Journal of Polish CIMAC
Gdansk University of Technology
The Faculty of Ocean Engineering and Ship Technology

SPATIAL APPROXIMATION OF IMPACT TEST RESULTS OF UNIT LOAD

Tomasz Piątkowski, Janusz Sempruch, Tomasz Tomaszewski

University of Technology and Life Sciences
ul. Prof. S. Kaliskiego 7, 85-789 Bydgoszcz, Poland
tel.: +48 52 3408145
e-mail: topiat@utp.edu.pl

Abstract

The paper presents an application proposal of B-spline surface of third order in the approximation of test results of unit load impact. The accelerations appearing during tests of free fall of the object from the height on undeformable ground were recorded by means of tri-axial acceleration sensor placed inside the tested load. An approximation was realized with the help of numeric optimization using the SQP method. The control points, forming initially the sphere surface, are the decision variables of the optimization task. The spatial visualization of data (using B-spline surface) is developed to streamline an interpretation and analysis of results recorded during tests. It allows precise load places determination, which require introduction of constructional modifications to improve the protective packaging properties.

Keywords: *unit load, free fall test, B-spline surface, numerical optimization, impact*

1. Introduction

All consumer goods, before they are delivered to the end user, have to travel the distance between the supplier and the final recipient. In order to improve their transportability, the goods are enclosed in protecting packaging – the packaging having the form of cuboidal parcels – that are treated as transport units. During travel, the unit loads are subjected to manipulation operations (e.g. loading, unloading, picking, positioning, sorting, etc.), which create a danger of mechanical damage of the package and its content. Among the most dangerous causes of the load safety infringement are the mechanical hazards of the impact character caused by fall of the load on the ground or on other loads, as well as the impacts that take place when the load interacts with manipulators – for example, realizing sorting process of load streams transported on conveyors [13], [15], [16].

Protection of the load content against mechanical damage can be achieved by properly packaging application that alleviates the dynamic effect of impact [10].

Scientific works dealing with the problem of investigation and analysis of the constructional packaging properties focus mainly on the use of computational numeric methods – the finite element method especially [12]. Packaging and their content are treated, as non-homogenous bodies. However, this attitude is not commonly applied in engineering practice. On the needs of classic packaging design, during their structural features modelling, it is sweeping simplification accepted and used guidelines and recommendations determined on the basis of manufacturers' practical experiences [7].

In view of the lack of the possibility of the precise and exact prediction of the actual mitigating impact properties of the packaging, while their design, the experimental investigations of the unit loads prototypes is necessary carrying out.

The free fall tests of loads from height are one of the main methods of the investigations of the packaging's protective ability. The programme of tests is defined on the basis of guidelines presented in standards, e.g. [8], [9], [5], [4]. This program consists in fall of the load (with suitable oriented walls, edges, and corners) from assumed height on smooth and undeformable ground. The visual inspection of the packaging and its content, performed after series of tests, make up the basis of effectiveness assessment of packaging's protective property. An appearance of any damages of the load content, causing lack of the addressee acceptance (or final consumer), is connected with an introduction necessity of the constructional changes to the packaging (or to the packaging content also).

The experimental investigations of the free fall do not require applying any specialist laboratory devices which are necessary, e.g. in case of the product damage boundary determination [3] or during the cushion curve development for packaging material [2]. In the simplest version of the free fall tests, the load can be positioned manually. In order to improve the repeatability of the test results, the positioning of unit loads before the fall, can be aided by simple devices – drop testers [17].

The constructional-design-test process is realized iteratively, on the basis of the trial-and-error method, till the packaging's protective properties are accepted. This process is considerably more predictable and precise, if the course of the experimental investigations of the impact is recorded by the tri-axial acceleration recorder installed in the load inside [14]. The recorder's data permit the packaging places identification, which require additional introduction of springy-damping or stiffening elements. It is also possible to indicate places, where the cushion material is applied in excess, causing the groundless growth of cost and mass of the packaging. The packaging property investigations with the use of the acceleration recorder make it possible to replace the load content with the substitute material of equivalent mass, geometry and consistency. This approach has economic meaning, especially in case of packaging design for costly or dangerous articles. The analysis of non-processed data written by tri-axial acceleration recorder during series of tests is difficult and non-effective. The spatial data approximation (proposed in presented work) can help to solve this problem. The recorder's data are subjected to approximation by means of B-spline surface of third order (m=n=3) using numeric optimization. This surface takes into consideration the recorder orientation with relation to the load edges, enabling the intuitive identification of packaging's characteristic places.

2. Basic description of B-spline surface

The B-spline surface is one of more often applied surface representation in the engineering graphics. There is capable of presenting complex, 3D solids using a little number of variables. The B-spline surface of order (n, m) is assigned through the grid of control points $P_{i,j}$ (Fig. 1) according to the expression [6], [11], [19]:

$$p(s,t) = \sum_{i=0}^{z_n} \sum_{j=0}^{z_m} P_{i,j} N_i^n(s) N_j^m(t) \tag{1}$$

where:

$s \in [0,1]$, $t \in [0,1]$ – parameters,

z_n+1, z_m+1 – number of control points P placed along rows and columns of the grid (Fig. 1),

m, n – order of B-spline surface,

$P_{i,j}$ – control points,

i=0,1,2,…,z_n, j=0,1,2,…,z_m – indexes of row and column grid of control points (Fig. 1),
$N_i^n(s)$, $N_j^m(t)$ – basis B-spline functions.

The function $N_i^n(s)$ (and $N_j^m(t)$) can be effectively assigned on the basis of recurrent Mansfield-de Boor-Cox formula [18]:

$$\begin{cases} N_i^n(s) = \dfrac{s-u_i}{u_{i+t-1}-u_i} N_i^{n-1}(s) + \dfrac{u_{i+n}-s}{u_{i+n}-u_{i+1}} N_{i+1}^{n-1}(s) \quad if \quad n>1 \\ N_i^1(s) = \begin{cases} 1 & dla \quad s \in [u_i, u_{i+1}) \\ 0 & otherwise \end{cases} \qquad otherwise \end{cases} \tag{2}$$

where:

u_k – knot of B-spline curve (k=0,1,2,…,z_n+n):

$$u_k = \begin{cases} 0 & k<n \\ \dfrac{k-n+1}{z_n-n+2} & n \le k \le z_n \\ 1 & otherwise \end{cases} \tag{3}$$

Knots determined according to formula (3) are uniformly spaced out in the range of $u_k \in$ [0; 1] in non-descending order. First and last knot are duplicated as many times as B-spline curve order is. This condition allows to obtain the tangency of beginning and end of B-spline curve to the grid of control points.

The steps in determining the function $N_j^m(t)$ are analogous.

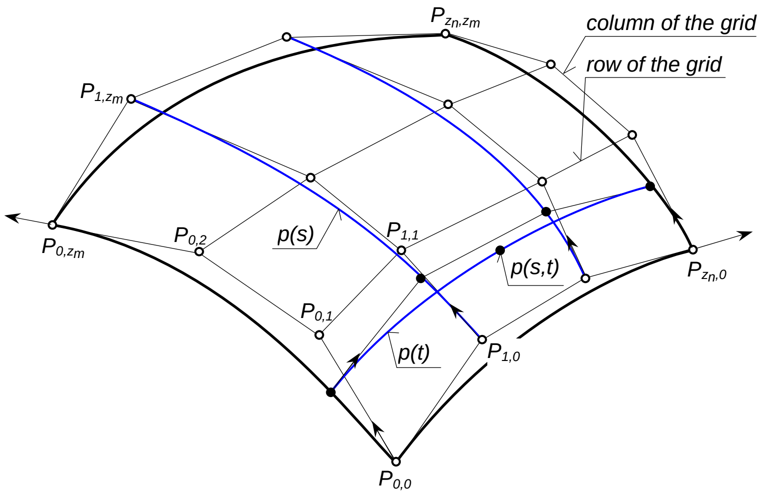

Fig. 1. The surface point p(s,t) as a point on the curve [11]

The point *p(s,t)* assignment, which lies on B-spline surface, can be reduced to the calculation of point on the B-spline curves. The points of surface can be determined using any algorithm designed for calculation of point on the curve, e.g. de Boor algorithm [11].

3. Method of experimental impact tests

For recording the course of the impact process during free fall of the object onto rigid ground, a tri-axial acceleration recorder type SAVER 3L30 made by Lansmont, was placed inside the load. The recorder, equipped with a built-in operational memory, was powered from a lithium battery. The device makes it possible to record up to100 courses of acceleration whose values do not exceed 100 g (g – acceleration of gravity). Acceleration signal is sampled with the rate of 1 kHz, and the error of measurement does not exceed 0.1 g. The recorder communicates with a PC computer through a serial port RS-232, which allows for data acquisition and control. The instrument is enclosed in an aluminium casing having the form of rectangular prism of dimensions 0.076x0.076x0.04 m and mass of 0.4 kg. When registering the course of events during the impact process, the recorder placed within the load is set to operate in event recording mode, and works as an autonomous system, without the need of external control and supply.

To perform the free fall drop tests we prepared unit load of dimensions 0.136x0.136x0.1 m and total mass 0.5 kg (Fig. 2). The load consisted of a box made of corrugated board, inside of which was filled with expanded polystyrene (EPS). The cushion material, whose function was shock protection, had a thickness of 0.03 m.

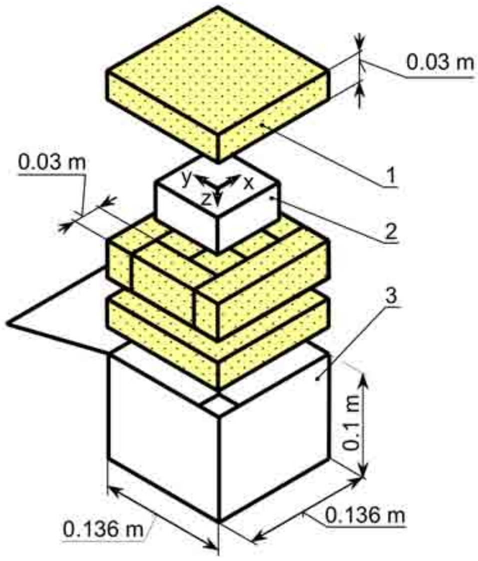

Fig. 2. Unit load prepared to the tests of free fall on rigid ground; 1 – cushion elements, 2 – acceleration recorder, 3 – cardboard box

The load with the acceleration recorder embedded inside it, was subjected to a series of tests of free fall on a smooth, rigid ground. Taking into account the condition of not exceeding the ±100 g acceleration value admissible for the recorder, we selected drop heights for load: h=[0.30; 0.45; 0.60] m. The hazard of falling from the assumed heights was consistent with real hazards in the actual process of transportation from the supplier to the final recipient [7]. The load during tests was manipulated manually and it was dropped four times from the selected height h, on each of corner, edge and wall. Moreover, in case of the load fall on the edge or corner, it was tried to place the gravity centre of the load on the normal of impact, and in case of the fall on the wall – the surface of the chosen wall of the load was parallel positioned to the rigid ground. The height of the free fall was related to the distance between the lowest point of the packaging and the surface of the rigid ground.

4. Proposition of spatial approximation of experimental test results

The data written in the acceleration recorder memory are represented in the rectangular coordinate system (Fig. 3b). Due to planned approximation of the results of experimental investigations, the spherical coordinate system is the more effective representation of these data (Fig. 3a, Fig. 4).

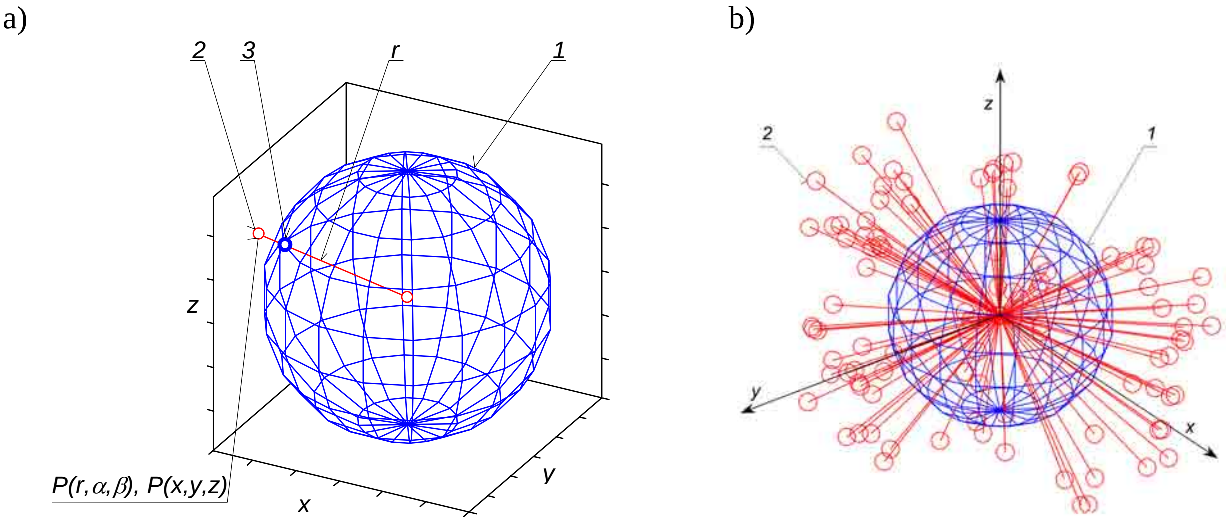

Fig. 3. Data registered during experimental investigations: a) singly impact test, b) series of 100 impact tests; 1 – initial grid of control points of B-spline surface, 2 – acceleration registered during experimental investigations, 3 – projection of point (2) on B-spline surface in radius-vector r direction

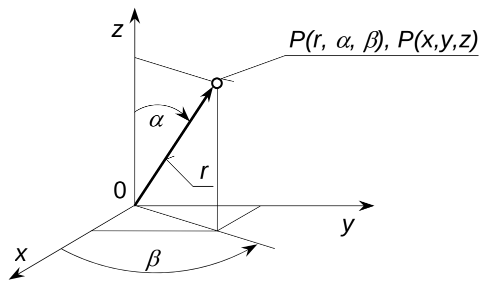

Fig. 4. Spherical and rectangular systems of coordinates

The conversion from the rectangular coordinate system to spherical is defined by dependences:

$$r = \sqrt{x^2 + y^2 + z^2}\,,\ \ \beta = arc\,tg\left(\frac{y}{x}\right),\ \ \alpha = arc\cos\left(\frac{z}{r}\right) \tag{4}$$

$$r \geq 0\,,\ 0 \leq \alpha \leq \pi\,,\ 0 \leq \beta \leq 2\pi \tag{5}$$

and renewed transition from the spherical system to rectangular is described by formulas:

$$x = r\sin\alpha\cos\beta\,,\ \ y = r\sin\alpha\sin\beta\,,\ \ z = r\cos\alpha \tag{6}$$

The results (x_i, y_i, z_i) of experimental investigations of the unit load impact (data in Fig. 3 marked out by the reference 2) are subjected to approximation by means of B-spline surface of third order (1÷3). This approximation was performed using the numeric optimization, whose objective function is the minimization of the sum of squares of relative differences between real accelerations (registered during investigations) and their approximations [1]:

$$\min Q(X) = \sum_{i=1}^{w}\left(\frac{\hat{r}_i - r_i}{\hat{r}_i}\right)^2 \tag{7}$$

where:

$X = [r_1,\ r_2,\ \ldots,\ r_v]$ – decision variables (radius-vectors of independent control points of B-spline surface in spherical co-ordinate system),

$v=(z_n - 1)(z_m - 1) + 2$ – number of independent control points (v=146, if $\Delta\alpha=\pi/9$ and $\Delta\beta=\pi/9$)

$z_m+1=\pi/\Delta\alpha + 1$ – number of control points of meridian,

$z_n+1=2\pi/\Delta\beta+ 2$– number of control points of equator,

w=100 – number of registered impact tests,

$r_i, \hat{r}_i$ – i-th resultant of object acceleration assigned during experimental research and projection of this resultant on B-spline surface.

During optimization, the SQP method (Sequential Quadratic Programming) was used – offered in the Matlab environment.

The initial values of the vector components of decision variables define the grid of control points assigning the sphere surface (Fig. 3a – marked by reference 1). The poles of this grid are common points for all meridians. From this reason, among control points, we can distinguish so-called dependent and independent points.

It is assumed, during the optimization, that the position of the control points of B-spline surface can be modified only through length change of the radius-vectors. The α and β angles of these radius-vectors' position are constant – they are accepted during decision variables vector X defining.

5. Research results

In Fig. 5÷Fig. 10 the investigation results of the free fall of the unit load from the height h=0.3 m, h=0.45 and h=0.6 m are shown. The data registered during experimental investigations are presented by means of markers in the shape of circles. An approximation of these data is represented with the help of B-spline surface. The grid of control points is also shown in these figures. The surface colours are connected with the value of an acceleration resultant. The bigger an acceleration of the load content, the more intensive red colour of the surface. To do correlation of B-spline surface position with respect to the load walls, the drawings contain the load edges and an indication of one of its corners (point in the black colour). The Fig. 5, Fig. 7 and Fig. 9 represent the oblique projection of the surface, and Fig. 6, Fig. 8, Fig. 10 – their orthogonal projections (main, top and left-side views). Numerical values showed in the graphs concern object accelerations in m/s^2.

From the analysis of presented data results, that the effectiveness of mitigation of overloads exerted on the load content is bigger, when the packaging impacts in the corners, than in the walls. The packaging shows larger stiffness of walls (the wall bottom especially – Fig. 8, Fig. 10) than corners. Increased flexibility of the load corners can be caused by the fact that cushion elements and the recorder don't constitute an uniform whole.

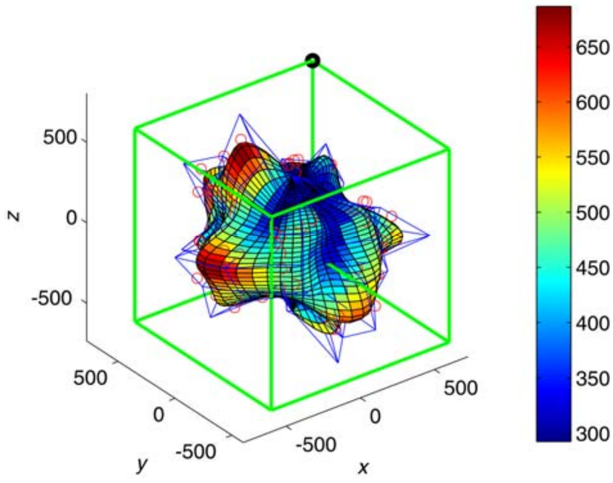

Fig. 5. Result of approximation (in isometric projection) of the of load accelerations registered during free fall from height h=0.3 m

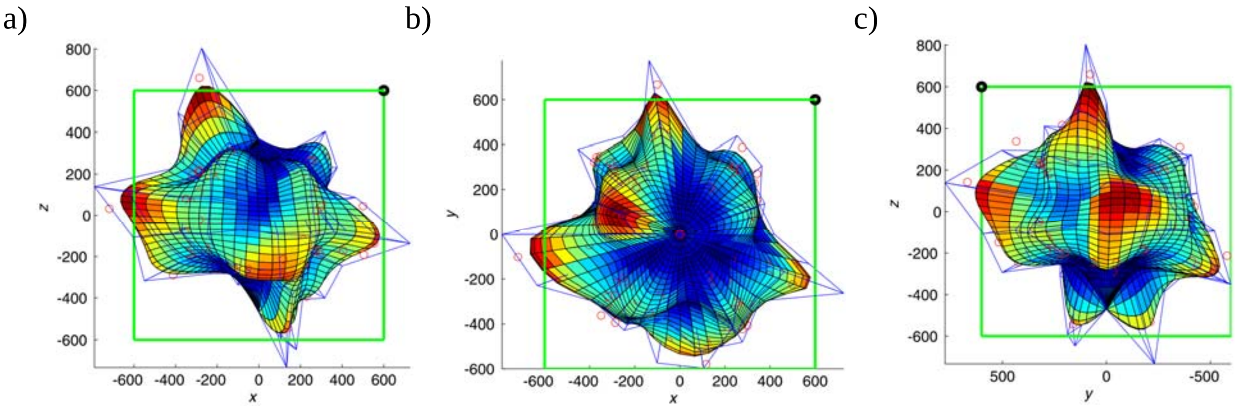

Fig. 6. Orthogonal projection of chart from Fig. 5: a) in the xz plane (main view), b) in the xy plane (top view), c) in the yz plane (left-side view)

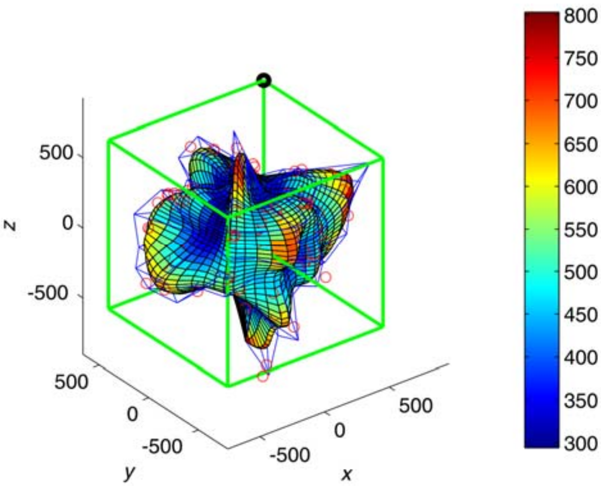

Fig. 7. Result of approximation (in isometric projection) of load accelerations registered during free fall from height h=0.45 m

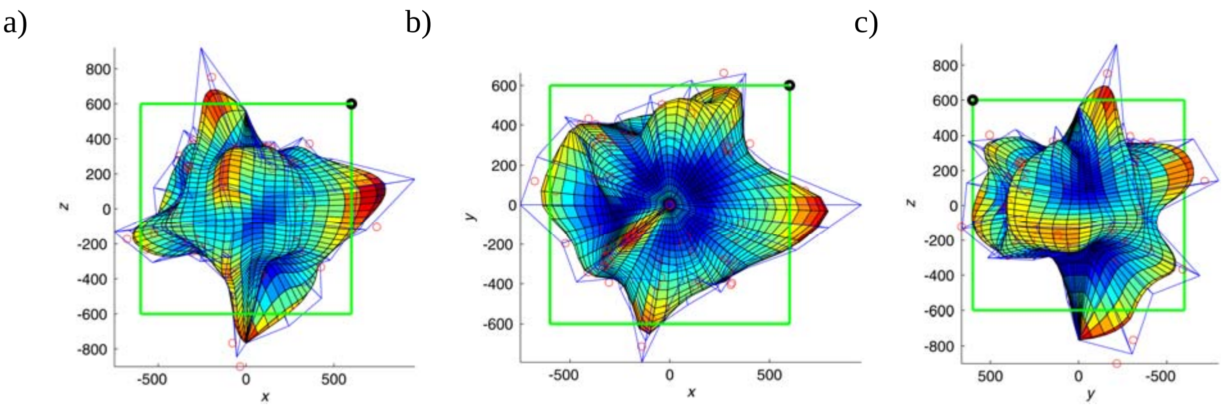

Fig. 8. Orthogonal projection of chart from Fig. 7: a) in the xz plane (main view), b) in the xy plane (top view), c) in the yz plane (left side view)

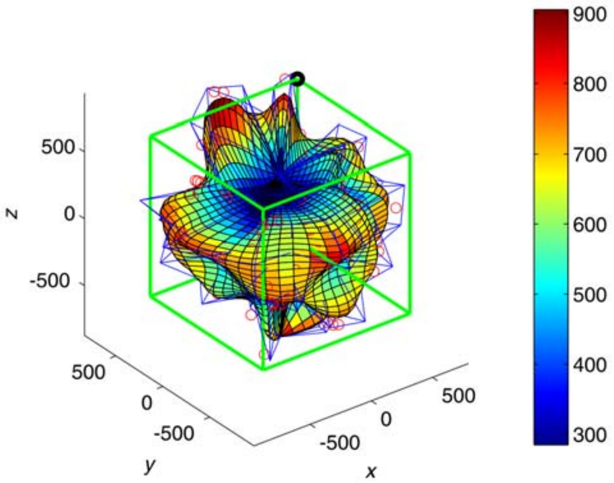

Fig. 9. Result of approximation (in isometric projection) of load accelerations registered during free fall from height h=0.6 m

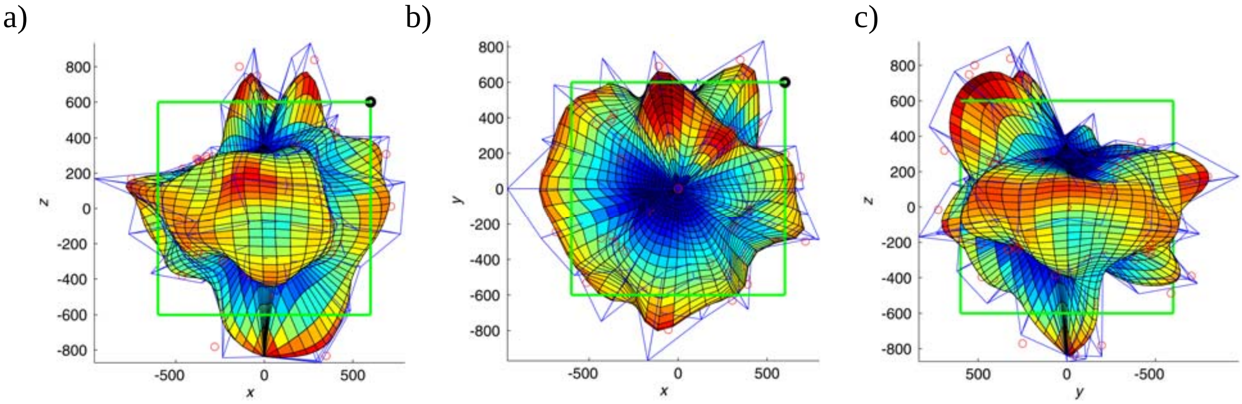

Fig. 10. Orthogonal projection of chart from Fig. 9: a) in the xz plane (main view), b) in the xy plane (top view), c) in the yz plane (left side view)

6. Summary

The following final attentions were formulated:

- Use of B-spline surface makes possible to reproduce complex solid geometry with an application of little number of decision variables.
- B-spline surface approximation enables an intuitive analysis of data obtained during experimental free fall tests of unit loads.
- 3D visualization of acceleration allows precise packaging places identification (in the three-dimensional space) which are critical for the safety of the load content and require constructional improvements.
- Proposed approximation of discreet data recorded by tri-axial sensor enables assigning the closed continuous surface that permits to determine acceleration of load during impact against an obstacle in any direction in 3D space.

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