



**Eugeniusz Ranatowski** 

*University of Technology and Life Sciences, Faculty of Mechanical Engineering, Al. Prof. S. Kaliskiego 7, Pl - 85-789 Bydgoszcz, Poland e-mai: ranatow@utp.edu.pl*

#### *Abstract*

*After formulating a simplified model of under- and overmatched welded joints an analysis was made of stress at interface for the cases of perpendicular and non - perpendicular orientation of the zones (soft layer and hard layer) relative to the load action direction under tension. Conclusions from the theoretical analysis form a basis to an assessment of effort and fracture of the mismatched welded joints. Conditions for producing brittle and ductile fracture in mismatched welded joints in relation to geometrical conditions of the layer (W), expressed by* κ*, and the*  mechanical properties of the layer materials,  $R_e^W$ ,  $R_0$  and equivalent stresses  $\sigma_H$ ,  $\sigma_v$  are established in further *experiments.* 

*Key words: stress state, fracture resistance, mismatched weld joints* 

# **1. Introduction**

Considerable local diversification of the material structure and consequently, of the mechanical properties may occur in the weld or in the heat affected zone (HAZ). This dissimilarity is caused by different mechanical and chemical features of the weld and base materials as well as by the thermal and strain cycles during welding and may occur through the fusion line and HAZ of welds. The effect of strength mis-match in steel weldments has received much attention over the last years and a main reason for this interest is the increased use of steel with higher strength and the difficulties in specified weld metal toughness. The mechanical properties and effects of strength mis-match depends on the presumption, and many misunderstandings have occurred because the purpose with the examinations and the selection of boundary conditions have not been clearly specified. Considering the above - mentioned problem of heterogeneous weld joints we will focus our attention on a simplified model with thin layer-soft or hard- which are presented on Fig. 1. Considering the above - mentioned problem of a mis-match welded joint, it is essential that a model which shows the real conditions of the joint is presented. It should be assumed that the respective model presents the physical reality precisely enough to ensure the physical or technical sense of the models analysis. Thus, the physical model is a simplification of the real welding system and only matches the system in respect of its essential features.

The essential physical phenomena affecting the mechanical properties of these models occur at the interface of zones (B) and (W). Determination of change in the state of stress occurring in this area is of primary importance for correct interpretation and estimation of mechanical properties and fracture resistance of these models. The main difficulty adequately estimating the state of stress is that the material of an undermatched weld joint undergoes heterogeneous deformations which result in non-uniform stress pattern. It is possible for discontinuities of stress to arise, but these should not disturb the equilibrium state.



*Fig. 1. Characteristic of the models of the mismatched weld joints: a. geometrical configuration - layer W is incline to external load, b. geometrical configuration - layer W is perpendicular to external load, c. change of the yield point R<sup>e</sup> in the overmatched weld joint, d. change of the yield point R<sup>e</sup> in the undermatched weld joint.* 

### **2. Characterisation of the state of stress at interface of zones (B) and (W)**

Components of the state stress in mis-matched weld joints under static tension are determined by the equilibrium equations and the equation of the plasticity condition which fulfilling the loundary condition of the interfaces of zones B and W (Fig.1). A suitable analytic models are presented in Figs.2 and 3.



*Fig. 2. Characteristic of analytic models of mismatched weld joints with inclined layer to force 2P: a. undermatching case, b. overmatching case.* 

The components  $\sigma_{xx}$ ,  $\sigma_{yy}$  and  $\sigma_{xy}$  for the undermatched and overmatched weld joints with inclined layer W to be determined as follows [1]:

undermatched weld joints (Fig.1a, d and  $2a$ ):

$$
\sigma_{x(\text{rel})}^{un} = \frac{\sigma_{xx}^{un}}{k} = \frac{1}{1 - \gamma} \left( \frac{\pi}{2} - \gamma \sqrt{1 - \gamma^2} - \arcsin \gamma \right) + \frac{1 - \gamma}{2} \frac{\xi}{\kappa} - \frac{1}{2\gamma} \sqrt{1 - \left( \frac{1 + \gamma}{2} + \frac{1 - \gamma}{2} \frac{\eta}{\kappa} \right)^2} \tag{1}
$$

$$
\sigma_{y(\text{rel})}^{un} = \frac{\sigma_y^{un}}{k} = \frac{1}{1 - \gamma} \left( \frac{\pi}{2} - \gamma \sqrt{1 - \gamma^2} - \arcsin \gamma \right) + \frac{1 - \gamma}{2} \frac{\xi}{\kappa},\tag{2}
$$

$$
\sigma_{xy(\,rel\,)}^{un} = \frac{\sigma_{xy}^{un}}{k} = \frac{1+\gamma}{2} + \frac{1+\gamma}{2} \frac{\eta}{\kappa},\tag{3}
$$

 $\gamma =$ k  $\frac{k_1}{k}$ ,  $|\gamma| \le 1$ ,  $k = \frac{R_e^{W(un)}}{\sqrt{3}}$  $k = \frac{R_e^{\text{W}}(un)}{r}$ =  $\frac{1}{3}$ ,  $-k \le k_1 \le k$ ,  $R_e^{\text{W}(un)} \le R_e^{\text{B}}$ .

- overmatched weld joints (Figs 1a, c and 2b):

$$
\sigma_{xx(\text{rel})}^{\text{ov}} = \frac{\sigma_{xx}^{\text{ov}}}{k} = -\left[\frac{1}{1-\gamma}\left(-\frac{\pi}{2} + \gamma\sqrt{1-\gamma^2} + \arcsin\gamma\right) + \frac{1-\gamma}{2}\frac{\xi}{\kappa} + \right. \\
\left. + 2\sqrt{1-\left(\frac{1+\gamma}{2} + \frac{1-\gamma}{2}\frac{\eta}{\kappa}\right)^2} \right],
$$
\n(4)

$$
\sigma_{yy(\,rel)}^{ov} = \frac{\sigma_{yy}^{ov}}{k} = \frac{1}{1-\gamma} \left( -\frac{\pi}{2} + \gamma \sqrt{1-\gamma^2} + \arcsin\gamma \right) + \frac{1-\gamma}{2} \frac{\xi}{\kappa},\tag{5}
$$

$$
\sigma_{xy(\text{rel})}^{\text{ov}} = \frac{\sigma_{xy}^{\text{ov}}}{k} = \frac{1+\gamma}{2} + \frac{1-\gamma}{2}\frac{\eta}{\kappa},\tag{6}
$$

 $\gamma = k_1 / k, |\gamma| \le 1, k = R_e^{W(ov)}/\sqrt{3}, R_e^{W(ov)} \ge R_e^B$  $R_e^{\text{W(ov)}} \ge R_e^{\text{B}}$ ,

 $k=\frac{2}{3}$ 2

t

$$
\frac{h}{t}, \eta = \frac{2y}{2t}, \xi = \frac{2x}{2t}, \kappa \ge \eta.
$$
\n
$$
\xrightarrow{\frac{k}{t} + \frac{4}{y} + \frac{4}{z}} \xi
$$
\n
$$
\xrightarrow{\frac{k}{t} + \frac{4}{z} + \frac{4}{z}} \xi
$$
\n
$$
\xrightarrow{\frac{k}{t} + \frac{4}{z}} \xi
$$
\n

*Fig. 3. Characteristic of analytic models of mismatched weld joints with perpendicular layer to force 2P. a. undermatching case, b. overmatching case.* 

If the layer is perpendicular to the tension force 2P the parameter  $\gamma$  is eq. (- 1) and equation  $(1) \div (3)$ ,  $(4) \div (6)$  are transformed in the following shape:

undermatched case (Fig. 1b, d and 3a):

$$
\sigma_{xx(rel)}^{un} = \frac{\sigma_{xx}^{un}}{k} = \frac{1}{2}\pi + \frac{\xi}{\kappa} - 2\sqrt{1 - \frac{\eta^2}{\kappa^2}},
$$
\n(7)

$$
\sigma_{yy(\text{rel})}^{un} = \frac{\sigma_{yy}^{un}}{k} = \frac{\pi}{2} + \frac{\xi}{\kappa},\tag{8}
$$

$$
\sigma_{xy(\,rel\,)}^{un} = \frac{\sigma_{xy}^{un}}{k} = \frac{\eta}{\kappa},\tag{9}
$$

overmatched case (Figs 1b, c and 3a):

$$
\sigma_{xx(\text{rel})}^{\text{ov}} = \frac{\sigma_{xx}^{\text{ov}}}{k} = \frac{\pi}{2} - \frac{\xi}{\kappa} + 2\sqrt{I - \frac{\eta^2}{\kappa^2}},\tag{10}
$$

$$
\sigma_{yy\text{(rel)}}^{ov} = \frac{\sigma_{yy}^{ov}}{k} = -\frac{\pi}{2} + \frac{\xi}{\kappa},\tag{11}
$$

$$
\sigma_{xy(\,rel\,)}^{ov} = \frac{\sigma_{xy}^{ov}}{k} = \frac{\eta}{\kappa},\tag{12}
$$

These above equations  $(7) \div (9)$  assumes the form previously determined by Prandtl for undermatched case. It were reveals a non-linear stress state of  $\sigma_{xx}$ ,  $\sigma_{yy}$  in inclined soft and hard layers and some analytical examples are presented in Figs 4 and 5.



*Fig. 4. Characteristic of stresses,*  $\sigma^{un}_{xx (rel)}$ ,  $\sigma^{un}_{yy (rel)}$ ,  $\sigma^{un}_{xy (rel)}$ *at* κ *= 0,3; 0,5; 0,75; 0,99 and* η *= 0,3;* ξ *= 0,1.* 



Fig. 5. *Characteristic of stresses*  $\sigma_{xx}^{ov}$   $(\text{rel})$ ,  $\sigma_{yy}^{ov}$   $(\text{rel})$ ,  $\sigma_{xy}^{ov}$ *at* κ *= 0,3; 0,5; 0,75; 0,99 and* η*=0,3;* ξ*=0,1.* 

# **3. Influence of the state of stress on the mechanical properties and fracture resistance of mismatched weld joint models**

From the practical point of view, the effect of the change in the state of stress on the mechanical properties of the welded joint model at static tension is very interesting. It can be expressed by the average values of stresses that can be transferred by a joint with a soft or hard layer as [2]:

$$
\sigma_{ever}^{un} = \frac{2R_e^{W(un)}}{\sqrt{3}} \left\{ \frac{1}{4(l-q)} \left[ \frac{\pi}{2} + 2(l-2q)\sqrt{(q(l-q))} - arc\sin(2q-l) \right] + (l-q)\frac{l}{4\kappa} \right\},\tag{13a}
$$

$$
\sigma_{ever}^{ov} = \frac{2R_e^{W(ov)}}{\sqrt{3}} \left\{ \frac{1}{4(l-q)} \left[ -\frac{\pi}{2} - 2(l-2q)\sqrt{(q(l-q))} + arc\sin(2q-l) \right] + (l-q)\frac{l}{4\kappa} \right\},\tag{13b}
$$

where:

$R_{\text{ever}}^{W(\text{un})}$	The tensile yield point of the layer (W) for under- and overmatched case,
q	The factor which allows for the effect of interface normalised tangential stresses $0 \leq q < 1$ .

By converting the above equation and introducing the following ratio:

$$
K_W^{un} = \frac{2}{\sqrt{3}} \left( \frac{1}{4(l-q)} \left[ \frac{\pi}{2} + 2(l-2q)\sqrt{q(l-q)} - arc\sin(2q-l) \right] + (l-q)\frac{1}{4\kappa} \right),\tag{14a}
$$

$$
K_W^{ov} = \frac{2}{\sqrt{3}} \left( \frac{l}{4(l-q)} \left[ -\frac{\pi}{2} - 2(l-2q)\sqrt{q(l-q)} + \arcsin(2q-l) \right] + (l-q)\frac{l}{4\kappa} \right),\tag{14b}
$$

we can evaluate the effect of the change of mechanical properties of the soft or hard (zone) (W) as a result of the change in the state of stress as a constraint factors  $K_W^{un}$ ,  $K_W^{ov}$ . The above data indicate that the greater value of  $K_W$ , the smaller the value of  $\kappa$  and q but different for under- and overmatched case. If  $q = 0$  (2Q=0,  $\alpha$ =0), equation (13a) assumes the form previously determined by Kačanov for undermatched case:

$$
\sigma_{ever}^{un} = \frac{2}{\sqrt{3}} R_e^{W(un)} \left( \frac{\pi}{4} + \frac{1}{4\kappa} \right),\tag{15}
$$

For overmatched case equation (13b) takes the following form:

$$
\sigma_{ever}^{ov} = \frac{2}{\sqrt{3}} R_e^{W(ov)} \left( -\frac{\pi}{4} + \frac{1}{4\kappa} \right),\tag{16}
$$

The theoretical values of  $K_W^{un}$  indicate that the mechanical properties of the so-called soft layer can be considerably improved due to the change in stresses of that area. Apart from the geometrical conditions, the upper limit of the strength is determined by the mechanical properties of the zones (B) and (W).

If the mechanical properties of the material in the zone (B), determined as  $R_m^B$  (tensile strength) and  $R_e^B$  (tensile yield strength), correspond in principle with the mechanical properties of the material in its initial state before welding, and it is assumed that  $\sigma_{\text{ever}} = R_m^B$ , then the relative thickness of the layer (W) which has no negative effect on the whole strength of the welded joint can be calculated from the following equation ( $K_s = R_e^B / R_e^{W(un)}$ ,  $\gamma^B = R_m^B / R_e^B$ e B m  $\gamma^{\rm B} = R_{\rm m}^{\rm B}/R_{\rm e}^{\rm B}$  ):

$$
\kappa_{cr} = \frac{1-q}{2\sqrt{3(1-q)K_s \cdot \gamma^B - \left[\pi/2 + 2(1-2q)\sqrt{q(1-q)} - \arcsin(2q-1)\right]}}\,,\tag{17}
$$

Conditions which cause brittle fracture can be determined based on the conception of Pełczyński [3] as:

- undermatching case

$$
\frac{\sigma_{ever}^{un}}{R_0^{un}} = \frac{\sigma_H^{un}}{\sigma_v^{un}},
$$
\n(18a)

overmatching case

$$
\frac{\sigma_{\text{ever}}^{\text{ov}}}{R_0^{\text{ov}}} = \frac{\sigma_H^{\text{ov}}}{\sigma_v^{\text{ov}}},\tag{18b}
$$

where:

 $R_0$  - cohesive strength,  $\sigma_H$ ,  $\sigma_v$ - the equivalent stress according to Huber-Mises and Saint-Venant, respectively.

The equivalent stresses  $\sigma_H^{un}$ ,  $\sigma_H^{ov}$  can be calculated from:

- undermatching case

$$
\sigma_H^{un} = \sqrt{\left(\sigma_{xx}^{un} - \sigma_{yy}^{un}\right)^2 + \sigma_x^{un} \sigma_y^{un} + 3\sigma_{xy}^{un^2}}\,,\tag{19a}
$$

- overmatching case

$$
\sigma_H^{ov} = \sqrt{\left(\sigma_{xx}^{ov} - \sigma_{yy}^{ov}\right)^2 + \sigma_x^{ov}\sigma_y^{ov} + 3\sigma_{xy}^{ov^2}}\,,\tag{19b}
$$

Instead of  $\sigma_{v}^{\text{un}}, \sigma_{v}^{\text{ov}}$  we can write:

- undermatching case

$$
\sigma_v^{un} = \frac{\left(\sigma_{xx}^{un} + \sigma_{yy}^{un}\right)\left(1 - \nu\right)}{2} + \frac{1 + \nu}{2} \sqrt{\left(\sigma_{xx}^{un} + \sigma_{yy}^{un}\right)^2 + 4\sigma_{xy}^{un^2}}\,,\tag{20a}
$$

- overmatching case

$$
\sigma_v^{ov} = \frac{\left(\sigma_{xx}^{ov} + \sigma_{yy}^{ov}\right)\left(1 - \nu\right)}{2} + \frac{1 + \nu}{2} \sqrt{\left(\sigma_{xx}^{ov} + \sigma_{yy}^{ov}\right)^2 + 4\sigma_{xy}^{ov^2}},\tag{20b}
$$

Therefore, with regard to equation (15), (16) and (18a), (18b) we can evaluate the geometrical conditions of the layer (W) expressed by the parameter κ which can be calculated from:

- undermatching case

$$
\kappa = \frac{R_e^{W(un)} \sigma_v^{un}}{2\sqrt{3}\sigma_H^{un} R_0^{un} - \pi R_e^{W(un)} \sigma_v^{un}},
$$
\n(21a)

- overmatching case

$$
\kappa = \frac{R_e^{W(ov)} \sigma_v^{ov}}{2\sqrt{3}\sigma_H^{ov} R_0^{ov} + \pi R_e^{W(ov)} \sigma_v^{ov}},
$$
\n(21b)

The solution of the above equation gives a value for parameter  $\kappa$  after which brittle fracture may occur. Mismatched weld joints fail in ductile mode in soft or hard layer when:

- undermatching case

$$
\frac{\sigma_{\text{ever}}^{un}}{R_0^{un}} < \frac{\sigma_H^{un}}{\sigma_v^{un}},\tag{22a}
$$

- overmatching case

$$
\frac{\sigma_{\text{ever}}^{\text{ov}}}{R_0^{\text{ov}}} < \frac{\sigma_H^{\text{ov}}}{\sigma_v^{\text{ov}}},\tag{22b}
$$

We can now evaluate the geometrical conditions of the layer (W) expressed by parameter  $\kappa$  when the mismatched weld joints fail in the ductile mode:

undermatching case

$$
\kappa > \frac{R_e^{W(un)}(I-q)\sigma_v^{un}}{2\sqrt{3}\sigma_H^{un}R_0^{un} - R_e^{W(un)}\sigma_v^{un}\left[\pi/2 + 2(I-2q)\sqrt{q(I-q) - arc\sin(2q-1)}\right]},
$$
(23a)

overmatching case

$$
\kappa > \frac{R_e^{W(ov)}(1-q)\sigma_v^{ov}}{2\sqrt{3}\sigma_H^{ov}R_0^{ov} - R_e^{W(ov)}\sigma_v^{ov}[-\pi/2 - 2(1-2q)\sqrt{q(1-q) + \arcsin(2q-1)}}\right],
$$
(23b)

Regarding the requirements of, e.g., standards concerning the estimation of  $K_{IC}$ , it should be noted that in the zone (W) favourable conditions for passing  $K_C \rightarrow K_{IC}$  occur when the value of  $K_W^{\text{un/ov}}$  is increased.  $K_C$  and  $K_{IC}$  are the critical values of stress intensity factors adequate for plane stress and strain [3].

# **Conclusions**

The following revealed features of undermatched and overmatched weld joints models were established:

- state of stress is mismatched weld joint models under static tension,
- constraint factor  $K_{\rm w}^{\rm un/ov}$  which described change of mechanical properties of the soft or hard zones (W) as a result on the conversion in the state of stress,
- relative thickness  $\kappa_{cr} = f(K_s, \gamma^B, q)$  of the layer (W) for undermatched weld joint model which has no negative effect on the strength at static tension,
- conditions for producing brittle and ductile fracture in mismatched weld joint models in relation to geometrical conditions of the layer (W), expressed by  $\kappa$ , and the mechanical properties of the layer materials,  $R_e^{W(\text{un})/(\text{ov})}$ ,  $R_0^{\text{un}/\text{ov}}$  and equivalent stresses  $\sigma_H^{\text{un}/\text{ov}}$ ,  $\sigma_{\rm v}^{\rm un/ov}$  are established.

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