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THE USAGE OF MULTI-EQUATION MODELS IN THE ANALYSIS OF DYNAMIC PROCESS IN MARINE DIESEL ENGINE RESEARCH

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Abstract

Contemporary empirical researches on the object, which is combustion engine, are processed basing on the theory of experiment. Available software applications to analyze the experimental data commonly use the multiple regression models, which enables studying effects and interactions between input values of the model and single output variable. Using multi-equation models gives free hand at analyzing measurement results because it enables analysis of effects and interaction of many output variables. It also allows analysis of the measurement results during dynamic process. In this paper author presents advantages of using the multidimensional regression model on example of researches conducted on engine test stand.

Keywords: *diagnostic, theory of experiments, marine diesel engine, exhaust gas toxicity, multi-equation models*

1. Introduction

During the working process of the engine, its structure parameters are changing. It doesn't affect its performance, described by a set of output parameters. The reciprocal relationship between the parameters of the structure and parameters of the motor output allows under certain conditions to treat the symptoms of the output parameters as engine condition, measured without dismantling, because the physicochemical processes occurring during the working process and figures describing them can generally be observed and measured from the outside. These figures include the value of the emission of exhaust components.

This simple combination is of interest to writers and aims to analyze the suitability and performance indicators to evaluate the emission parameters of the engine structure. At this point however, a comment is in place, as in the classical sense, output parameter can be regarded as diagnostic only while meeting the characteristics, that is: uniqueness, of sufficient width of the change field and availability. Thus, should the indicators and emission characteristics be considered a diagnostic parameter?

Given the complicated measure, the cost of equipment, and the ambiguity of (the presence of extremes), and the characteristics of toxic compounds, a negative answer comes to mind. Nevertheless, the rapid development of measurement methods, progressively more advanced analyzers with increasing measurement power, ie: speed and capacity of archiving items, which make the signal of changes in emissions of toxic compounds more useful, carrying more and more information useful for subsequent analysis.

The above approach goes as far as to impose a need to use the theory in empirical research experiment. The primary objective of this research work is to demonstrate the relationship between the input signals (introduced by the investigator), and the output signals (seen by him). The ultimate goal of statistical analysis of measurement results is to define a function of the test object and an empirical model of a functional engine. Very extensive calculations using probability theory, stochastic processes, and calculus that are associated with this task are very labor intensive and without the use of computer technology and specialized software, it is impracticable. In the process of solving problems of inter-linkages and complementary aspects of approximation, the correlation statistics, assessment of the relevance and measurement uncertainty as well as the adequacy of test object's functions, including questions of mathematical and graphical determination of singular points, available computer programs are used, including the package STATISTICA PL.

It should be emphasized that the statistical computer analysis can involve a number of models that do not include interaction and do not take into account the interactions of varying degrees of involvement adopted to describe the model input variables. At the same time there is a possibility in the statistical analysis to reject (ignore) both freely chosen input variables describing the object of research as well as the various types of interactions. This means that choosing the right (most appropriate) model depends on the operator, and their expertise and specialized knowledge of the theoretical basis of the discussed issue.

While assuming a less accurate representation of reality, practically determining the nature of change (trend) output quantities, there is the possibility of significant simplification of approximating polynomials by considering only the input variables and their only statistically significant interactions. The complexity of the model and the degree of entanglement of the basic volumes are also strongly determined by the degree of approximating polynomials. Hence it is reasonable to seek to create models of a possibly simple form, and most preferably linear models. It is assumed that due to the possibility of errors, it is better to describe the studied problem of non-linear nature with small linear segments than a non-linear description of a complex whole.

The software commonly used in the field of experimental design and subsequent analysis does not provide the freedom to analyze the collected material, and uses ready analysis diagrams described above. Thus, interfering with the program (software package) itself is not possible. The recently observed development of the social sciences, medicine and economics has caused rapid progress in the application of statistical methods, securing planning of the experiment [1,2,3,5,6]. In this area, econometrics in particular has some great achievements, and the new approach to statistical analysis used there can be successfully implemented in technical studies [1,2]. Among other things, the use of multi-equation models (models with interdependent equations) makes it possible to study not only the correlation between the input and output, but also take into account the feedback between output variables and thereby give the possibility of direct analysis. This assumption, as opposed to the commonly used multiple linear regression, is closer to reality even while taking into account the dilemma of diesel, that is the relationship between the concentration of CO, HC and NOx concentration.

This approach was presented by the authors including the earlier works, where the results of research on the engine fuel supply system (fuel injection) and charge exchange system (with particular emphasis on TPC) were presented using a divalent fractional plan and a multi-equation model [7, 8, 9, 10, 11].

The multi-equation model relations between input signals and output signals can be described by a system of linear equations

$$\begin{aligned}
y_1 &= b_{12}y_2 + b_{13}y_3 + b_{14}y_4 + \cdots + b_{1M}\, y_M \;\; + a_{10} + a_{11}x_1 + a_{12}x_2 + \cdots + a_{1N}x_N + \xi_1 \\
y_2 &= b_{21}y_1 + b_{23}y_3 + b_{24}y_4 + \cdots + b_{2M}\, y_M \;\; + a_{20} + a_{21}x_1 + a_{22}x_2 + \cdots + a_{2N}x_N + \xi_2 \\
y_3 &= b_{31}y_1 + b_{32}y_2 + b_{34}y_4 + \cdots + b_{2M}\, y_M \;\; + a_{30} + a_{31}x_1 + a_{22}x_2 + \cdots + a_{3N}x_N + \xi_3 \\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
y_M &= b_{M1}y_1 + b_{M2}y_2 + \cdots + b_{MM-1}y_{M-1} + a_{M0} + a_{M1}x_1 + a_{M2}x_2 + \cdots + a_{MN}x_N + \xi_M
\end{aligned} \tag{1}$$

where:

$y_i, i = 1,2,\ldots,M$ - explained variable (output),

$x_j, j = 1.2,\ldots,N,$ - the explanatory variables (input),

b_{ij} – is a factor present in the i-th equation with j-th being the explained variable (output), $i, j = 1,2,\cdots,M$

a_{ij} - is a factor present in the i-th equation with j-th being the explanatory variable (input), $i = 1,2,\cdots,N, \quad j = 0,1,\cdots,N$,

ξ_i - is a non-observable random component in the i-th equation.

The solution of the equation (1) is reduced to its matrix form

$$\mathbf{BY} = \mathbf{AX} + \xi \tag{2}$$

where A, B, ξ – matrix of coefficients,

and the selection of the coefficients in the equations (1) with the values of the input signals known from measurements on the real object. The next step is to bring the equation (2) to the reduced form

$$\mathbf{Y} = \mathbf{\Pi X} + \boldsymbol{\eta} \tag{3}$$

where: $\Pi := \mathrm{B}^{-1}\mathrm{A}, \quad \eta := \mathrm{B}^{-1}\xi$.

As a result, after verifying the significance of coefficients and, consequently, the rejection of insignificant values, correlations between equations describing the output variables, both the input variables and the remaining output variables, are obtained.

Multi-equation models, as demonstrated by the earlier works of the authors, show a significant adjustment to the value obtained in the experiment [7,8,9,10,11]. However, they describe the changes in the output parameters (indicators of toxic compounds) in steady states of the engine, when the influence of structural parameters is not the greatest. Hence the problem with the wide variety of changes in the output parameter. The situation changes when we do an analysis of the changes in output parameters during transient processes. In the course of its duration, due to, among other things, imperfections of control systems, there is a chance of a repeated, though usually short-term instances when the parameter values are exceeded in comparison to the set state. The effect of structural parameters is significantly larger then, thereby the issue with the variety of changes in the output parameter is less severe.

Using these somewhat detrimental to the engine operating states, it was decided to implement tested in steady-state multi-equation models for analysis of dynamic processes.

2. Identification of a dynamic process of multi-equation model

Assuming that the process of changing the exhaust emissions occurs over time, which means it is dynamic, the multi-equation model can be described with a system of linear differential equations. Since the measurement of the concentration of toxic compounds is a discrete measurement, the time-discrete signal (time sequence) is a function whose domain is the congregation of integers. Thus, a discrete-time signal is a sequence of numbers. This kind of sequences will continue to be recorded in the functional notation.

Discrete-time signal $x[k]$ is often determined by sampling $x(t)$, a continuous signal in time. If the sampling is uniform, then $x[k] = x(kT)$. Constant T is called the sampling period. Course of the dynamic process in time depends not only on the value of extortion at a given time but also the value of extortion in the past. Thus, the dynamic process (system) has a memory where it stores consequences of past interactions.

The relations between the input signals $x_1[k], x_2[k], \cdots, x_n[k]$, and output signals $y_1[k], y_2[k], \cdots, y_m[k]$, $k = 0,1,2,\ldots$, will be described by a system of linear differential equations.

$$\begin{cases} y_1[k+1] = a_{11}y_1[k] + a_{12}y_2[k] + \cdots + a_{1m}\,y_m[k] \; + b_{11}x_1[k] + b_{12}x_2[k] + \cdots + b_{1n}x_n[k] + \xi_1 \\ y_2[k+1] = a_{21}y_1[k] + a_{22}y_2[k] + \cdots + a_{2m}\,y_m[k] \; + b_{21}x_1[k] + b_{22}x_2[k] + \cdots + b_{2n}x_n[k] + \xi_2 \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ y_m[k+1] = a_{m1}y_1[k] + a_{m2}y_2[k] + \cdots + a_{mm}y_m[k] + b_{m1}x_1[k] + b_{m2}x_2[k] + \cdots + b_{mn}x_n[k] + \xi_m \end{cases} \tag{4}$$

where:

$y_i[k], i = 1,2,\ldots,m$ - output signal values at k,

$x_j[k]$, $j = 1,2,\ldots,n$ - input signal values at k,

a_{ij} – is a coefficient found in i-th equation with j-th output signal, $i, j = 1,2,\cdots,m$

b_{ij} - is a coefficient found in i-th equation with j-th input signal, $i = 1,2,\cdots,m,\quad j = 0,1,\cdots,n$,

ξ_i - is a non-observable random component in i-th equation.

In analogy to (1), the system of equations (4) can be written in matrix form

$$\mathbf{y}[k+1] = \mathbf{A}\mathbf{y}[\mathbf{k}] + \mathbf{B}\,\mathbf{x}[\mathbf{k}] + \boldsymbol{\xi} \tag{5}$$

where:

$$\mathbf{B} = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1m} \\ b_{21} & b_{22} & \cdots & b_{2m} \\ \cdots & \cdots & \cdots & \cdots \\ b_{m1} & b_{m2} & \cdots & b_{mm} \end{bmatrix}, \qquad \mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m1} & \cdots & a_{mn} \end{bmatrix},$$

$$\mathbf{y}[k]=\begin{bmatrix} y_1[k] \\ y_2[k] \\ \cdots \\ y_m[k] \end{bmatrix}, \quad \mathbf{y}[k+1]=\begin{bmatrix} y_1[k+1] \\ y_2[k+1] \\ \cdots \\ y_m[k+1] \end{bmatrix}, \quad \mathbf{x}[\mathbf{k}]=\begin{bmatrix} x_1[k] \\ x_2[k] \\ \cdots \\ x_n[k] \end{bmatrix}, \quad \boldsymbol{\xi}=\begin{bmatrix} \xi_1 \\ \xi_2 \\ \cdots \\ \xi_m \end{bmatrix} \quad .$$

Later denoting:

$$\mathbf{C}=\begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1m} & c_{1m+1} & c_{1m+2} & \cdots & c_{1m+n} \\ c_{21} & c_{22} & \cdots & c_{2m} & c_{2m+1} & c_{2m+2} & \cdots & c_{2m+n} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ c_{m1} & c_{m1} & \cdots & c_{mn} & c_{mm} & c_{mm+1} & \cdots & \pi_{mm+n} \end{bmatrix}, \quad \boldsymbol{\eta}=\begin{bmatrix} \eta_1 \\ \eta_2 \\ \vdots \\ \eta_m \end{bmatrix},$$

$$c_{i\nu}[k]=b_{i\nu}[k], \text{ dla } \nu=1,2,\ldots,m, \qquad c_{i\nu}[k]=a_{i\nu}[k], \quad \text{dla } \nu=m+1,m+2,\ldots,m+n, \tag{6}$$

$$\mathbf{z}[k]=\begin{bmatrix} z_1[k] \\ \vdots \\ z_m[k] \\ z_{m+1}[k] \\ \vdots \\ z_{m+n}[k] \end{bmatrix},$$

$$z_\nu[k]=y_\nu[k], \text{ dla } \nu=1,2,\ldots,m, \qquad z_\nu[k]=x_{\nu-m}[k], \quad \text{dla } \nu=m+1,m+2,\ldots,m+n,$$

system of equations (4) and functionals (6) are shown in reduced form

$$\mathbf{y}[k+1]=\mathbf{C}\,\mathbf{z}[k]+\boldsymbol{\eta} \quad . \tag{7}$$

By identifying the system of equations (1), (4) we get to understand a problem of selecting coefficients using the values determined by real property measurements

$\widetilde{x}_1[k],\widetilde{x}_2[k],\cdots,\widetilde{x}_n[k], \quad k=0,1,2,\cdots,N$ input signals $x_1,x_2,\ldots,x_n$

and values

$\widetilde{y}_1[k],\widetilde{y}_2[k],\cdots,\widetilde{y}_m[k], \quad k=0,1,2,\cdots,N+1$ input signals $y_1,y_2,\cdots,y_m$,

in $t_k=kT$ instants.

Measured values can be written in matrix form (8):

$$\widetilde{\mathbf{X}}=[e_1\,|\,e_2\,|\ldots|\,e_m\,|\,e_{m+1}\,|\,e_{m+2}\,|\ldots|\,e_{m+n}]=\begin{bmatrix} \widetilde{y}_1[0] & \widetilde{y}_2[0] & \cdots & \widetilde{y}_m[0] & \widetilde{x}_1[0] & \cdots & \widetilde{x}_n[0] \\ \widetilde{y}_1[1] & \widetilde{y}_2[1] & \cdots & \widetilde{y}_m[1] & \widetilde{x}_1[1] & \cdots & \widetilde{x}_n[1] \\ \vdots & \vdots & \vdots & \vdots \; \vdots & \vdots & \vdots & \vdots \\ \widetilde{y}_1[N] & \widetilde{y}_2[N] & \cdots & \widetilde{y}_m[N] & \widetilde{x}_1[N] & \cdots & \widetilde{x}_n[N] \end{bmatrix}$$

Coefficients $b_{i1},\ldots,b_{im},a_{im+1}\ldots,a_{im+n}, \ i=1,2,\ldots,m$, of the above system of equations are chosen specifically so functional (9)

$$J_i(b_{i1},\ldots b_{im,}a_{im+1},\ldots,a_{i(m+n)})=\sqrt{\sum_{k=1}^{N}(a_{i1}\widetilde{y}_1[k]+\ldots+a_{im}\widetilde{y}_m[k]+b_{im+1}\widetilde{x}_1[k]\ldots+b_{i(m+n)}\widetilde{x}_n[k]-\widetilde{y}_i[k+1])^2},$$

reaches a minimum for $i=1,2,\ldots m$.

In the denotations adopted above (6), functionals (9) can be written as (10)

$$J_i(c_{i1},c_{i2},\ldots,c_{i(m+n)})=\sqrt{\sum_{k=0}^{N}(c_{i1}\widetilde{z}_1[k]+\ldots+c_{im}y\widetilde{z}_m[k]+c_{im+1}\widetilde{z}_{m+1}[k]\ldots+c_{i(m+n)}\widetilde{z}_{m+n}[k]-\widetilde{y}_i[k+1])^2},$$

$i=1,2,\ldots m$, $\widetilde{z}_\nu[k]=\widetilde{y}_\nu[k]$, dla $\nu=1,2,\ldots,m,$ $\widetilde{z}_\nu[k]=\widetilde{x}_{\nu-m}[k],$ dla $\nu=m+1,m+2,\ldots,m+n$.

Matrix (8) is essentially a system of linearly independent vectors in Hilbert space

$$\mathbf{e}_1=\begin{bmatrix}\widetilde{y}_1[0]\\ \widetilde{y}_1[1]\\ \vdots\\ \widetilde{y}_1[N]\end{bmatrix},\mathbf{e}_2=\begin{bmatrix}\widetilde{y}_2[0]\\ \widetilde{y}_2[1]\\ \vdots\\ \widetilde{y}_2[N]\end{bmatrix},\cdots,\mathbf{e}_m=\begin{bmatrix}\widetilde{y}_m[0]\\ \widetilde{y}_m[1]\\ \vdots\\ \widetilde{y}_m[N]\end{bmatrix},\mathbf{e}_{m+1}=\begin{bmatrix}\widetilde{x}_1[0]\\ \widetilde{x}_1[1]_1\\ \vdots\\ \widetilde{x}_1[N]\end{bmatrix},\cdots,\mathbf{e}_{m+n}=\begin{bmatrix}\widetilde{x}_n[0]\\ \widetilde{x}_n[1]\\ \vdots\\ \widetilde{x}_n[N]\end{bmatrix}.$$

The issue of choosing the best model out of the class of equations (11) in the sense of minimizing the quality coefficient of identification was solved using the orthogonal projection theorem [3,6]. Given the vastness of the issue, the shift leading to the equation in matrix form was omitted

$$\mathbf{G}(\mathbf{C}_i^0)^T=\mathbf{W}_i, \tag{12}$$

where:

$$G=\begin{bmatrix}\sum_{k=0}^{N}\widetilde{y}_1[k]\widetilde{y}_1[k] & \cdots & \sum_{k=0}^{N}\widetilde{y}_m[k]\widetilde{y}_1[k] & \sum_{k=0}^{N}\widetilde{x}_1[k]\widetilde{y}_1[k] & \cdots & \sum_{k=0}^{N}\widetilde{x}_n[k]\widetilde{y}_1[k]\\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots\\ \sum_{k=0}^{N}\widetilde{y}_1[k]\widetilde{y}_m[k] & \cdots & \sum_{k=0}^{N}\widetilde{y}_m[k]\widetilde{y}_m[k] & \sum_{k=0}^{N}\widetilde{x}_1[k]\widetilde{y}_m[k] & \cdots & \sum_{k=0}^{N}\widetilde{x}_n[k]\widetilde{y}_m[k]\\ \sum_{k=0}^{N}\widetilde{y}_1[k]\widetilde{x}_1[k] & \cdots & \sum_{k=0}^{N}\widetilde{y}_m[k]\widetilde{x}_1[k] & \sum_{k=0}^{N}\widetilde{x}_1[k]\widetilde{x}_1[k] & \cdots & \sum_{k=0}^{N}\widetilde{x}_1[k]\widetilde{x}_1[k]\\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots\\ \sum_{k=0}^{N}\widetilde{y}_1[k]\widetilde{x}_n[k] & \cdots & \sum_{k=0}^{N}\widetilde{y}_m[k]\widetilde{x}_n[k] & \sum_{k=0}^{N}\widetilde{x}_1[k]\widetilde{x}_n[k] & \cdots & \sum_{k=0}^{N}\widetilde{x}_1[k]\widetilde{x}_n[k]\end{bmatrix}=\widetilde{\mathbf{X}}^T\widetilde{\mathbf{X}}$$

$$
(\mathbf{C}_i^0)^T = \begin{bmatrix} c_{i1}^0 \\ \vdots \\ c_{im}^0 \\ c_{im+1}^0 \\ \vdots \\ c_{im+n}^0 \end{bmatrix}, \qquad \mathbf{W}_i = \begin{bmatrix} \sum_{k=0}^{N} \tilde{y}_i[k+1]\tilde{y}_1[k] \\ \vdots \\ \sum_{k=0}^{N} \tilde{y}_i[k+1]\tilde{y}_n[k] \\ \sum_{k=0}^{N} \tilde{y}_i[k+1]\tilde{x}_1[k] \\ \vdots \\ \sum_{k=0}^{N} \tilde{y}_i[k+1]\tilde{x}_n[k] \end{bmatrix} = \tilde{\mathbf{X}}^T \tilde{\mathbf{y}}_i[k+1]
$$

Thus, the matrix equation (12) can be expressed as

$$
(\tilde{\mathbf{X}}^T \tilde{\mathbf{X}})(\mathbf{C}_i^o)^T = \tilde{\mathbf{X}}^T \tilde{\mathbf{y}}_i[k+1] \tag{13}
$$

Which gives

$$
(\mathbf{C}_i^o)^T = (\tilde{\mathbf{X}}^T \tilde{\mathbf{X}})^{-1} \tilde{\mathbf{X}}^T \tilde{\mathbf{y}}_i[k+1], \quad i = 1,2,\cdots,m \quad . \tag{14}
$$

Thus, the optimal coefficients

$$
c_{ij}^0, \quad i = 1,2,\ldots,m, \quad j = 1,\ldots,m,m+1,\ldots,m+n .
$$

of the reduced model form (10) can be determined from the equation

$$
(\mathbf{C}^0)^T = (\tilde{\mathbf{X}}^T \tilde{\mathbf{X}})^{-1} \tilde{\mathbf{X}}^T \tilde{\mathbf{Y}}[k+1] \tag{15}
$$

$\tilde{\mathbf{X}}_{(N+1)\times(m+n)}$ - matrix of measured values of signals $y_1, y_2, \ldots, y_n, x_1, x_2, \ldots, x_m$,

$\tilde{\mathbf{X}}^T_{(m+n)\times(N+1)}$ - transposed matrix to the matrix of measured values of input signals,

$(\tilde{\mathbf{X}}^T \tilde{\mathbf{X}})^{-1}_{(N+1)\times(N+1)}$ - inverse to the Gram matrix $\mathbf{G}_{(N+1)\times(N+1)}$,

$\tilde{\mathbf{Y}}_{(N+1)\times m}[k+1]$ - matrix of measured values of output signals $y_1, y_2, \ldots, y_m$,

$\tilde{\mathbf{Y}}^T_{m\times(N+1)}$ - transposed matrix to the matrix of measured values of output signals,

N - number of measurements, n- number of input signals, m- number of output signals.

3. The study of dynamic process in engine fuel supply system through multi-equation models

The object of this research was the engine fuel supply system (fuel injection) of a single-cylinder test engine 1-SB installed in the Laboratory of the Exploitation of Marine Power Plants at the Naval Academy (10). The experimental material was collected by a bivalent fractional plan. The implementation of specific measuring systems (measuring points) of the above experiment design were performed using a programmable controller, which allowed a high repeatability of dynamic processes. The period between an onset of the clipping of injection system components and the re-stabilization of output quantities was adopted as the duration of the dynamic process. This period was chosen through a series of experiments, and it averaged to about 320 seconds.

In order to identify the impact of the technical condition of the fuel supply system on the parameters of the engine power during dynamic processes, sets of input quantities (preset parameters) and output quantities (observed parameters) were defined. For the purpose of this study a set of input quantities X was limited to three elements, that is: x_1 - engine speed *n [r/min];* x_2 – engine torque T_{tq} *[N·m];* x_3 – coking of the spray nozzle S_k *[μm²].*

Similar treatment was applied to the set *Y* of output quantities, limiting the number of its elements to only the primary toxic compounds in exhaust manifold: y_1 - concentration of carbon monoxide in the exhaust manifold $C_{CO(k)}$ *[ppm]; y_2* – concentration of hydrocarbons in the exhaust manifold $C_{HC(k)}$ *[ppm]; y_3* – concentration of nitrogen oxides in the exhaust manifold $C_{NOx(k)}$ *[ppm]*. Changes of the input and output quantities during the dynamic process are shown in Fig. 1.

Statistical identification was made using GRETL [2]. Estimation of the equation coefficients for specific output variables was performed using the least-squares method and it had to verify the significance of its parameters and, consequently, the rejection of insignificant values, which consequently led to a significant simplification of the models. Equations describing the changes in concentration of hydrocarbons (y_2) and the concentration of nitrogen oxides (y_3) have undergone the greatest simplification. (Table 2, 3). In the case of the equation describing the change of hydrocarbons in a way that they depend significantly on the structure parameter, which represents coking of the spray nozzle (x_3). The case of a model describing changes in carbon monoxide (y_1) is similar.

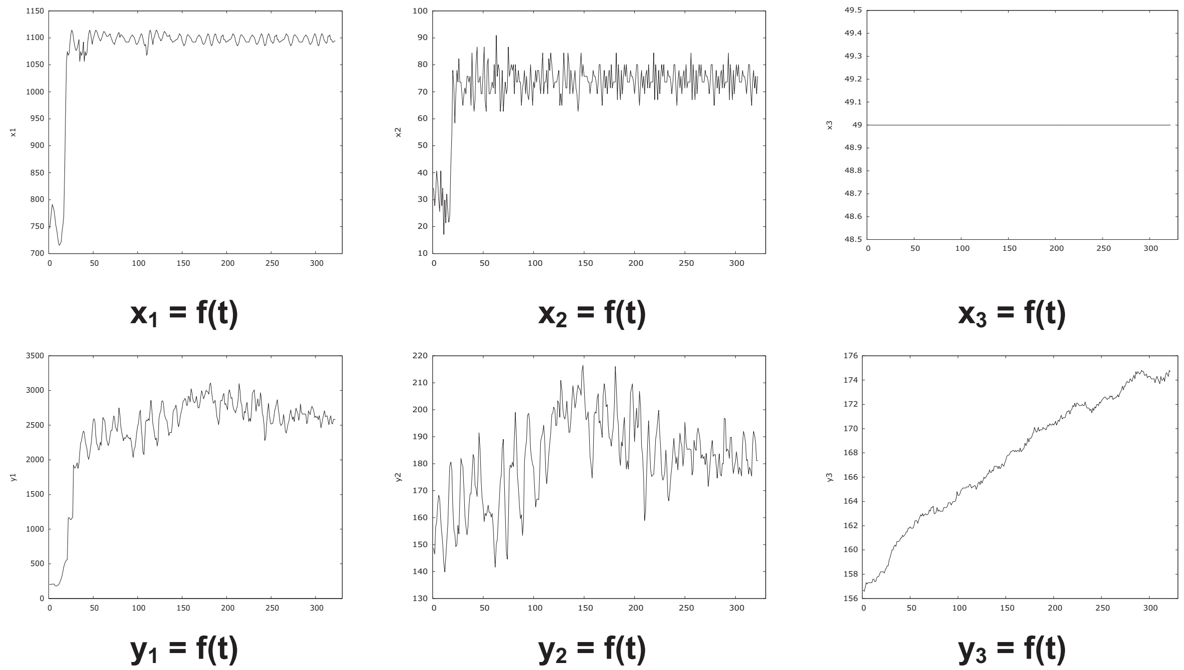


Fig. 1. Changes of the input and output quantities during the dynamic process
where: x_1 – engine speed n [r/min]; x_2 – engine torque T_{tq} [N·m]; x_3 – coking of the spray nozzle S_k [μm^2]; y_1 - concentration of carbon monoxide in the exhaust manifold $C_{CO(k)}$ [ppm]; y_2 – concentration of hydrocarbons in the exhaust manifold $C_{HC(k)}$ [ppm]; y_3 – concentration of nitrogen oxides in the exhaust manifold $C_{NOx(k)}$ [ppm]; t – duration of the process [s]

Table 1. Least-squares estimation of the dependent variable y_1

	Coefficient	*Mean error*	*Student t*	*p value*	
y3_1	2,91309	1,43271	2,0333	0,04286	**
x1_1	0,644317	0,130189	4,9491	<0,00001	***
x3_1	-18,2606	5,31195	-3,4376	0,00067	***
y1_1	0,884015	0,0193211	45,7538	<0,00001	***

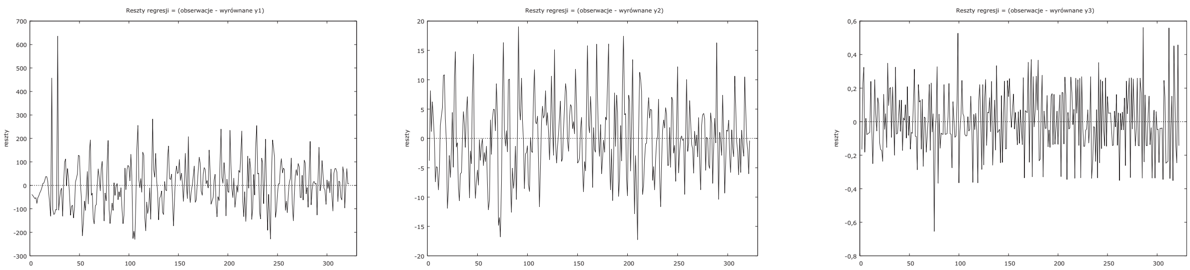
Table 2. Least-squares estimation of the dependent variable y_2

	Coefficient	*Mean error*	*Student t*	*p value*	
y1_1	0,00120044	0,00067661	1,7742	0,07699	*

x3_1	0,412034	0,0899356	4,5814	<0,00001	***
y2_1	0,873606	0,0279092	31,3017	<0,00001	***

Table 3. Least-squares estimation of the dependent variable y_3

	Coefficient	*Mean error*	*Student t*	*p value*	
y1_1	-5,65987e-05	2,41816e-05	-2,3406	0,01987	**
x1_1	0,000396798	0,000206581	1,9208	0,05565	*
y3_1	0,998597	0,00112836	884,9995	<0,00001	***



reszty y_1 = f(t) **reszty y_2 = f(t)** **reszty y_3 = f(t)**

Fig. 2. Graph of the regression residuals for output variables where:]; y_1 - concentration of carbon monoxide in the exhaust manifold $C_{CO(k)}$ [ppm]; y_2 – concentration of hydrocarbons in the exhaust manifold $C_{HC(k)}$ [ppm]; y_3 – concentration of nitrogen oxides in the exhaust manifold $C_{NOx(k)}$ [ppm]; t – duration of the process [s]

An even distribution of residuals from the regression of mean values may be indicative of being a good fit model to the values obtained from an experiment on the engine.

4. Summary

Presented description of the active experiment space by the multidimensional models gives great possibilities in analysis of measurement data and scientific conclusions. Furthermore, assuming that coefficients' matrix $(C^o)^T$ is orthogonal, there is a possibility of fulfilling reverse task, that is assessing, with complex relevance at known input variables, which describe work point i.e. engine rotational speed n and torque load T_{tq}, the other input quantities. In the nearest future authors will work on this issue.

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