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Swarm optimization of stiffeners locations in 2-D structures

M. SZCZEPANIK[1] and T. BURCZYŃSKI[1,2]*

[1] Department for Strength of Materials and Computational Mechanics Silesian University of Technology
18a Konarskiego St., 44-100 Gliwice, Poland
[2] Cracow University of Technology, Institute of Computer Science, Computational Intelligence Department
24 Warszawska St., 31-155 Cracow, Poland

Abstract. The paper is devoted to the application of the swarm methods and the finite element method to optimization of the stiffeners location in the 2-D structures (plane stress, bending plates and shells). The structures are optimized for the stress and displacement criteria. The numerical examples demonstrate that the method based on the swarm computation is an effective technique for solving the computer aided optimal design. The additional comparisons of the effectiveness of the particle swarm optimizer (PSO) and evolutionary algorithms (EA) are presented.

Key words: swarm algorithms, optimization, finite element method, bars, plane stress, bending plates, shells.

1. Introduction

Reinforced structures are often used in practice because they are resistant, stiff and stable. A typical area of application of such structures is an aircraft industry where light, stiff and highly resistant structures are required. Many aircraft elements are made as thin panels reinforced by stiffeners. The choice of an optimal shape of the structure or of the proper stiffeners arrangement in a domain of the structure decides about the effectiveness of the construction or about the effectiveness of reinforcement. Optimal properties of structures can be searched using the computer aided optimization tools.

The stiffeners layout is usually achieved by modifying the thickness of each element of the finite element mesh or using the homogenization method. However, the results obtained using these approaches do not give a clear stiffeners layout. Bendsoe and Kikuchi [1] analyzed composites with perforated microstructures by the use of the homogenization method. As the results of the topology optimization, the gray-scaled structures emerged. Cheng and Olhoff [2] considered the problem of the stiffener layout using a method based on thickness distribution to maximize the stiffness of rectangular and axisymmetric plates. Ding and Yamazaki [3] generated stiffener layout patterns by introduction a growing and branching tree model and the topology optimization method. Diaz and Kikuchi [4] searched for the optimal reinforcement layout for the plates by adding a declared amount of reinforcing material to increase the fundamental frequency. Bojczuk and Szteleblak [5] proposed the heuristic algorithm in order to find the optimal reinforcement layout. This algorithm consist of two stages – first the initial localization of new fiber or rib is determined using information from sensitivity analysis (analogous to the topological derivative approach of Sokołowski and Zochowski [6]), next – the gradient optimization method is performed to correct their positions. Another method is based on the optimization of the layout of isogrid stiffeners applied as special triangular patterns. Due to their efficiency, these isogrid members have been applied for example in launch vehicles and spacecraft components [7].

In the present paper, coupling FEM with a swarm algorithm in optimization of statically loaded reinforced structures is presented. The structures are optimized using the criteria dependent on displacements or stresses. Recently, swarm methods have found various applications in mechanics, and also in structural optimization. The PSO algorithm realizes directed motion of the particles in n-dimensional space to search for a solution for the n-variable optimisation problem. The optimization process using PSO is based on finding the better and better locations in the search-space (in the natural environment that are for example hatching or feeding grounds). The main advantage of the bio-inspired method is the fact that these approach do not need any information about the gradient of the fitness function and give a strong probability of finding the global optimum. The main drawback of these approaches is the long time of calculations.

2. Formulation of the problem, parameterization

Consider a 2-D structure (a plate in plane stress, a bending plate or a shell) which is stiffened by several bars. The domain of the 2-D structure and the domains of the bars are filled by a homogeneous and isotropic material of a Young's modulus E and a Poisson ratio ν. The location and shape of the bars can change for each iteration t of the swarm process. The stiffened structures are considered in the framework of the theory of elasticity. The swarm process proceeds in an

*e-mail: tburczyn@pk.edu.pl

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environment in which the structure fitness is described by the minimization of the stress functional

$$J = \int_{\Omega} \psi(\sigma) d\Omega, \tag{1}$$

where ψ is an arbitrary function of stress tensor σ, or maximization of the structure stiffness by minimization of the displacement functional

$$J = \int_{\Omega} \xi(\boldsymbol{u}) d\Omega, \tag{2}$$

where ξ is an arbitrary function of displacements $\boldsymbol{u}$.

Two different types of optimization tasks are considered:

- optimization of the location of the straight stiffeners (Fig. 1a),
- optimization of the location and shape of curved stiffeners (Fig. 1b).

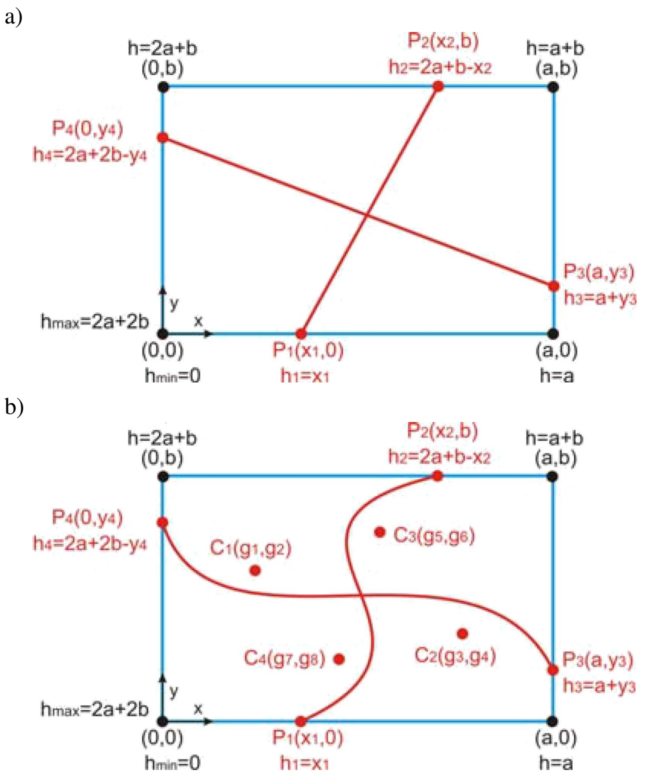

Fig. 1. Particle representation: a) straight stiffeners in 2-D structure geometry, b) curved stiffeners in 2-D structure geometry

The locations and shapes of the stiffeners in the domain of 2-D structures are controlled by particle parameters which create a particle. In order to reduce the number of the particle parameters, the particle representations, presented in the Fig. 1, has been introduced. The connection of the stiffeners ends with the 2-D structures boundary has been assumed, therefore the location of the stiffener in the 2-D structure domain is determined by two points P_i - beginning and end of the stiffener (Fig. 1a).

In order to minimize the number of parameters the curved stiffener is defined by means of NURBS curve (Non Uniform Rational B-Spline) [8]. The shape of this curve is defined by the control points C_k, $k = 1, 2, \ldots, L$; $C_k \subset \Omega_{2D}$ (L – number of the control points). The location of the stiffeners in the domain of 2-D structures is controlled by particle parameters h_i, $i = 1, \ldots, N$ and the shape of them by particle parameters g_j, $j = 1, \ldots, M$ (Fig. 1b). The set of the particle parameters creates a particle

$$\begin{gathered} par = [h_1, h_2, ..., h_i, ..., h_N, g_1, g_2, ..., g_j, ..., g_M], \\ h_i^{\min} \le h_i \le h_i^{\max}, \qquad g_j^{\min} \le g_j \le g_j^{\max}, \end{gathered} \tag{3}$$

where $h_i^{\min}$, $h_i^{\max}$ – the minimal and maximal value of the particle parameter h, respectively; $g_j^{\min}$, $g_j^{\max}$ – the minimal and maximal value of the particle parameter g, respectively.

In order to solve the formulated problems, the finite element models of the structures are considered [9]. The 2-D structure domain Ω_{2D} is divided into triangular finite elements Ω_s, $s = 1, 2, ..., R$ (for plane stress, bending plate or shell) [10], according to the geometry mapped on the basis of the particle. The edges of the triangular finite elements which belong to the curves mapped on the basis of the particle and playing the role of the stiffeners, create the bar elements Ω_b, $b = R + 1, R + 2, ..., C$ (Fig. 2).

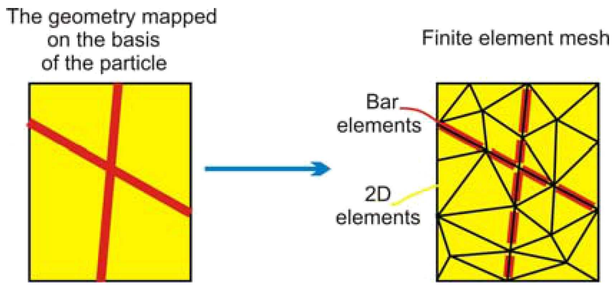

Fig. 2. Mesh of 2-D structure and bar finite elements

After the geometry discretization, the finite element analysis is performed by means of MSC NASTRAN and node displacements are calculated by solving a system of linear algebraic equations

$$\mathbf{KU} = \mathbf{F}, \tag{4}$$

where $\mathbf{U}$ is a column matrix of unknown displacements, $\mathbf{F}$ is a known column matrix of acting forces and $\mathbf{K}$ is a known global stiffness matrix of the structure which elements are given as follows:

$$\mathbf{k_s} = \int_{A} \mathbf{B_s^T D_s B_s} dA \tag{5}$$

for 2-D structure elements, and

$$\mathbf{k_b} = \int_{l} \mathbf{B_b^T D_b B_b} dl \tag{6}$$

for the bar elements, where $\mathbf{D_s}$, $\mathbf{B_s}$ and $\mathbf{D_b}$, $\mathbf{B_b}$ are the known elasticity and geometrical matrices for the 2-D structure and

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bar elements, respectively, l represents the length of the bar element, A represents the area of the finite element.

After the finite element analysis, the values of the fitness functions given by (1) or (2) are evaluated and the swarm algorithm is applied.

3. Particle swarm optimizer

The particle swarm algorithms, similarly to the evolutionary [11–13] and immune algorithms [14–16], are developed on the basis of the mechanisms discovered in the nature. The swarm algorithms are based on the models of the animals social behaviours: moving and living in the groups. The animals relocate in the three-dimensional space in order to change their stay place, the feeding ground, to find the good place for reproduction or to evading predators. We can distinguish many species of the insects living in swarms, fishes swimming in the shoals, birds flying in flocks or animals living in herds (Fig. 3a, 3b).

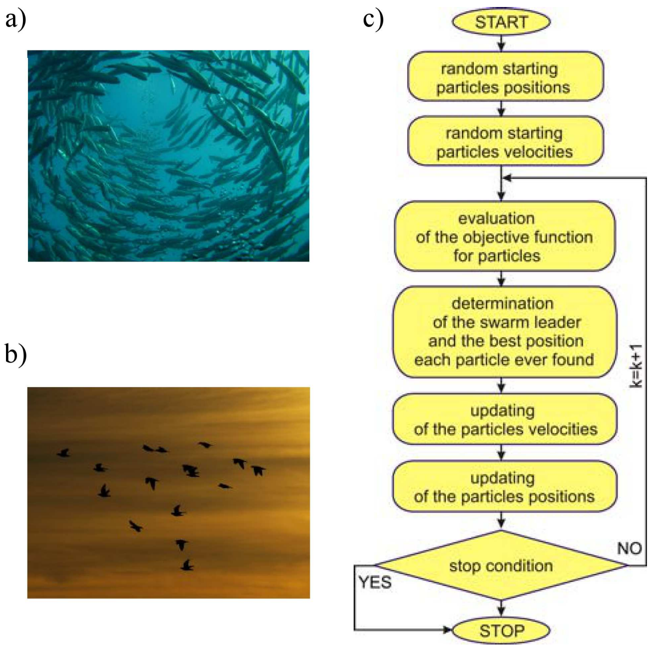

Fig. 3. Particles swarms:
a) fish shoal (http://www.sxc.hu/photo/1187373),
b) bird flock (http://www.sxc.hu/photo/1095384),
c) particle swarm optimiser – block diagram

A simulation of the bird flocking was published by Raynolds [17]. They assumed that this kind of the coordinated motion is possible only when three basic rules are fulfilled: collision avoidance, velocity matching of the neighbours and flock centring. The results of the biological researches where used by Kennedy and Eberhart [18], who proposed Particle Swarm Optimizer – PSO. This algorithm realizes directed motion of the particles in n-dimensional space to search for solution for n-variable optimization problem. PSO works in an iterative way. The location of one individual (particle) is determined on the basis of its earlier experience and experience of whole group (swarm) (Fig. 4). Moreover, the ability to memorize and, in consequence, returning to the areas with convenient properties, known earlier, enables adaptation of the particles to the life environment. The optimization process using PSO is based on finding the better and better locations in the search-space (in the natural environment that are for example hatching or feeding grounds).

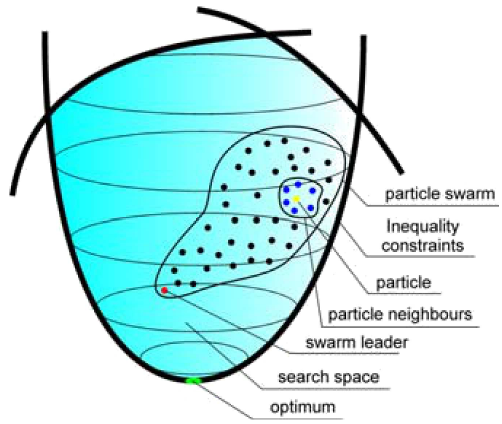

Fig. 4. The idea of the particle swarm

The position of the i-th particle is changed by stochastic velocity v_i, which is dependent on the particle distance from its earlier best position and position of the swarm leader. This approach is given by the following equations:

$$v_{ij}(k+1) = wv_{ij}(k) + \phi_{1j}(k)\left[q_{ij}(k) - h_{ij}(k)\right] + \phi_{2j}(k)\left[\widehat{q}_{ij}(k) - h_{ij}(k)\right], \tag{7}$$

$$h_{ij}(k+1) = h_{ij}(k) + v_{ij}(k+1), \quad i = 1,2,...,m, \quad j = 1,2,...,n, \tag{8}$$

where

$$\phi_{1j}(k) = c_1 r_{1j}(k), \qquad \phi_{2j}(k) = c_2 r_{2j}(k),$$

m – number of the particles,
n – number of design variables (problem dimension),
w – inertia weight,
c_1, c_2 – acceleration coefficients,
r_1, r_2 – random numbers with uniform distribution [0,1],
$h_i(k)$ – position of the i-th particle in k-th iteration step,
$v_i(k)$ – velocity of the i-th particle in k-th iteration step,
$q_i(k)$ – the best found position of the i-th particle found so far,
$\widehat{q}_i(k)$ – the best position found so far by swarm – the position of the swarm leader,
k – iteration step.

The flowchart of the particle swarm optimizer is presented in Fig. 3c. At the beginning of the algorithm the particle swarm of assumed size is created randomly. Starting positions and velocities of the particles are created randomly. The objective function values are evaluated for each particle. In the next step the best positions of the particles are updated and the swarm leader is chosen. Then the particles velocities are modified by means of Eq. 7 and particles positions are modified according to Eq. (8). The process is iteratively repeated until the stop condition is fulfilled. The stop condition is typically expressed as the maximum number of iterations.

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The modified version of PSO algorithm with additional procedure has been used in presented research. The additional procedure consists of two stages. First the clones of the swarm leader are created. The clones substitute the particles with the worst fitness. Next the clones parameters are mutated with the declared probability by adding random numbers with uniform distribution from the range of the design parameters variation

$$\widehat{q}_{ij}(k+1) = \widehat{q}_{ij}(k) + rand\left[h_j^{\min}, h_j^{\max}\right], \qquad (9)$$

where $rand\left[h_j^{\min},\ h_j^{\max}\right]$ – function which returns the random numbers with uniform distribution from the range of the design parameters variation $\left[h_j^{\min},\ h_j^{\max}\right]$.

4. Examples of swarm optimization of structures

Two numerical examples of stiffeners optimization in 2-D structures are considered. Example 1 represents optimization of a plate in plane stress stiffened with 2 curved ribs. Example 2 is devoted to optimization of a shell structure stiffened with 5 ribs. The domain of 2-D structures and domains of the bars in each example are filled by an elastic homogeneous and isotropic material of a Young's modulus $E = 2 * 10^5$ MPa and a Poisson ratio $\nu = 0.3$. The results of the examples are obtained by use of an optimization method based on the swarm algorithm with parameters included in Table 1. The stiffeners in each of the numerical examples have rectangular cross-section of dimensions $d \times h$.

Table 1
Parameters of Particle Swarm Optimizer

number of particles	30–40
inertia weight w	0.73
acceleration coefficient c_1	1.47
acceleration coefficient c_2	1.47
number of the clones	5
probability of mutation	50%

4.1. Example 1. The optimization task of two stiffeners location and shape in a plate in plane stress with boundary conditions shown in Fig. 5a is considered. The optimal positions of stiffeners are searched in order to maximize stiffness of the plate. The function J_1 dependent on maximal nodal displacement in the structure is minimized:

$$J_1 = \max\sqrt{u_x^2 + u_y^2},\ J_1 \to \min. \qquad (10)$$

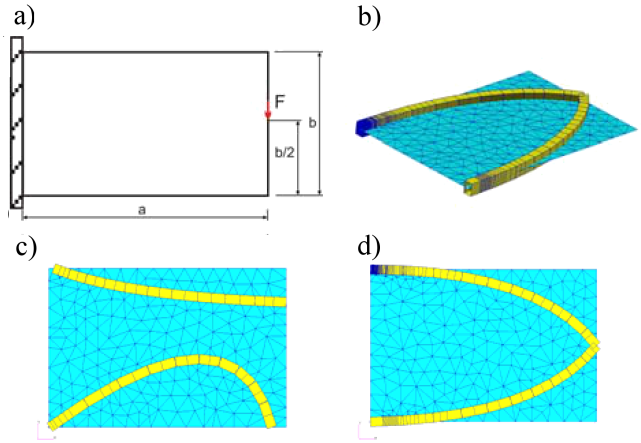

Fig. 5. Plate in plane stress (example 1): a) geometry and boundary conditions, b) the optimal solution, c) 1^{st} iteration, d) 339^{th} iteration

Shapes of stiffeners are parameterized by 3-point NURBS curves. The value of weight of each control point is 1 (no influence on distance between the control point and the NURBS curve). Input data to the optimization program and the parameters of the swarm algorithm are included in Table 2 and 1, respectively. The results of the optimization process are presented in the Fig. 5.

Table 2
Input data to the optimization program for example 1

a × b [mm]	F [N]	Number of stiffeners	Number of particle parameters	Rectangular cross-section of dimensions $d \times h$ [mm]	Thickness of the plate [mm]
400×600	1000	2	8	10×20	8

4.2. Example 2. The optimization task of five stiffeners location by the minimization of the stress functional J_2 (dependent on von Misses equivalent stresses σ_{eq}) in a cylindrical shell is considered

$$J_2 = \int\limits_{\Omega_{shell}} \sigma_{eq} d\Omega_{shell}, \qquad J_2 \to \min. \qquad (11)$$

The structure is stretched with continuous load q and is fixed as presented in the Fig. 6a. Input data to the optimization program and the parameters of the swarm algorithm are included in Table 3 and 1, respectively. The results of the optimization process are presented in the Fig. 6.

Table 3
Input data to the optimization program for example 2

a × b [mm]	q [N/mm]	Number of stiffeners	Number of particle parameters	Rectangular cross-section of dimensions $d \times h$ [mm]	Thickness of the shell [mm]
300×200	450	5	10	10×20	10

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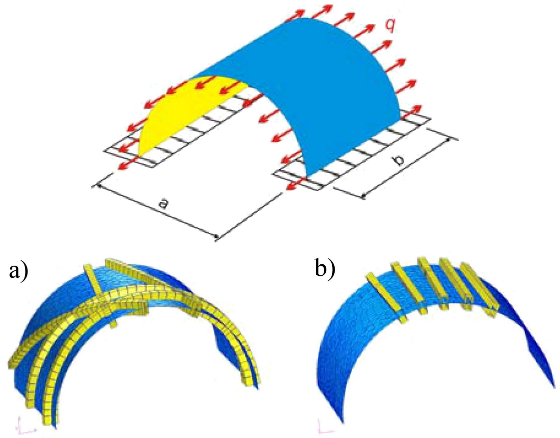

Fig. 6. The cylindrical shell (example 2): a) geometry and boundary conditions, b) 1^{st} iteration , b) 186^{th} iteration

4.3. Comparison of the effectiveness between PSO and DEA. The main drawback of the bio-inspired approaches is the long time of calculations. So the choose of the effective method seems to be quite important. The comparison of the particle swarm optimiser (PSO) and distributed evolutionary algorithm (DEA) [19] with parameters included in Table 4 has been made. The results of the comparison obtained for the presented above numerical examples are included in the Table 5. The stiffeners arrangement obtained for examples 1 is consistent for both applied algorithms and different for example 2 (Fig. 7). Fitness function value for the result obtained using PSO is better.

Table 4
Parameters of distributed evolutionary algorithm

Number of subpopulations	2
Number of chromosomes in each subpopulation	10
Probability of Gaussian mutation	100%
Probability of simple crossover	100%
Selection method	rang selection

Table 5
Results of the comparison of PSO and DEA

	Example 1		Example 2	
	DEA	PSO	DEA	PSO
Fitness function value	0.001642	0.001640	1675702	1589670
Number of iterations	1563	339	463	186
Number of individuals in each iteration	20	40	20	30
Number of fitness function evaluations	31260	13560	9260	5580

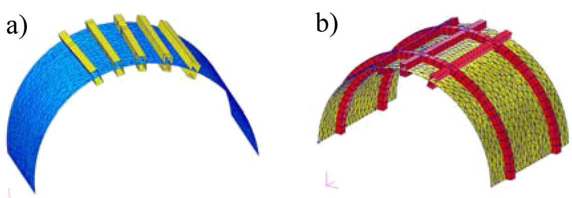

Fig. 7. Location of five stiffeners in the cylindrical shell obtained using: a) PSO, b) DEA

5. Conclusions

An effective tool of swarm optimization of 2-D structures stiffened with several ribs is presented. Using this approach the optimal arrangement of the stiffeners in geometry of 2-D structures can be found. Implementation of the swarm algorithm to this approach gives a strong probability of finding the global optimal solutions. This approach is free from limitations connected with classic gradient optimization methods referring to the continuity of the objective function, the gradient or hessian of the objective function and the substantial probability of getting a local optimum. The swarm algorithm performs multidirectional optimum searching by exchanging information between particles and finding better and better particles positions. The result presented for example 1 is consistent with the one obtain by Bojczuk and Szteleblak with application of heuristic algorithm [5], what proves the accuracy of the presented swarm approach. Comparison between PSO and DEA proves good effectiveness of particle swarm optimization method.

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REFERENCES

[1] M.P. Bendsoe and N. Kikuchi, "Generating optimal topologies in structural design using a homogenization method", *Comput. Methods Appl. Mech. Eng.* 71, 197–224 (1988).

[2] K.T. Cheng and N. Olhoff, "An investigation concerning optimal design of solid elastic plates", *Int. J. Solids Struct.* 17, 305–323 (1981).

[3] X. Ding and K. Yamazaki, "Stiffener layout design for plate structures by growing and branching tree model (application to vibration-proof design)", *Struct. Multidisciplinary Optim.* 96, 99–110 (2004).

[4] A.R. Diaz and N. Kikuchi, "Solution to shape and topology eigenvalue optimization problems using a homogenization method", *Int. J. Numer. Methods Eng.* 35, 1487–1502 (1992).

[5] D. Bojczuk and W. Szteleblak, "Optimization of layout and shape of stiffeners in 2D structures", *Computers and Structures* 86, 1436–1446, (2008).

[6] J. Sokołowski and A. Żochowski, "On topological derivative in shape optimization", *SIAM J. Control. Optim.* 37 (4), 1251–72 (1999).

[7] E. Silverman, M. Rhodes, and M. Dyer, "Composite isogrid structures for spacecraft components", *SAMPE J.* 35, 51–59 (1999).

[8] L. Piegel and W. Tiller, *The NURBS Book*, second ed., Springer, Berlin, 1995.

[9] O.C.Zienkiewicz and R.L.Taylor, *The Finite Element Method*, Butterworth Heinemann, Oxford, 2000.

[10] J.R. Shewchuk, "Triangle: engineering a 2d quality mesh generator and delaunay triangulator", *First Workshop on Applied Computational Geometry, Association for Computing Machinery* 1, 124–133 (1996).

[11] Z. Michalewicz, *Genetic Algorithms + Data Structures = Evolutionary Programs*, Springer, Berlin, 1992.

Authenticated | 195.187.97.1
Download Date | 10/31/12 11:44 AM

[12] T. Burczyński, A. Poteralski, and M. Szczepanik, "Topological evolutionary computing in the optimal design of 2D and 3D structures", *Eng. Optimisation* 39 (7), 811–830, (2007).

[13] T. Burczyński, W. Kuś, A. Długosz, A. Poteralski, and M. Szczepanik, "Sequential and distributed evolutionary computations in structural optimisation", *Lecture Notes on Artificial Intelligence* 3070, 1069–1074 (2004).

[14] S.T. Wierzchoń, *Artificial Immune Systems,Ttheory and Applications*, EXIT, Warszawa, 2001, (in Polish).

[15] T. Burczyński, M. Bereta, A. Poteralski, and M. Szczepanik, "Immune computing: intelligent methodology and its applications in bioengineering and computational mechanics", *Computer Methods in Mechanics*, *Advanced Structured Materials* 1, 165–181 (2010).

[16] A. Poteralski, M. Szczepanik, G. Dziatkiewicz, W. Kuś, and T. Burczyński, "Immune identification of piezoelectric material constants using BEM", *Inverse Problems in Science and Eng.* 19, 103–116 (2010).

[17] C.W. Reynolds, "Flocks, herds, and schools, A distributed behavioral model", *Computer Graphics* 21, 25–34 (1987).

[18] J. Kennedy and R.C. Eberhart, *Swarm Intelligence*, Morgamn Kauffman, San Francisco, 2001.

[19] R. Tanese, "Distributed Genetic Algorithms", *Proc. 3rd ICGA* 1, 434–439 (1989).

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