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ANALYSIS OF THE QUALITY OF ELECTRICITY ON THE BASIS OF THE RESULTS OF THE RESEARCH MODEL OF LOW VOLTAGE LINE

Piotr Kolber, Daniel Perczyński, Bogdan Landowski

Uniwersytet Technologiczno-Przyrodniczy
ul. S.Kaliskiego 7, 85-796 Bydgoszcz, Polska
tel.: 052 3408297, fax: 052 3408495
e-mail: pkolber@utp.edu.pl

Summary

The work shows a model of a low voltage line (LV), loaded asymmetrical (asymmetry load), feeding scattered customers. The object of the research is an LV line supplying rural customers. Analysis of the tests results of the built model allows among others, evaluation of the values that characterise the quality of the electricity supplied to customers and the parameters that describe the objective of the research. In order to assess the quality of electricity (provided to customers) the algorithm calculation was compiled and on the basis of the developed model, it is possible to calculate the values of voltage sags and deviations on the individual points of the line. In addition, it enables the designation of the asymmetry of voltage (the coefficients of voltage asymmetry). The figures characterising the quality of electricity can be appointed to different parameters describing an LV, inter alia, distribution of customers along the line, what type of customers are connected to the line (productive and unproductive), the nature and extent of the load and the degree of load asymmetry.

Keywords: *low voltage line, loads asymmetry, voltage drops, voltage deviations, voltage asymmetry*

1. Introduction

The reasons for the presence of currents and voltages asymmetry in three-phase grids are primarily non-symmetrical loads and non-symmetrical transmission devices. The work three-phase electrification system is called asymmetrical or non-symmetrical when the work conditions of one or all phases are different. In three-phase systems there are short-term and long-term operational, asymmetrical modes of work. Short-term asymmetry is usually caused by emergency processes. These are primarily asymmetrical short circuit and disconnections of one phase in the cycle of automatic re-connection. Long-term asymmetry in a transmission system can occur when you enable the network an asymmetrical load, with the presence of asymmetrical elements or asymmetric work of transmission system.

Asymmetry caused by an asymmetrical load is called the transverse asymmetry. Asymmetry caused by an asymmetrical transmission element (dissimilar own impedances and interactions of the lines) or asymmetric and cross currents flowing in this element is called the longitudinal asymmetry [3].

Asymmetrical states of work are present especially in rural distribution networks of low-voltage power systems. Such states of work follow from asymmetry of loads in phases, caused by an unbalanced power distribution for each phase of single-phase receivers and random enable receivers to the network.

In the designing and construction of a rural network efforts are made to proportionally and evenly connect all single-phase receivers to the individual phases, but it does not always meet this criterion.

Even in conditions of balanced connection of receivers, an asymmetry occurs in low-voltage lines. This is because each of receivers can be enabled or disabled from the network depending on changing situation. Usually a load of each phase changes in time, regardless of other phases load.

Asymmetry of line caused by dissimilar reciprocal impedances has caused a negative impact on the quality of electricity, particularly in the case of overhead low-voltage lines, built with flat laid cables. In the lines of higher voltages (above 15 kV) the asymmetry is reduced by changes in the mutual positions of the cables.

2. The object of research

The object of research is a low voltage line, loaded asymmetrical, supplying rural customers. The analysis adopted a line model of 2^{nd} type, characterised by zero values of crosswise parameters in the substitute scheme (crosswise parameters: unit conductance of G_o line, unit susceptance of B_o line, takes into account medium and high-voltage lines) [1]. This type of line is characteristic of rural areas; four-cables and overhead line, with flat laid cables, and as the most widespread, is also the most unfavourable, due to mutual impedance of line in terms of asymmetry loads, the quality of the power is thus supplied to customers. Different values of mutual impedances make that line as an asymmetrical element of electricity transmission. In addition, assumed that receptions have a resistance-induction character.

Test Object Model is presented in the Figure 1.

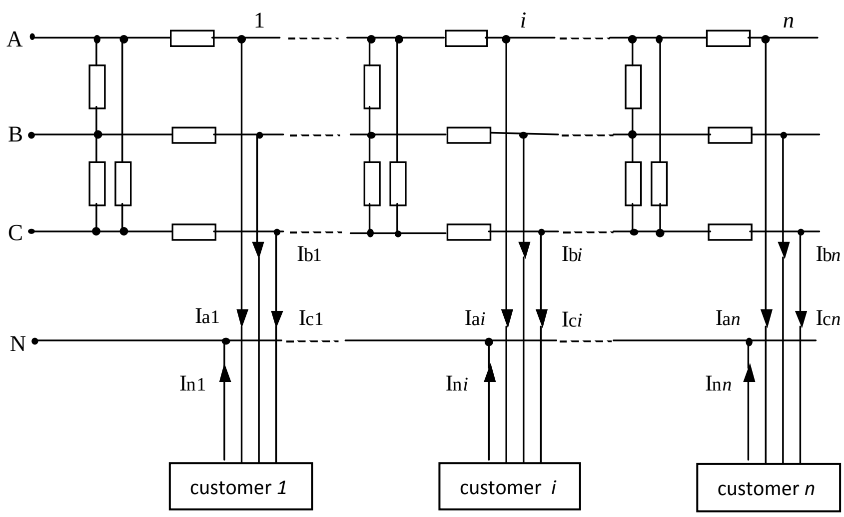

Fig.1. Model of low voltage line asymmetrical loaded

3. Model of low voltage line

Model of low voltage line include, inter alia:

1) Fixed parameters:

f – network frequency (f = 50 Hz),

U_{st} – voltage phase at the beginning of the line (in a station); U_{st} = 240 V,

γ - wire conductivity (for Al. γ =34,8 m/Ω·mm^2),

s , s_N – cross-section of the phase and neutral wires:,
b_{AN}, b_{BN}, b_{CN} – distance between the wires A i N, B i N, C i N,
b_{AB}, b_{BC}, b_{AC} – distance between wires A and B, B and C, A and C,

2) Data determining the topology of the line:
   l – length of the line [m],
   n – number of receipt points,
   l_i – distance of *i-th* receipt point from the beginning a line; [m].

3) Resistance values on sections between receipt points; [Ω]
   a) for a phase wire

$$R_{(k-1)k} = \frac{l_k - l_{k-1}}{\gamma S} \tag{1}$$

   b) for a neutral wire

$$R_{N(k-1)k} = \frac{l_k - l_{k-1}}{\gamma S_N} \tag{2}$$

4) Unit reacatances, own and mutual of loops wire; [Ω/m]
   a) reactances of loops wire respectively A and N, B and N, C and N

$$X_{AN} = \left(0{,}15\log\frac{b_{AN}^2}{r_A \cdot r_N}\right)\cdot 10^{-3} ,$$
$$X_{BN} = \left(0{,}15\log\frac{b_{BN}^2}{r_B \cdot r_N}\right)\cdot 10^{-3} , \tag{3}$$
$$X_{CN} = \left(0{,}15\log\frac{b_{CN}^2}{r_C \cdot r_N}\right)\cdot 10^{-3},$$

   where:

   r_A, r_B, r_C, i r_N – radii of cross-sections phase and zero wires

   b) reactances of loop wire A and B, B and C, C and A

$$X_{ANB} = \left(0{,}15\log\frac{b_{AN} \cdot b_{BN}}{b_{AB} \cdot r_N}\right)\cdot 10^{-3},$$
$$X_{BNC} = \left(0{,}15\log\frac{b_{BN} \cdot b_{CN}}{b_{BC} \cdot r_N}\right)\cdot 10^{-3}, \tag{4}$$
$$X_{ANC} = \left(0{,}15\log\frac{b_{AN} \cdot b_{CN}}{b_{AC} \cdot r_N}\right)\cdot 10^{-3}.$$

5) The power values at individual receipt points
   In every group of value of three powers phase at receipt points, you can separate: P_{maxi} – power of the most loaded phase, P_{pi} – power of indirectly loaded phase , P_{mini} – power of the least loaded phase.

$$P_{\max i} = \frac{1}{1 + k_{1i} + k_{2i}} P_{si} = w_i P_{si} , \tag{5}$$

where:

- maximum load factor [2]

$$w_i = \frac{P_{\max i}}{P_{si}}, \tag{6}$$

- intermediate load factor

$$k_{1i} = \frac{P_{pi}}{P_{\max i}}, \tag{7}$$

- minimum load factor

$$k_{2i} = \frac{P_{\min i}}{P_{\max i}}, \tag{8}$$

- peak power in the *i-th* receipt point

$$P_{si} = \sum_{f=1}^{3} U_{fi} I_{fi} \cos\varphi_{fi} = P_{\max i} + P_{pi} + P_{\min i}, \tag{9}$$

where:

U_{fi}, I_{fi}, $\cos\varphi_{fi}$ – phase values of: voltage, current and the cosine at the *i-th* receipt point (f = A,B,C).

6) The values of phase currents measured in individual connections:

$$\underline{I}_{Ai} = \frac{P_{Ai}}{U_n \cdot \cos\varphi_{Ai}} e^{-j\varphi_{Ai}},$$

$$\underline{I}_{Bi} = \frac{P_{Bi}}{U_n \cdot \cos\varphi_{Bi}} e^{-j\varphi_{Bi}} e^{j240^o}, \tag{10}$$

$$\underline{I}_{Ci} = \frac{P_{Ci}}{U_n \cdot \cos\varphi_{Ci}} e^{-j\varphi_{Ci}} e^{j120^o},$$

where:

U_n – value of nominal voltage.

7) The phase current values in a line between (*k*-1) and *k-th* receipt point

$$\underline{I}_{A(k-1)k} = \sum_{i=k}^{n} \underline{I}_{Ai} \quad , \quad \underline{I}_{B(k-1)k} = \sum_{i=k}^{n} \underline{I}_{Bi} \quad , \quad \underline{I}_{C(k-1)k} = \sum_{i=k}^{n} \underline{I}_{Ci} \ . \tag{11}$$

8) Resistance phase losses of voltage on the section: phase wire – neutral wire between (*k*-1), and *k-th* receipt point

$$\Delta\underline{U}^{R}_{AN(k-1)k} = \Delta\underline{U}^{R}_{A(k-1)k} - \Delta\underline{U}^{R}_{N(k-1)k},$$

$$\Delta\underline{U}^{R}_{BN(k-1)k} = \Delta\underline{U}^{R}_{B(k-1)k} - \Delta\underline{U}^{R}_{N(k-1)k}, \tag{12}$$

$$\Delta\underline{U}^{R}_{CN(k-1)k} = \Delta\underline{U}^{R}_{C(k-1)k} - \Delta\underline{U}^{R}_{N(k-1)k},$$

where:

- resistance phase losses of voltage between receipt points

$$\Delta\underline{U}^{R}_{A(k-1)k}=\underline{I}_{A(k-1)k}\cdot R_{(k-1)k},$$
$$\Delta\underline{U}^{R}_{B(k-1)k}=\underline{I}_{B(k-1)k}\cdot R_{(k-1)k},\tag{13}$$
$$\Delta\underline{U}^{R}_{C(k-1)k}=\underline{I}_{C(k-1)k}\cdot R_{(k-1)k},$$

- resistance losses of voltage in neutral wire

$$\Delta\underline{U}^{R}_{N(k-1)k}=-(\underline{I}_{A(k-1)k}+\underline{I}_{B(k-1)k}+\underline{I}_{C(k-1)k})\cdot R_{N(k-1)k}.\tag{14}$$

9) Inductive phase losses of voltage on the section: phase wire – neutral wire (*k*-1) and *k-th* receipt point

$$\Delta\underline{U}^{X}_{AN(k-1)k}=\Delta\underline{U}^{X}_{A(k-1)k}-\Delta\underline{U}^{X}_{N(k-1)k},$$
$$\Delta\underline{U}^{X}_{BN(k-1)k}=\Delta\underline{U}^{X}_{B(k-1)k}-\Delta\underline{U}^{X}_{N(k-1)k},\tag{15}$$
$$\Delta\underline{U}^{X}_{CN(k-1)k}=\Delta\underline{U}^{X}_{C(k-1)k}-\Delta\underline{U}^{X}_{N(k-1)k},$$

- inductive phase losses of voltage between neighboring receipt points

$$\Delta\underline{U}^{X}_{A(k-1)k}=\frac{l_{(k-1)k}}{2}\left(\underline{I}_{A(k-1)k}\cdot jX_{AN}+\underline{I}_{B(k-1)k}\cdot jX_{ANB}+\underline{I}_{C(k-1)k}\cdot jX_{ANC}\right),$$
$$\Delta\underline{U}^{X}_{B(k-1)k}=\frac{l_{(k-1)k}}{2}\left(\underline{I}_{A(k-1)k}\cdot jX_{ANB}+\underline{I}_{B(k-1)k}\cdot jX_{BN}+\underline{I}_{C(k-1)k}\cdot jX_{BNC}\right),\tag{16}$$
$$\Delta\underline{U}^{X}_{C(k-1)k}=\frac{l_{(k-1)k}}{2}\left(\underline{I}_{A(k-1)k}\cdot jX_{ANC}+\underline{I}_{B(k-1)k}\cdot jX_{BNC}+\underline{I}_{C(k-1)k}\cdot jX_{CN}\right),$$

- inductive loss of voltage in neutral wire on the section as above

$$\Delta\underline{U}^{X}_{N(k-1)k}=-\frac{l_{(k-1)k}}{2}\left(\underline{I}_{A(k-1)k}\cdot jX_{AN}+\underline{I}_{B(k-1)k}\cdot jX_{BN}+\underline{I}_{C(k-1)k}\cdot jX_{CN}\right).\tag{17}$$

10) Phase losses of voltage from the beginning of the line to the *i-th* receipt point

$$\Delta\underline{U}_{ANi}=\sum_{k=1}^{i}\left(\Delta\underline{U}^{R}_{AN(k-1)k}+\Delta\underline{U}^{X}_{AN(k-1)k}\right),$$
$$\Delta\underline{U}_{BNi}=\sum_{k=1}^{i}\left(\Delta\underline{U}^{R}_{BN(k-1)k}+\Delta\underline{U}^{X}_{BN(k-1)k}\right),\tag{18}$$
$$\Delta\underline{U}_{CNi}=\sum_{k=1}^{i}\left(\Delta\underline{U}^{R}_{CN(k-1)k}+\Delta\underline{U}^{X}_{CN(k-1)k}\right).$$

11) Phase voltages and between wires in the *i-th* point of receipt point

a) phases voltages

- vectors / - modules

$$\underline{U}_{Ai}=U_{st}-\Delta\underline{U}_{ANi},\qquad U_{Ai}=\left|\underline{U}_{Ai}\right|,$$
$$\underline{U}_{Bi}=U_{st}\cdot e^{j240^{o}}-\Delta\underline{U}_{BNi},\qquad U_{Bi}=\left|\underline{U}_{Bi}\right|,\tag{19}$$
$$\underline{U}_{Ci}=U_{st}\cdot e^{j120^{o}}-\Delta\underline{U}_{CNi},\qquad U_{Ci}=\left|\underline{U}_{Ci}\right|,$$

b) voltages between wires

- vectors / - modules

$$\underline{U}_{ABi}=\underline{U}_{Ai}-\underline{U}_{Bi},\qquad U_{ABi}=\left|\underline{U}_{ABi}\right|,$$
$$\underline{U}_{BCi}=\underline{U}_{Bi}-\underline{U}_{Ci},\qquad U_{BCi}=\left|\underline{U}_{BCi}\right|,\tag{20}$$

$$\underline{U}_{CAi} = \underline{U}_{Ci} - \underline{U}_{Ai} \ , \qquad U_{CAi} = \left|\underline{U}_{CAi}\right| .$$

12) Phase drops and voltage deviations [4]

a) voltage drops

b) voltage deviations

$$\Delta U_{Ai} = U_{st} - U_{Ai} , \qquad \delta U_{Ai} = \frac{U_{Ai} - U_n}{U_n} \cdot 100\% ,$$

$$\Delta U_{Bi} = U_{st} - U_{Bi} , \qquad \delta U_{Bi} = \frac{U_{Bi} - U_n}{U_n} \cdot 100\% , \tag{21}$$

$$\Delta U_{Ci} = U_{st} - U_{Ci} , \qquad \delta U_{Ci} = \frac{U_{Ci} - U_n}{U_n} \cdot 100\% .$$

13) Coefficients of voltage asymmetry [3]

a) opposite order

b) zero order

$$\alpha_{U_2}(i) = \frac{U_{2i}}{U_{1i}} \cdot 100\% , \qquad \alpha_{U_0}(i) = \frac{U_{0i}}{U_{1i}} \cdot 100\% , \tag{22}$$

where:

U_{2i}, U_{1i}, U_{0i} – symmetrical components of voltage; in opposite order, matching, and zero order

$$U_{2i} = \frac{1}{3}\sqrt{U_{Ai}^2 + U_{Bi}^2 + U_{Ci}^2 - 2U_{Ai}U_{Bi}\cos(\alpha_i - \frac{\pi}{3}) - 2U_{Bi}U_{Ci}\cos(\beta_i - \frac{\pi}{3}) - 2U_{Ci}U_{Ai}\cos(\alpha_i + \beta_i + \frac{\pi}{3})} ,$$

$$U_{1i} = \frac{1}{3}\sqrt{U_{Ai}^2 + U_{Bi}^2 + U_{Ci}^2 - 2U_{Ai}U_{Bi}\cos(\alpha_i + \frac{\pi}{3}) - 2U_{Bi}U_{Ci}\cos(\beta_i + \frac{\pi}{3}) - 2U_{Ci}U_{Ai}\cos(\alpha_i + \beta_i - \frac{\pi}{3})} ,$$

(23)

$$U_{0i} = \frac{1}{3}\sqrt{U_{Ai}^2 + U_{Bi}^2 + U_{Ci}^2 + 2U_{Ai}U_{Bi}\cos\alpha_i + 2U_{iB}U_{Ci}\cos\beta_i + 2U_{Ci}U_{Ai}\cos\beta_i} ,$$

$$\alpha_i = \arccos\frac{U_{Ai}^2 + U_{Bi}^2 - U_{ABi}^2}{2U_{Ai}U_{Bi}} , \qquad \beta_i = \arccos\frac{U_{Bi}^2 + U_{Ci}^2 - U_{BCi}^2}{2U_{Bi}U_{Ci}} . \tag{24}$$

In the case of an angle symmetry: $\alpha = 2\pi/3$, $\beta = 2\pi/3$

According to PN-EN 50160 standards, values of deviations should be within the limits of:

$$- 10\% < \delta U_f < +10\% \tag{25}$$

while the value of the coefficient of voltage asymmetry (for the network, restrictions concern only the values of the asymmetry coefficient of the opposite order)

$$\alpha_{U2} < \alpha_{dop} = 2\% \tag{26}$$

4. Selected results of simulations

Examples of calculations carried out for overhead line LV with simple track, specified cross-section wires ($s = s_n = 50$ mm^2) and a fixed topology, so the length $l = 1080$ m, the number of receipt points of $n = 10$. Assume customers evenly along the line. At the beginning, values of the peak power P_{si} were generated for the individual connections and its average value was $P_{sg} = 5{,}753$ kW (on the basis of research results in rural customers). Distribution of the power on

time stages was implemented by determining the values of coefficients described in formulas (6), (7), (8).
For several values of the maximum load factor obtained on the basis of research results concerning loads in rural customers:

$$w_i = \overline{w} - \sigma_w = 0{,}406 \quad w_i = \overline{w} = 0{,}541 \quad w_i = \overline{w} + \sigma_w = 0{,}675 \quad w_i = \overline{w} + 2\sigma_w = 0{,}709$$

values of coefficient k_{1i} were generated. On the basis of mutual relations between the asymmetry loads factors resulting from the formula (5), fixed the value of the coefficient k_{2i} in individual receipt points of the line. On this basis, obtained values of coefficients, values of peak power and phase powers were calculated in individual connections. Values of phase coefficients were generated on the basis of measurements carried out in a transformer station MV/LV (average value of the cosφ = 0,923). This enabled the calculation of phase currents in receipt points according to the formula (10). These currents corresponding to the phase powers at random, were attributed to the phases of the line. This allowed us to define the value of currents in the line, drops and levels of phase voltages and therefore values of voltage deviations and coefficients of voltage asymmetry, which are among the parameters which characterise the quality of electricity in low-voltage power line. Table 1. shows the values of coefficient voltage asymmetry and voltage deviations for the most loaded phase and at the end of the line for a specific level of loads asymmetry in customers, which largely are characterised by the coefficient of maximum load w_i (it determines a level of phase load as dominant in relation to the remaining phase loads).

Tab. 1. Results of qualitative parameters of electricity at the end of the line for a specific level of asymmetry loads in customers for the peak of load.

w_i	α_{U2}	α_{Uo}	$\delta U_{f\max}$
$\overline{w} - \sigma_w = 0{,}406$	1,12	5,41	-11,9
$\overline{w} = 0{,}541$	2,00	9,85	-20,1
$\overline{w} + \sigma_w = 0{,}675$	2,08	10,67	-21
$\overline{w} + 2\sigma_w = 0{,}709$	2,77	14,39	-23,1

where:

$\overline{w}$, σ_w - mean value and standard deviation of the maximum load factor (w_i) on the basis of research concerning phase loads in rural customers for the peak of load,

α_{U2}, α_{Uo} - values of coefficients of voltage asymmetry for opposite order and zero-order,

$\delta U_{f\max}$ - value of voltage deviations in the most loaded stage.

5. Conclusions

Analysis of the results of simulation studies allows us to determine the sensitivity of the system of low voltage line, on a degree of asymmetry loads of customers in terms of the quality of supplied electricity. For a relatively moderate asymmetry of loads occuring in customers (w = 0,406; for the symmetry of the phase loads w = 0.33), value of the coefficient of voltage asymmetry, ranged limits (α_2 = 1,12% α_{dop} = 2%), which took place, overrun limit of voltage deviations in the phase of the most loaded (δU_{fmax} =-11.9% < -10%). It must be noticed that asymmetry loads overlapped the asymmetry of line as a transmission element. In the case of very explicit asymmetry customer loads (w = 0.709) the value of coefficient exceeded the limit value of the asymmetry voltage (α_{U2} = 2.77% > α_{dop} = 2%) and moreover, the limit value of voltage deviations in two phases were exceeded as well, while in one of them - very strongly (δU_{fmax} = -23,1% < -10%).

Due to the complexity of the phenomenon of load asymmetry, it is necessary to develop simulation models that using the data of a network, customers and the results of measurements which representative customers, allows us to assess risk of exceeded limit values of selected parameters of electricity in the LV line. Then, it enables us to take into consideration the asymmetry loads to design a new network (e.g. choice cross-section of wires), quality control of electricity in existing and potential of its modernisation.

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