$$

The first derivative $g_2'(t)$ decreases from $+\infty$ to 0 at point p, whereas $d\,(ET_1 - ET_2) < 0$. Now we conclude, that the equation $g'(t) = 0$ has exactly one solution. By assumption (5), the function g(t) approaches exactly one maximum on the interval (0, p). Consequently, we proved:
Proposition 1. If $ET_1 < ET_2$ and the assumptions (3), (4) and (5) are fulfilled, then the objective function g(t) approaches the exactly one maximum on the interval (0, p).

3. Examples

Example 1. In this example, we assume that the lifetime o T_1 has two-point distribution

$$P\{T_1 = 0\} = r, \quad P\{T_1 = 1\} = 1 - r,$$

and T_2 has an exponential distribution with the density function

$$f(t) = \lambda \exp(-\lambda t) \text{ for } t \geq 0$$

The expected value of T is following:

$$ET = p(1-r) + (1-p)/\lambda$$

We assume that the function $g_2(t)$ has the form: $g_2(t) = -a/t - b\,t + c$
It can be seen that $g_2(0^+) = -\infty$ and

$$g_2(p^-) = 0 \text{ if and only if } -a/p - b\,p + c = 0 \qquad (6)$$

The first derivative of $g_2'(t)$ is $g_2'(t) = a/t^2 - b$
It can be seen that $g_2'(0^+) = +\infty$ and $g_2'(p^-) = 0$ if and only if $p = \sqrt{a/b} \leq 1$, hence $a \leq b$.
Substituting $p = \sqrt{a/b}$ to (6), we obtain $c = 2\sqrt{ab}$.
It is easily seen that: if $a > 0$, $b > 0$, $c = \sqrt{ab}$, then the function $g_2(t)$ fulfills the assumption (3), (4) and (5). The objective function approaches the maximum at the point t_0,

$$t_0 = \sqrt{\frac{a}{b + d/\lambda - (1-r)d}}$$

Fig. 1 demostrates the dependence of profit on the the value parameter p for the different values of the parameter a.

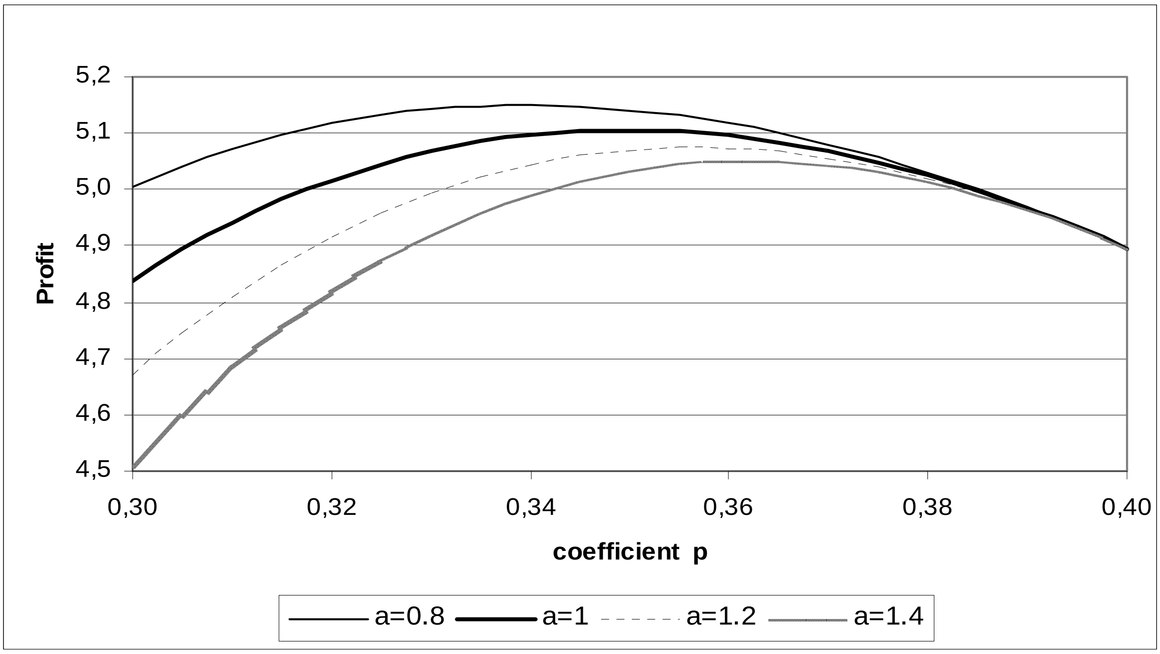

Fig. 1. The profit as function of the parameter p for different values of parameter a

Example 2. We assume that the lifetime T_1 is two parameters Weibull distribution with the reliability function

$$R(t) = \exp(-\alpha t^{\beta}) \text{ for } t \geq 0$$

and T_2 has an exponential distribution with $ET_2 = 1/\lambda$. For the expected value of T, we have

$$ET = p\alpha^{-\frac{1}{\beta}}\Gamma\left(\frac{1}{\beta}\right)/\beta + (1-p)\lambda,$$

where Γ(q) is gamma function $\Gamma(q) = \int_0^\infty t^{q-1}e^{-t}dt$.

We assume that the function $g_2(t)$ has the form: $g_2(t) = -a\ln(t) - b\,t + c$. If $b > 0$, $a = b\,p$, $c = b\,p\ln(e/p)$, then the function $g_2(t)$ fulfills the assumption (3), (4) and (5). The function $g_2(t)$, we can describe in the form: $g_2(t) = (b / p)\,(\ln(t) - t / p + \ln(e / p))$. In this example the objective function has the form:

$$g(t) = t d\alpha^{-\frac{1}{\beta}}\Gamma\left(\frac{1}{\beta}\right)/\beta + d(1-t)/\lambda - a/t - bt + c$$

Fig. 2 shows a graphics of the objective function for the different values of the parameter a.

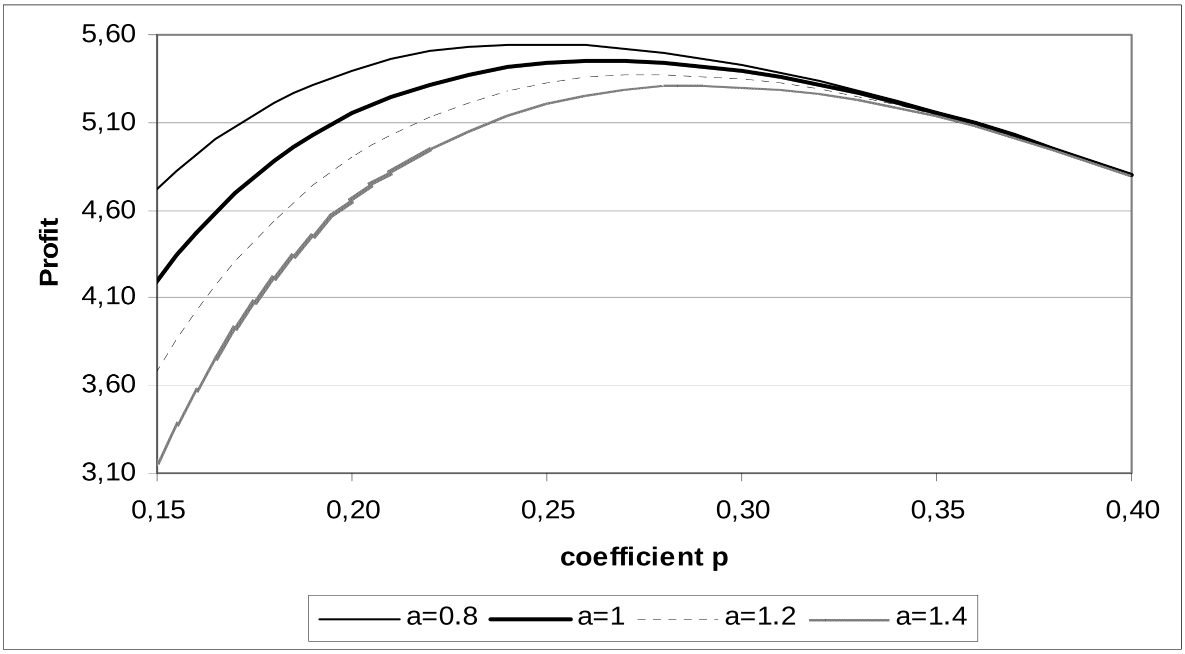

Fig. 2. The profit as function of the parameter p for different values of parameter a

4. Conclusions

In this paper, we study the profit maximization model while the number instantaneous failures is decreasing. The objective function is the profit from the work of a technical object. We show under general assumptions that this function approaches the maximum. Two numerical examples show that this model may be applicable in practice. In both causes of the objective function approaches exactly one maximum.

References

1. Knopik L., *Mixture of distributions as a lifetime distribution of a technical object,* Scientific Problems of Machines Operation and Maintenance, vol.45, 2(165), 2010, pp. 53-60.

2. Wdzięczny A. Woropay and M. Muślewski Ł., *Division and investigation of damages to technical object and their influence on the reliability of operation and maintenance systems* Scientific Problems of Machines Operation and Maintenance, vol.44, 2, 2008, pp. 31-43.