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A PROPOSAL OF MODIFICATION OF THE ZENNER'S FATIGUE CRITERION FOR THE CASE OF NON-PROPORTIONAL LOADING

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Abstract

In this paper, a modification of the Zenner's fatigue criterion has been proposed. The modification changes the value of "effective shear stress" amplitude for the case of non-proportional load. In this work there's also verification of proposed modification has been made for 4 kinds of materials. In the conclusions, distinctive features of results obtained by using these methods have been pointed out.

Keywords: *fatigue of materials, fatigue curves, accelerated methods*

1. The Idea of Integral Criteria

The Integral approach assumes that an appropriate description of fatigue behavior of a structural element involves summing the damage parameters on all the physical planes, running through the considered point P of the material [3]. Location of these planes is described by means of a sphere representing the material elementary volume (fig.1b). The planes are tangent to the sphere surface in point P. Coordinates of point P are determined by radius vector $\bar{n}$. In turn, the location of vector $\bar{n}$ is determined by angles γ and φ (fig.1).

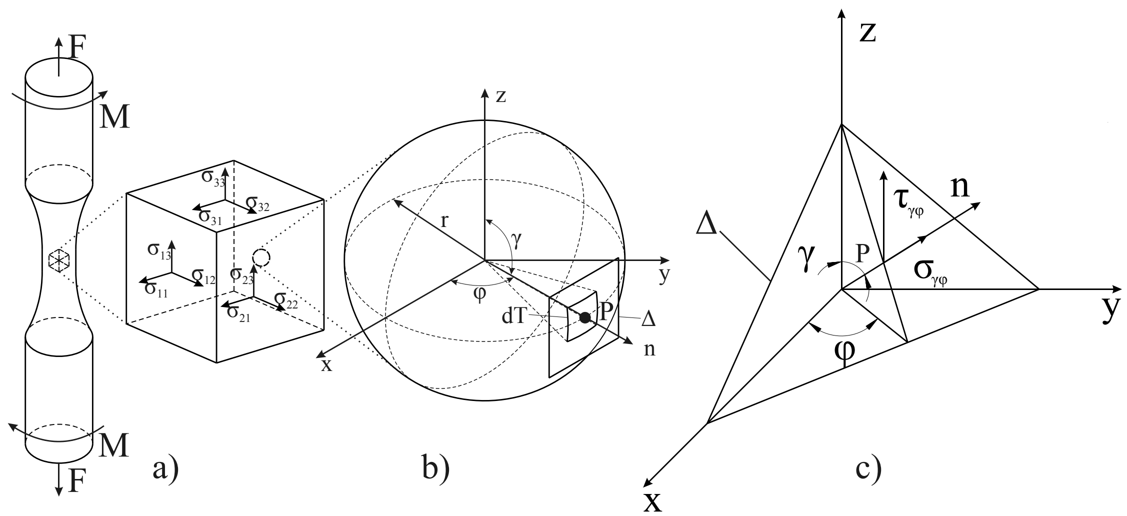

Fig. 1 - A way of description of a physical plane location

The integral approach was used for the first time by Novoshilov [8]. He identified the second invariant of the stress tensor deviator with the square mean from shear stresses acting on all planes running through a given point. The mean can be expressed in the following way:

$$\tau_{RMS} = \sqrt{\frac{1}{T}\int_T \tau_{\gamma\varphi}{}^2\, dT}. \tag{1}$$

If the location of a sectional plane is described in the way given in figure 1, then dT is an elementary surface of sphere with radius $r = 1$ and is equal to:

$$dT = r^2 \sin\gamma\, d\varphi d\gamma = \sin\gamma\, d\varphi d\gamma, \tag{2}$$

a T is the sphere equal surface, that is, the shear stress acting on plane as Δ has been denoted as $\tau_{\gamma\varphi}$. After taking into consideration all the notations, the root mean square from all the stresses has the form:

$$\tau_{RMS} = \frac{1}{4\pi}\int_{\gamma=1}^{\pi}\int_{\varphi=0}^{2\pi} \tau_{\gamma\varphi}{}^2 \sin\gamma\, d\varphi d\gamma. \tag{3}$$

For a plane stress state the formula for $\tau_{\gamma\varphi}{}^2$ can be given as:

$$\begin{aligned}\tau_{\gamma\varphi}{}^2 = {} & \sin^2\gamma\left[\left(\sigma_x{}^2 + \tau_{xy}{}^2\right)\cos^2\varphi + \tau_{xy}{}^2\sin^2\varphi + 2\sigma_x\tau_{xy}\sin\varphi\cos\varphi\right] - \\ & - \sin^4\gamma\,[\sigma_x{}^2\cos^4\varphi + 4\sigma_x\tau_{xy}\sin\varphi\cos^3\varphi + 4\tau_{xy}{}^2\sin^2\varphi\cos^2\varphi].\end{aligned} \tag{4}$$

After substituting formula (4) to (3) and integration we receive:

$$\tau_{RMS} = \left[\frac{2}{15}\left(\sigma_x{}^2 + 3\tau_{xy}{}^2\right)\right]^{\frac{1}{2}}. \tag{5}$$

It can be noticed that the expression in round brackets is equal to square of equivalent stress according to the hypothesis of Huber-Mises-Hencky (HMH):

$$\sigma_{HMH} = \sqrt{\sigma_x{}^2 + 3\tau_{xy}{}^2}. \tag{6}$$

Formulas (3) and (5) can be compared in the following way:

$$\left[\frac{2}{15}\sigma_{HMH}\right]^{\frac{1}{2}} = \left[\frac{1}{4\pi}\int_{\gamma=0}^{\pi}\int_{\varphi=0}^{2\pi} \tau_{\gamma\varphi}{}^2 \sin\gamma d\varphi d\gamma\right]^{\frac{1}{2}}. \tag{7}$$

The formula of equivalent stress according to HMH hypothesis can be derived from the above equation, for constant-amplitude fatigue loads:

$$\sigma_{HMH} = \sqrt{\frac{15}{8\pi}\int_{\gamma=0}^{\pi}\int_{\varphi=0}^{2\pi} \tau_{\gamma\varphi}{}^2 \sin\gamma\, d\varphi d\gamma \le Z_{rc},} \tag{8}$$

where Z_{rc} is fatigue limit for fully reversed tension-compression.

2. Zenner's criterion in terms of integral criteria

The idea of integral approach to HMH hypothesis was developed by Zenner [9]. He notes that, firstly, HMH hypothesis assumes the same ratio $Z_{so}/Z_{rc} = 1/\sqrt{3}$ for all the materials. Actually, this ratio occupies interval $0{,}5 < Z_{so}/Z_{rc} < 0{,}8$ for ductile materials, which should be taken into account in the calculations. Secondly, Zenner says that notation (8) accounts only for shear stress. Experimental tests show that also mean stresses and normal stress amplitudes have an influence on the material fatigue life.

With regard to these observations Zenner formulated a criterion in which ${\tau_{\gamma\varphi}}^2$ replaces the so called 'effective amplitude'. The criterion can be written in the form [9]:

$$\sigma_Z = \sqrt{\frac{15}{8\pi}\int_{\gamma=0}^{\pi}\int_{\varphi=0}^{2\pi}\left(a{\tau_{\gamma\varphi,a}}^2\left(1+m{\tau_{\gamma\varphi,m}}^2\right)+b\sigma_{\gamma\varphi,a}\left(1-n\sigma_{\gamma\varphi,a}\right)^2\right)\sin\gamma\, d\gamma d\varphi} \le Z_{rc}. \qquad (9)$$

In the above notation *a, b, m* and *n* are coefficients dependent on the material constants. In order to calculate them it is necessary to have: tensile – compressive fatigue strength Z_{rc}, torsional fatigue strength Z_{so}, pulsating tensile strength Z_{rj}, pulsating torsional Z_{sj}. The constants are calculated on the basis of the formulas:

$$a = \frac{1}{5}\left(3\left(\frac{Z_{rc}}{Z_{so}}\right)^2 - 4\right), \quad b = \frac{2}{5}\left(3 - \left(\frac{Z_{rc}}{Z_{so}}\right)^2\right), \qquad (10)$$

$$a\cdot m = \frac{{Z_{rc}}^2 - \left(\frac{Z_{rc}}{Z_{so}}\right)^2\left(\frac{Z_{sj}}{2}\right)^2}{\frac{12}{7}\left(\frac{Z_{sj}}{2}\right)^4}, \quad b\cdot n = \frac{{Z_{rc}}^2 - \left(\frac{Z_{rj}}{2}\right)^2 - \frac{4}{21}am\left(\frac{Z_{rj}}{2}\right)^4}{\frac{15}{4}\left(\frac{Z_{rj}}{2}\right)^3} \qquad (11)$$

For load without mean stresses, expression (9) is reduced to the form:

$$\sigma_Z = \sqrt{\frac{15}{8\pi}\int_{\gamma=0}^{\pi}\int_{\varphi=0}^{2\pi}\left(a{\tau_{\gamma\varphi,a}}^2 + b{\sigma_{\gamma\varphi,a}}^2\right)\sin\gamma\, d\gamma d\varphi} \le Z_{rc}. \qquad (12)$$

Thanks to this, it is enough to know values of Z_{rc} and Z_{so}.

3. Zenner's criterion for nonproportional loads

In case of non-proportional loads, that is, when sinusoidal variables of stress are out-of-phase phase, the vector of shear stress on a sectional plane changes not only its module but also its direction, drawing a hodrograph (fig.2).

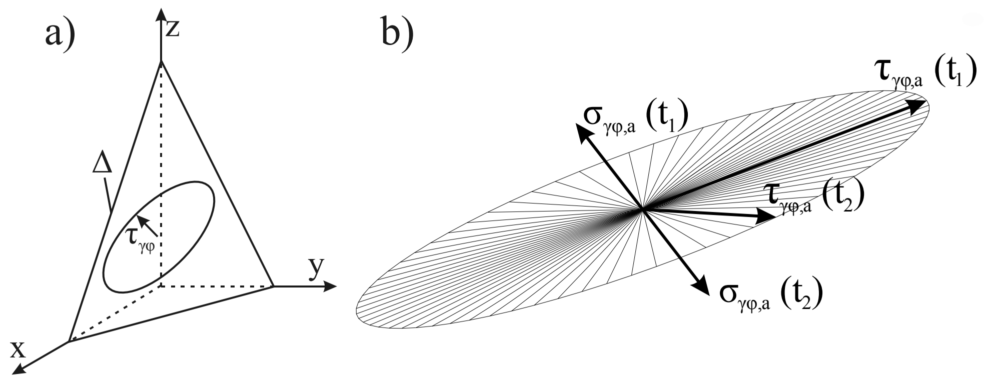

Fig. 2 - Changes of the and normal stress vector during a non-proportional load cycle: a) hodograph of a shear stress vector, b) subsequent positions of a shear stress vector

Zenner's criterion, given in form (12), accounts only for maximal values of modules for normal and shear stress vectors, whereas, tests of micro-structure and fractures of materials exposed to non-proportional load [3] indicate that not only the change of shear stress vector module but also the change of its direction has a large influence on fatigue behavior.

4. Modification of Zenner's criterion due to load non-proportionality

In the proposed modification of Zenner's criterion, the maximal module of shear stress $\tau_{\gamma\varphi,a}$ has been replaced in formula (12) with a quantity accounting for changes of the action course of the shear stress vector.

For tension-compression and torsion, when the components of load are sinusoidal and out-of-phase, the path drawn by the vector of shear stress is an ellipse, as in figure 2. In such a situation it is convenient to describe the changes of shear stress vector location in a system of coordinates, determined by the course of a vector with maximal module $\bar{k}$ and direction perpendicular to it $\bar{n}$ (fig. 3).

Coordinates of unit vectors determining these directions can be obtained from the formulas:

$$\bar{k} = \frac{\bar{\tau}_{\gamma\varphi,max}}{\left\|\bar{\tau}_{\gamma\varphi,max}\right\|}, \qquad \bar{n} = \bar{k}\begin{bmatrix} \cos\frac{\pi}{2} & -\sin\frac{\pi}{2} \\ \sin\frac{\pi}{2} & \cos\frac{\pi}{2} \end{bmatrix}. \tag{13}$$

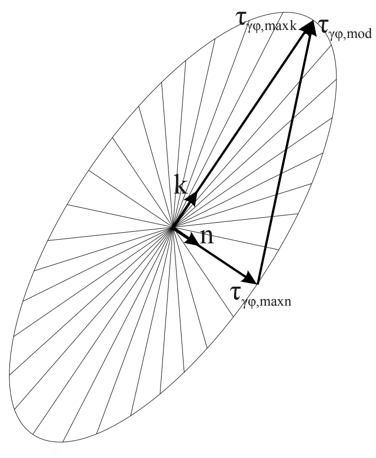

Fig. 3 - Unit vectors $\bar{k}$ and $\bar{n}$ determining directions used for a description of the tangent stress vector location changes and the proposed quantity $\tau_{\gamma\varphi,mod}$

The proposed quantity, substituting $\tau_{\gamma\varphi,a}$ in Zenner's criterion, is a module of vector which is the result of the sum of vectors whose projections on directions $\bar{k}$ and $\bar{n}$ reach the highest values during the cycle of loading (fig.3). This quantity can be expressed by the formula:

$$\tau_{\gamma\varphi,mod} = \left\|\bar{\tau}_{\gamma\varphi,max\bar{k}} + \bar{\tau}_{\gamma\varphi,max\bar{n}}\right\|. \quad (14)$$

When there is no phase shift between the components of loading, the value of $\tau_{\gamma\varphi,mod} = \tau_{\gamma\varphi,a}$ is identical as in an unchanged Zenner's criterion.

A similar solution to the description of non-proportionality was used by Freitas and others [5], as a measurement of the loading path non-proportionality on an octahedral plane.

Eventually, a modified Zenner's criterion is expressed by the dependence:

$$\sigma_Z = \sqrt{\frac{15}{8\pi}\int_{\gamma=0}^{\pi}\int_{\varphi=0}^{2\pi}\left(a\left\|\bar{\tau}_{\gamma\varphi,max\bar{k}} + \bar{\tau}_{\gamma\varphi,max\bar{n}}\right\|^2 + b\sigma_{\gamma\varphi,a}{}^2\right)\sin\gamma\, d\gamma d\varphi} \leq Z_{rc}. \quad (15)$$

5. Verification of Modified Zenner's criterion

Modified Zenner's criterion was verified for the aluminum alloy 7075-T651 [1], steel 1045 for the data from works [2] and [6] as well as tests of steel X2CrNiMo17-12-2 [7].

The verification involvs comparing experimental fatigue life with the computing one. Statistical parameters used for the comparison are: the mean scatter of fatigue life T_N and the mean-square error of fatigue life estimation T_{RMS}, which has been calculated according to formulas [7]:

$$T_N = 10^{\bar{E}}, \quad (16)$$

$$\bar{E} = \frac{1}{n}\sum_{i=1}^{n}\log\left(\frac{N_{exp,i}}{N_{cal,i}}\right), \quad (17)$$

$$T_{RMS} = 10^{E_{RMS}}, \quad (18)$$

$$E_{RMS} = \sqrt{\frac{\sum_{i=1}^{n} log^2\left(\frac{N_{exp,i}}{N_{cal,i}}\right)}{n}}. \quad (19)$$

It has been assumed that acceptable results should be included in interval 0,5÷2 for and 1÷2 for T_{RMS}. Verification results have been presented below in the form of charts of comparison between calculated life N_{cal} and experimental N_{exp} and tables with values T_N and T_{RMS}.

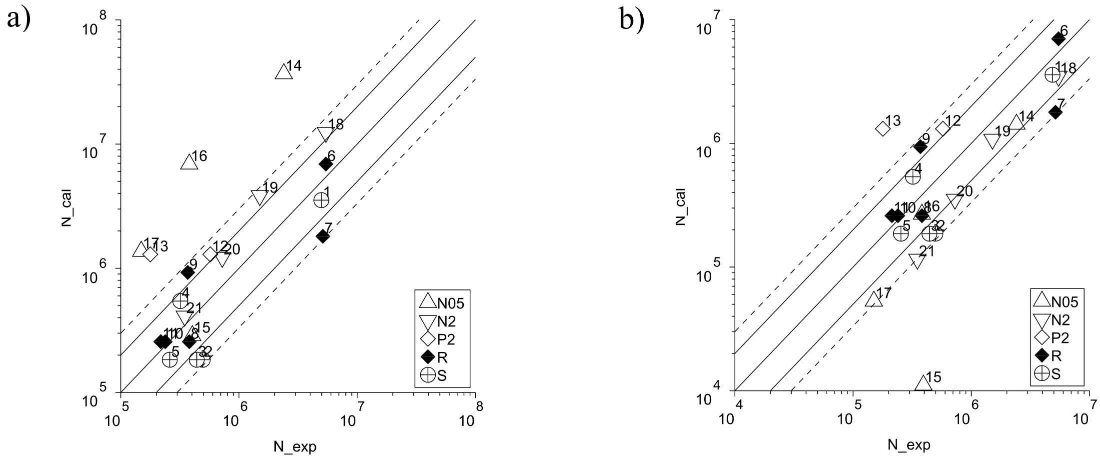

Fig. 4a - Chart of comparison between experimental fatigue life and calculated obtained for: a) Zenner's criterion for 1045 steel [6], b) modified Zenner's criterion for 1045 steel [6]

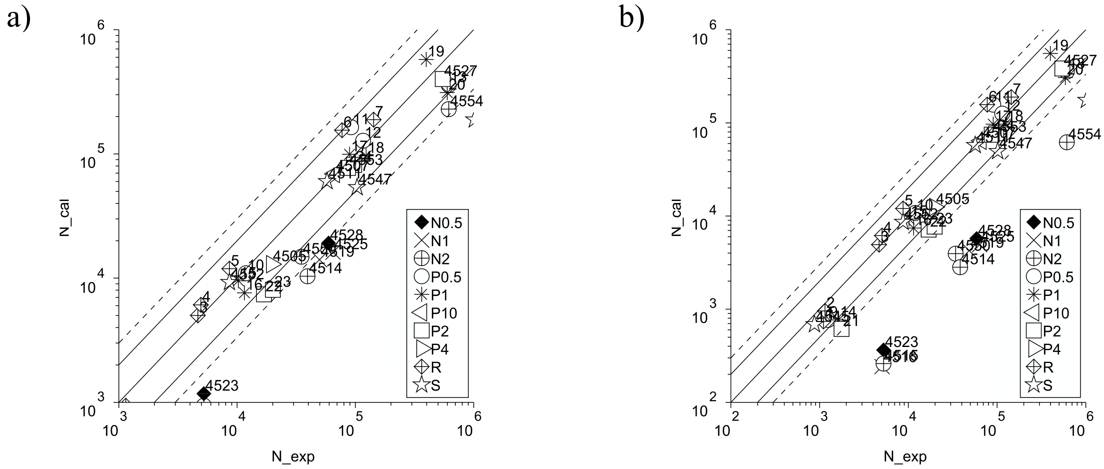

Fig. 5a - Chart of comparison between experimental fatigue life and calculated obtained for: a) Zenner's criterion for 1045 steel [2], b) modified Zenner's criterion for 1045 steel[2]

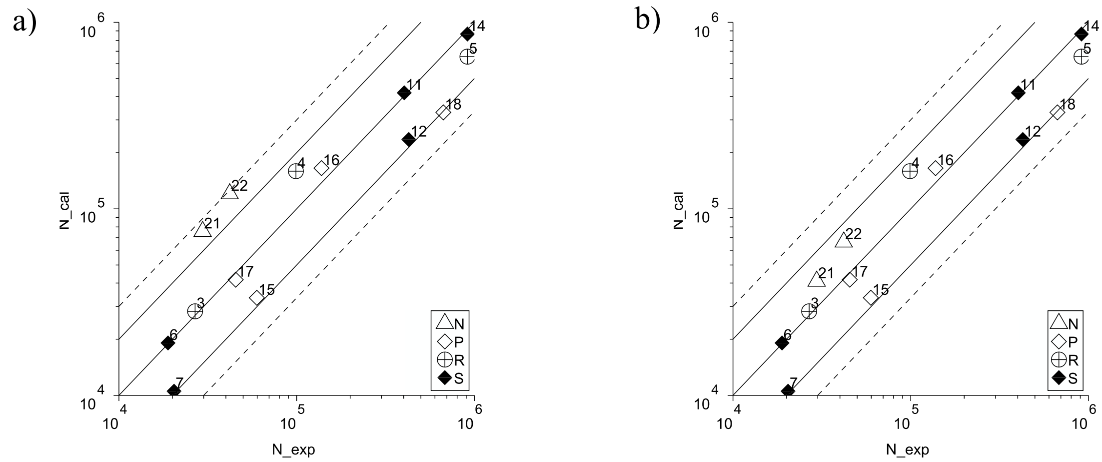

Fig. 6a - Chart of comparison between experimental fatigue life and calculated obtained for: a) Zenner's criterion for 7075-T651 aluminum alloy [1], b) modified Zenner's criterion for 7075-T651 aluminum alloy [1]

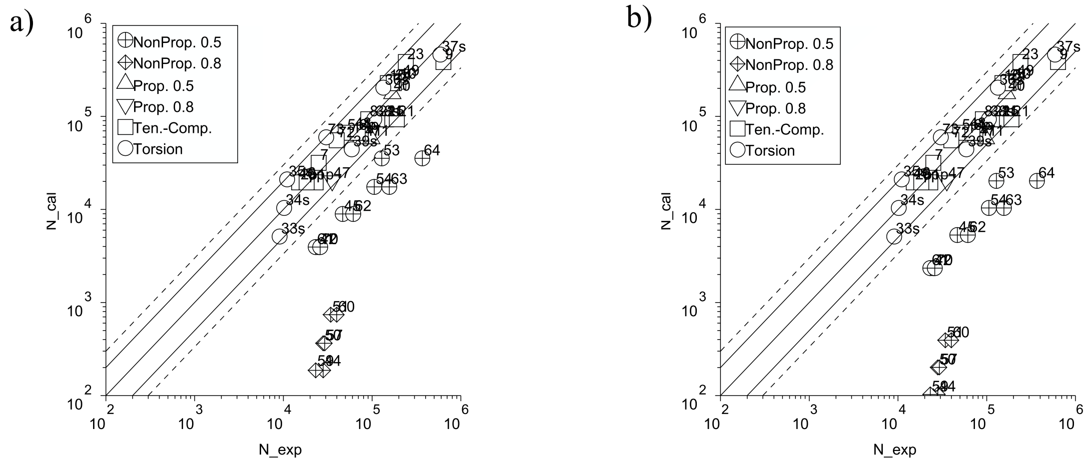

Fig. 7a - Chart of comparison between experimental fatigue life and calculated obtained for: a) Zenner's criterion for X2CrNiMo17-12-2 steel[7] b) modified Zenner's criterion for X2CrNiMo17-12-2 steel [7]

Tab 1. Comparison of T_N and T_{RMS} parameters for Zenner's criterion before and after modification for 1045 steel [6]

	Zenner's criterion		Modified Zenner's criterion	
	T_N	T_{RMS}	T_N	T_{RMS}
Tension	1,00	1,84	1,00	1,84
Torsion	1,49	1,95	1,49	1,95
Proportional load $\lambda = 2$	0,24	4,61	0,24	4,61
Non-proportional load $\lambda = 0,5$	0,15	9,73	3,97	6,63
Non-proportional load $\lambda = 2$	0,54	1,97	1,90	2,03

Tab 2. Comparison of T_N and T_{RMS} parameters for Zenner's criterion before and after modification for 1045 steel [2]

	Zenner's criterion		Modified Zenner's criterion	
	T_N	T_{RMS}	T_N	T_{RMS}
Tension	1,00	1,60	1,00	1,60
Torsion	1,62	2,25	1,73	2,34
Proportional load $\lambda = 0,5$	1,09	1,51	1,10	1,51
Proportional load $\lambda = 1$	1,27	1,56	1,3	1,57
Proportional load $\lambda = 2$	1,75	1,95	1,83	2,02
Proportional load $\lambda = 4$	1,54	1,54	1,60	1,60
Proportional load $\lambda = 10$	0,97	1,03	1,03	1,03
Non-proportional load $\lambda = 0,5$	3,72	3,77	12,16	12,24
Non-proportional load $\lambda = 1$	5,36	5,60	17,51	17,98
Non-proportional load $\lambda = 2$	3,39	3,54	12,44	12,70

Tab 3. Comparison of T_N and T_{RMS} parameters for Zenner's criterion before and after modification for 7075-T651 aluminum alloy [1]

	Zenner's criterion		Modified Zenner's criterion	
	T_N	T_{RMS}	T_N	T_{RMS}
Tension	1,00	1,36	1,00	1,36
Torsion	1,05	1,71	1,05	1,71
Proportional load	1,22	1,53	1,22	1,53
Non-proportional load	0,62	2,4	1,13	2,11

Tab 4. Comparison of T_N and T_{RMS} parameters for Zenner's criterion before and after modification for X2CrNiMo17-12-2 steel [7]

	Zenner's criterion		Modified Zenner's criterion	
	T_N	T_{RMS}	T_N	T_{RMS}
Tension	1,00	1,41	1	1,41
Torsion	0,94	1,54	0,94	1,54
Proportional load $\lambda = 0{,}5$	1,14	1,45	1,14	1,45
Proportional load $\lambda = 0{,}8$	1,09	1,40	1,09	1,40
Non-proportional load $\lambda = 0{,}5$	6,34	6,49	10,83	11,02
Non-proportional load $\lambda = 0{,}8$	99,76	101,92	185,43	188,97

6. Conclusions

On the basis of the carried out analysis it can be said that in case of steel 1045 [6] the proposed modification of Zenner's criterion improved the obtained values of equivalent stresses for non-proportional loads. In case of steel 1045 [2] there occurred a re-estimation of equivalent stresses which caused smaller values of computing fatigue lives as compared to the experimental ones. For aluminum alloy 7075-T651 [1] the results obtained with the use of the modification indicate improvement in the obtained equivalent values. However, for austenite steel X2CrNiMo17-12-2 [7] the obtained results turned out to be not acceptable.

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