



## ANALYTICAL VERIFICATION OF NUMERICAL ANALYSIS OF FORAGE TRAILER SUPPORTING STRUCTURE

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### **Abstract**

*The paper presents manner of analytical modelling of forage trailer supporting structure. Process of obtaining mathematical model from physical model was presented as well as final form of mathematical model in the form of equations describing deflections and stresses of the forage trailer construction. Results of analytical calculations were compared with results of numerical calculations with the method of complete elements. Good, 5%, compliance of the results of analytical and numerical calculations was achieved.*

**Keywords:** *agricultural machinery, mathematic modelling, Clebsch method,*

### **1. Introduction**

The paper [1] presents results of calculations with MES method of forage trailer supporting structure. Basic functions of forage trailers are: dosing of correct feed dose, feed mixing and transportation to pasture. It is a complex object – contains large number of constructional elements and many constructional nodes of complicated manner of stress transmission. Construction of numerical model for such structure is very time consuming and can be burdened with many errors. Modelling errors can be avoided by using the possibility of verification of numerical model with analytical calculations.

The paper presents analytical mathematical model of forage trailer supporting frame. With model application, analytical calculations were performed in order to specify stresses and deflections in the function of wagon frame length. Results of analytical calculations were compared with results of numerical calculations, obtaining good, 5%, compliance.

### **2. Object and testing conditions**

Based upon supplied constructional documentation of low forage trailer frame, simplification of its geometrical form was performed. Simplified form is presented at Fig.1. The figure presents dimensional scheme and locations of supports and application of external loads.

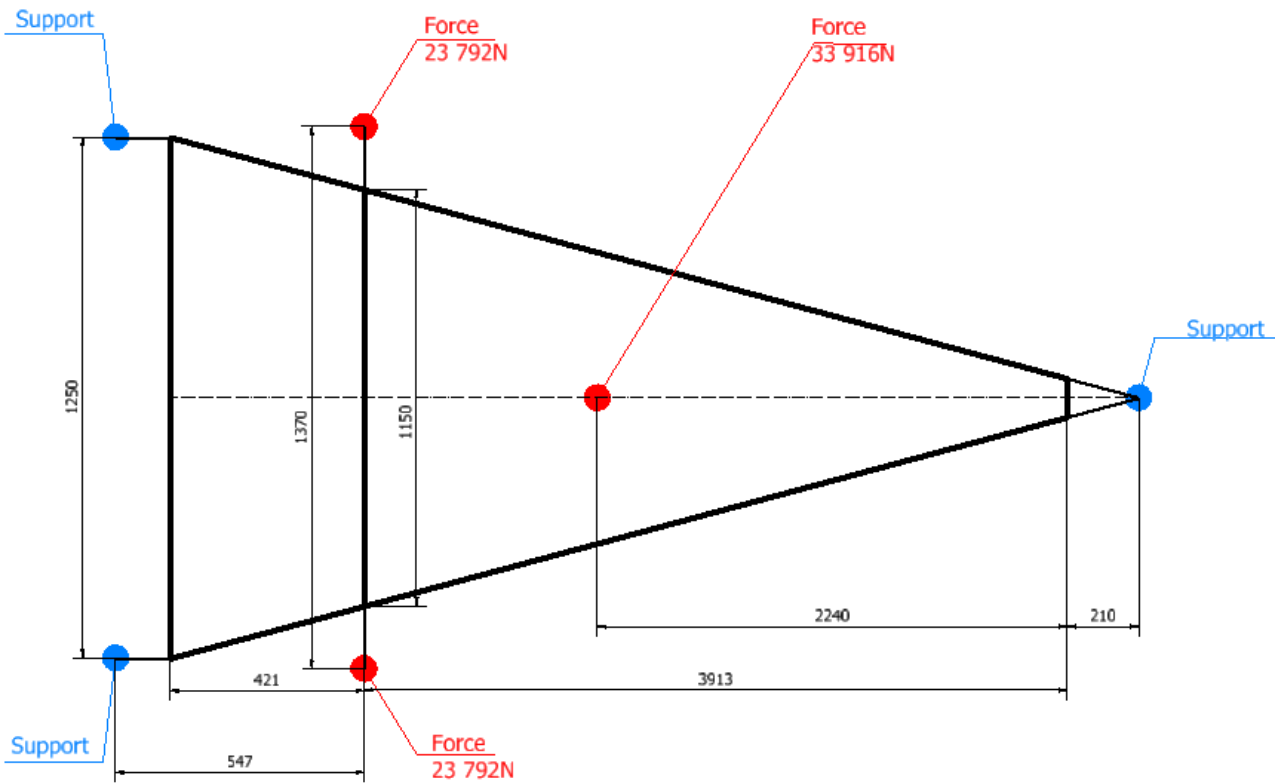


Fig. 1. Characteristic dimensions of EVO 12 frame

Based upon the scheme as of Figure 1, values of parameters present in analytical dependencies (1÷3) were specified. Based upon the scheme as of Figure 2 the form of dependency was set and value of moment of inertia of beams horizontal cross-section (4÷5) was calculated. Main supporting beams are made of closed sections 80x160mm and flat sections 180x14mm. Values of the aforementioned parameters are presented in Table 1.

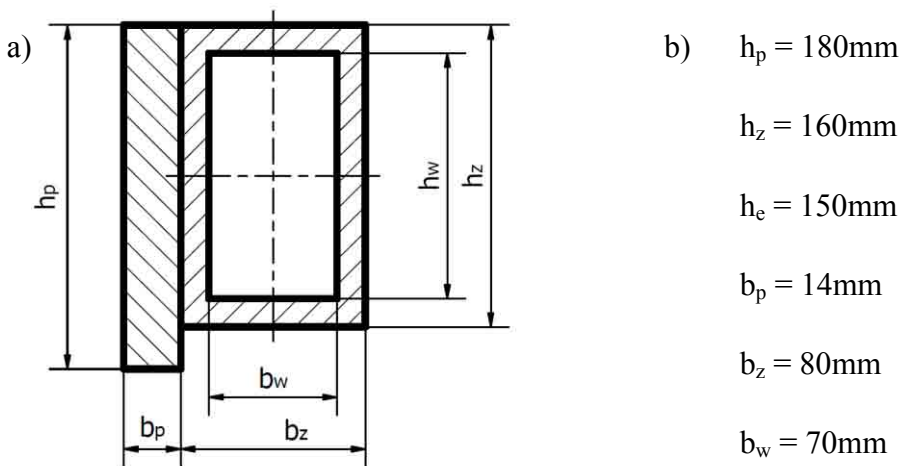


Fig. 2. Cross-section of main supporting beam a) scheme, b) values of parameters

### 3. Analytical calculations

Simplified geometry of forage trailer was modelled with beam supported on both sides (Fig. 3). After dismantling of knots and application of external loads mathematical model as on Figure 3 was obtained.

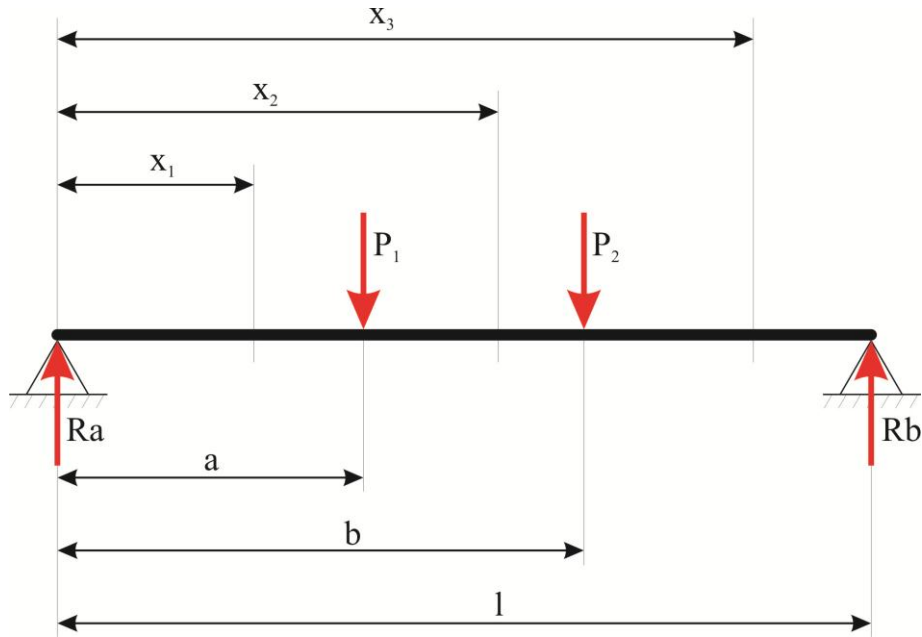


Fig. 3. Scheme of forces layout

Solving of layout presented at Figure 3 with Clebsch method [2] leads to setting the deflecting functions with dependency (1) and deflecting moments with dependency (2):

$$y = \frac{1}{EI} \left( \frac{R_A \cdot x^3}{6} \Big|_{x_1} - \frac{P_1 \cdot (x-a)^3}{6} \Big|_{x_2} - \frac{P_2 \cdot (x-b)^3}{6} \Big|_{x_3} + c_1 \cdot x + c_2 \right) \quad (1)$$

$$Mg = R_A \cdot x \Big|_{x_1} - P_1 \cdot (x-a) \Big|_{x_1} - P_2 \cdot (x-b) \Big|_{x_1} \quad (2)$$

Where integration constants and reaction values are described with dependencies (3):

$$c_1 = -\frac{R_A \cdot L^2}{6} + \frac{P_1 \cdot (L-a)^3}{6L} + \frac{P_2 \cdot (L-b)^3}{6L} \quad (3a)$$

$$c_2 = 0 \quad (3b)$$

$$R_A = \frac{P_1 \cdot (L-a)}{L} + \frac{P_2 \cdot (L-b)}{L} \quad (3c)$$

Crucial element of calculations of frame deflecting (1) is correct setting of inertia moments for supporting beam cross-section, presented as a scheme at Fig. 2.

After necessary transformations, location of gravity centre was described with dependency (4) and value of inertia moment with dependency (5):

$$y_c = \frac{b_p h_p \frac{h_p}{2} + b_z h_z \frac{h_z}{2} - b_w h_w \frac{h_w}{2}}{b_p h_p + b_z h_z - b_w h_w} \quad (4)$$

$$I_x = \frac{b_p h_p^3}{12} + b_p h_p \left( \frac{h_p}{2} - y_c \right)^2 + \frac{b_z h_z^3}{12} + b_z h_z \left( \frac{h_z}{2} - y_c \right)^2 - \frac{b_w h_w^3}{12} - b_w h_w \left( \frac{h_z}{2} - y_c \right)^2 \quad (5)$$

Thus, it was possible to specify value of stress in the most distant fibres from neutral axis (6):

$$\sigma = \frac{Mg * y_c}{I_x} \quad (6)$$

#### 4. Results of analytical calculations

Based upon dependencies (1)-(6) assuming values of individual parameters presented in Table 1, distributions of normal stresses and deflections of main beam, changing along wagon frame length, were set. Table of distributions specified by analytical and numerical means is presented for stresses as at Fig. 4 and for deflections as at Fig. 5. for such comparisons, in numerical models platform for drive gears fixing was skipped.

Table 1. Values of parameters specifying balance layout at Fig. 3

Parameter	Value
a	547 mm
b	2 220 mm
L	4 670 mm
P1	47 584 N
P2	33 916 N
I <sub>x</sub>	29 402 785 mm <sup>4</sup>
E	2e5 MPa

Very good compliance of numerical calculations results and analytical stresses is noticeable (Fig. 4). Differences result only from simplifications of analytical model based on skipping of local changes of stiffness caused by joining of main supporting structure of the forage trailer with other constructional elements. It is important that maximum values of stresses, which decide about the frame supporting ability, differ only by ca. 3%.

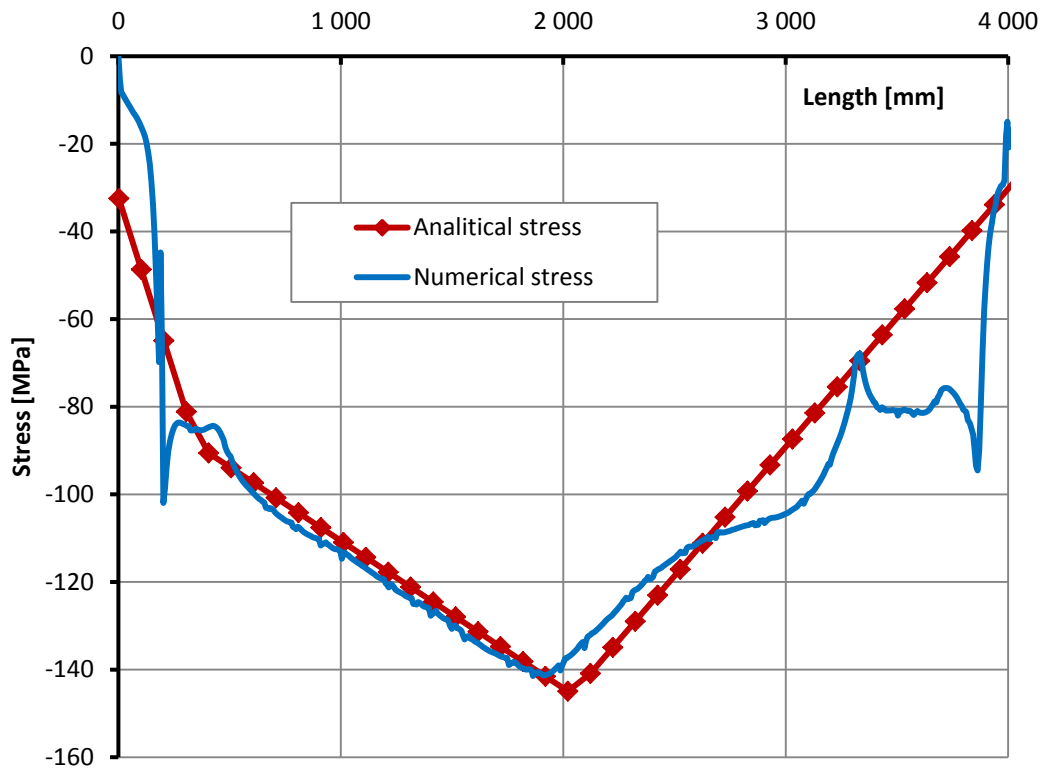


Fig. 4. Set of results of analytical calculations and numerical analyses for stresses in supporting beam of low EVO 12 frame

In case of deflections nature of changes in both cases of analyses is identical. Local stiffness do not influence the change of nature of function of forage trailer deflections. Value of deflections calculated numerically is only by 5% higher than the one obtained analytically.

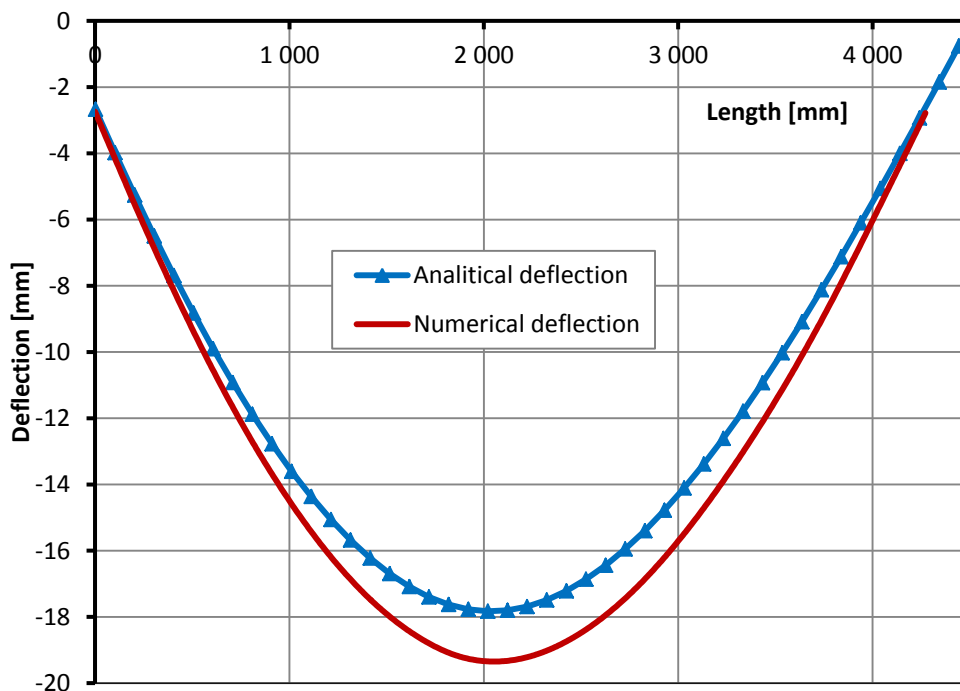


Fig. 5. Set of results of analytical calculations and numerical analyses for deflections in supporting beam of EVO 12 frame

## 5. Summary

The paper presents modelling manner and results of analytical analyses of modified forage trailer supporting structure. Calculation results were compared with results of numerical MES calculations. Very good compliance of analytically calculated stresses and deflections with the ones obtained based on MES calculations was achieved. Thus, it should be stated that even in case of complex technical objects, correctly performed modelling process of analytical model allows for obtaining correct results. Despite significant simplifications at construction of analytical mathematical model, achieved results are useful for initial verification of numerical model.

## References

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