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APPROXIMATE DETERMINATION OF A STRAIN-CONTROLLED FATIGUE LIFE CURVE FOR ALUMINUM ALLOY SHEETS

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Abstract

This paper deals with selected methods of approximate determination of a strain-controlled fatigue life curve for aluminium alloy sheets used in aircraft structures first of all. Authors based their analysis of those methods on the results of own research of 2024-T3 alloy and its Russian equivalent D16CzATW. The approximate strain-fatigue life curves were compared with the experimental curves. The influence of inconsistencies between those curves on the calculation results was analyzed on computational examples by means of the Palmgren-Miner's rule.

Keywords: *aluminium alloy, fatigue properties, fatigue life curve, estimation method, monotonic tensile test*

List of major symbols and abbreviations

$2N_f$ - reversals to failure (2 reversals = 1 cycle)
b - fatigue strength exponent
c - fatigue ductility exponent
E - Young's modulus, MPa
N_{cal} - calculating fatigue life obtained on the basis of the approximate strain-controlled fatigue life curve, cycles
N_{exp} - calculating fatigue life obtained on the basis of the experimental strain-controlled fatigue life curve, cycles
RA - reduction in area
S_u - ultimate tensile strength, MPa
ε_f - true fracture ductility
ε'_f - fatigue ductility coefficient
$\Delta\varepsilon$ - total strain range
$\Delta\varepsilon_e$ - elastic strain range
$\Delta\varepsilon_p$ - plastic strain range
σ_f - true fracture strength, MPa
σ'_f - fatigue strength coefficient, MPa

FPCM - Four-Point-Correlation Method
USM - Universal Slopes Method

MUSM - Modified Universal Slopes Method
UMLM - Uniform Material Law
MFPCM - Modified Four-Point-Correlation Method
MMM - Modified Mitchell's Method
MM - Median Method

1. Introduction

Fatigue curves serve as the basis for the calculation of fatigue strength [7, 17]. As fatigue tests are characterised by high labour and time consumption and are very costly, it is not always possible to perform full range fatigue tests or, for comparison purposes, it is sufficient to approximately determine the fatigue curve based e.g. on relatively simple and quick monotonic tensile tests or on available literature data. Such approach has been introduced, among others, to expert systems used to estimate fatigue properties [5].

A strain-controlled fatigue life curve is characterized by the following relationship:

$$\frac{\Delta\varepsilon}{2} = \frac{\Delta\varepsilon_e}{2} + \frac{\Delta\varepsilon_p}{2} = \frac{\sigma'_f}{E}\cdot(2N_f)^b + \varepsilon'_f\cdot(2N_f)^c\,. \tag{1}$$

One of the first methods for approximate determination of the relationship (1) based on the monotonous tensile test was proposed by Manson [8]. The first of them - Four–Point–Correlation Method has been later modified by Ong [13]. The second of them - Universal Slopes Method – has been modified by Muralidharan and Manson [11].

Socie, Mitchell and Caulfield [16] presented relationships designed to determine factors of the formula (1) for steel. Another method, intended particularly for steel grades with hardness value below 500 HB, was proposed by Mitchell [10]. Bäumel and Seeger [2] presented Uniform Material Law Method which is suitable for metals. Its coefficients are very similar to those of the Modified Universal Slopes Method. Roessle and Fatemi [15] proposed the method used for steel and based on the hardness value and the Young's modulus only. Whereas Hatscher, Marquardt and Zenner presented the Variable Slopes Method empirically verified for steel sheet [4]. Whereas as regards Polish literature, we can mention the study by Flasińska and Łagoda that includes attempts to find the relationship between selected static and fatigue properties [3].

Studies by Ong [12] (for 49 steel grades) as well as Kim, Chen, Han and Lee [6] (for 8 steel grades) also include empirical analysis of methods for approximate determination of a strain-controlled fatigue life curve. Park and Song [14] analysed several methods used for 138 types of material (116 steel grades, 16 aluminium alloys and 6 titanium alloys) and they concluded that methods proposed by Bäumel and Seeger [2], Muralidharan and Manson [11] as well as by Ong [13] provide better approximation of experimental data than the remaining ones. Song and Park [18] analysed 6 methods used for 5 groups of materials and they found out that universal slopes method seems to be the best for steel whereas the method developed by Bäumel and Seeger gives satisfactory results when monotonous properties exclude necking [2]. They also proposed a new method (a modified Mitchell's method) which better estimates fatigue properties of aluminium alloys [10]. Whereas Meggiolaro and Castro presented Medians Method based on statistical analysis of parameters used in the relationship (1) performed for 724 steel grades and 81 aluminium alloys [9].

2. Selected methods of fatigue curve determination for aluminium alloys

2.1. Four-Point-Correlation Method (FPCM) - (Manson, 1965)

The Four-Point-Correlation Method proposed by Manson [8] is based on plastic strain and elastic strain values represented by lines $\Delta\varepsilon_p$ and $\Delta\varepsilon_e$. Those lines are determined upon the basis of two points.

Coefficients used in the formula (1) for this method can be characterised by the following relationships:

$$\sigma'_f = \frac{E}{2} \cdot 10^{b \cdot \log 2 + \log\left[\frac{2{,}5 S_u \cdot (1+\varepsilon_f)}{E}\right]}, \tag{2}$$

$$\varepsilon'_f = \frac{1}{2} \cdot 10^{c \cdot \log\frac{1}{20} + \log\left(\frac{1}{4} \cdot \varepsilon_f^{3/4}\right)}, \tag{3}$$

$$b = \frac{\log\left[\frac{2{,}5 \cdot (1+\varepsilon_f)}{0{,}9}\right]}{\log\left(\frac{1}{4 \cdot 10^5}\right)}, \tag{4}$$

$$c = \frac{1}{3}\log\left(\frac{0{,}0132 - \Delta\varepsilon_e^*}{1{,}91}\right) - \frac{1}{3}\log\left[\frac{1}{4} \cdot \varepsilon_f^{3/4}\right], \tag{5}$$

where ε_f is dependent on reduction in the RA area of the specimen

$$\varepsilon_f = \ln\left(\frac{1}{1-RA}\right), \tag{6}$$

whereas $\Delta\varepsilon_e^*$ is the range of the elastic strain for 10 000 load cycles and it can be characterised by the following relationship

$$\Delta\varepsilon_e^* = 10^{b \cdot \log\left(4 \cdot 10^4\right) + \log\left[\frac{2{,}5 S_u \left(1+\varepsilon_f\right)}{E}\right]}. \tag{7}$$

2.2. Universal Slopes Method (USM) - Manson (1965)

Universal Slopes Method [8] assumes that inclination of lines $\Delta\varepsilon_p$ and $\Delta\varepsilon_e$ characterised by exponents b and c does not depend on material type. The fatigue strength coefficient as well as the fatigue ductility coefficient used in the formula (1) take the following form:

$$\sigma'_f = 1{,}9018 \cdot S_u, \tag{8}$$

$$\varepsilon'_f = 0{,}7579 \cdot \varepsilon_f^{0{,}6}, \tag{9}$$

where ε_f is determined according to the relationship (6), whereas exponents $b = -0{,}12$ and $c = -0{,}6$ assume constant values.

2.3. Modified Universal Slopes Method (MUSM) - Muralidharan and Manson (1988)

Like the original one, the Modified Universal Slopes Method [11] assumes that the exponents b and c do not depend on the material type. Coefficients used in the formula (1) can be calculated based on the following relationships:

$$\sigma'_f = E \cdot 0{,}623 \cdot \left(\frac{S_u}{E}\right)^{0{,}832}, \tag{10}$$

$$\varepsilon'_f = 0{,}0196 \cdot \varepsilon_f^{\;0{,}155} \cdot \left(\frac{S_u}{E}\right)^{-0{,}53}, \tag{11}$$

where ε_f is determined according to the relationship (6), whereas the exponents assume constant values: $b = -0{,}09$ and $c = -0{,}56$.

2.4. Uniform Material Law Method (UMLM) - Bäumel and Seeger (1990)

Uniform Material Law Method [2] assumes that the value of exponents b and c as well as the coefficient ε'_f is constant for the whole group of materials. Only the coefficient σ'_f depends on the material properties. Coefficients used in the formula (1) for this method for aluminium alloys can be characterised by the following relationships:

$$\sigma'_f = 1{,}67 \cdot S_u, \tag{12}$$

whereas constants value is assumed by: $b = -0{,}095$, $\varepsilon'_f = 0{,}35$, $c = -0{,}69$.

2.5. Modified Four-Point-Correlation Method (MFPCM) – Ong (1993)

Modified Four Point Correlation method (MFPC) proposed by Ong [13] differs slightly from the original method proposed by Manson [8]. According to Modified Four Point Correlation method, a strain-controlled fatigue life curve is determined by calculating the elastic strain amplitude at the load reversal level of 10^0 and 10^6 and the plastic strain amplitude at the load reversal level of 10^0 and 10^4. In this method, coefficients used in the formula (1) assume the following form:

$$\sigma'_f = S_u \cdot (1 + \varepsilon_f), \tag{13}$$

$$\varepsilon'_f = \varepsilon_f, \tag{14}$$

$$b = \frac{1}{6} \cdot \left[\log\left(0{,}16 \cdot \left(\frac{S_u}{E}\right)^{0{,}81} \right) - \log\left(\frac{\sigma_f}{E} \right) \right], \tag{15}$$

$$c = \frac{1}{4} \cdot \log \left(\frac{0{,}00737 - \frac{\Delta\varepsilon_e^{\,*}}{2}}{2{,}074} \right) - \frac{1}{4} \cdot \log \sigma_f, \tag{16}$$

where ε_f is determined according to the relationship (6), whereas the elastic strain range $\Delta\varepsilon_e^*$ for $2N_f = 10\,000$ reversals is calculated using the formula:

$$\frac{\Delta\varepsilon_e^{\,*}}{2} = \frac{\sigma_f}{E} \cdot 10^{\frac{2}{3} \cdot \left[\log\left(0{,}16 \cdot \left(\frac{S_u}{E}\right)^{0{,}81} \right) - \log\left(\frac{\sigma_f}{E} \right) \right]}. \tag{17}$$

2.6. Modified Mitchell's Method (MMM) - Song and Park (1996)

Song and Park modified Mitchell's method [10] by adapting it specially for aluminium alloys. This method assumes that coefficients used in the formula (1) can be calculated based on the following relationship:

$$\sigma'_f = S_u + 335, \tag{18}$$

$$b = -\frac{1}{6} \cdot \log\left(\frac{S_u + 335}{0{,}446 \cdot S_u} \right), \tag{19}$$

$$\varepsilon'_f = \varepsilon_f, \tag{20}$$

where ε_f is determined based on the relationship (6), whereas the fatigue ductility exponent assumes constant value $c = -0{,}664$.

2.7. Median Method (MM) - Meggiolaro and Castro (2004)

Based on properties of 81 aluminium alloys, it was assumed that only fatigue strength coefficient depends on strengths properties [9]:

$$\sigma'_f = 1{,}9 \cdot S_u, \tag{21}$$

whereas constant value is assumed by: $b = -0{,}11$, $c = -0{,}66$ and $\varepsilon'_f = 0{,}28$.

3. Strength properties of the selected aluminium alloys used for aircraft purposes

Strength properties of the selected aluminium alloys for aircraft purposes were determined in the Institute Laboratory for Material and Structure Testing at the University of Technology and Life Sciences in Bydgoszcz accredited by Polish Centre for Accreditation (Accreditation Certificate No. AB 372). The scope of accreditation includes, but is not limited to research methods used this study, such as: static tension tests (monotonic properties) and low-cycle fatigue tests for metals.

Tests of both alloy grades, 2024-T3 and D16CzATW, were performed using samples cut from 4 mm thick steel sheets. Basic mechanical properties for both alloy grades were determined in accordance with the norm ASTM E 8M - 04 Standard Test Methods for Tension Testing of Metallic Materials. Table 1 presents selected strength parameters that serve as the base for aforementioned methods of approximate determination of fatigue curves. Low-cycle tests of aluminium alloy grades were performed in accordance with the norm ASTM E 606 - 04 Standard Practice for Strain-Controlled Fatigue Testing. Parameters of fatigue curves determined experimentally were shown in the table 1. Achieved parameters are within the range specified in the literature [1].

4. Approximate fatigue curves determined using methods described herein

Figure 1 shows approximate fatigue curves determined according to abovementioned methods compared to experimental curve for analysed aluminium alloys. Whereas parameters of those curves are presented in Table 2.

All the presented approximate fatigue curves are shifted with regards to the experimental curve. The shape of approximate curves is similar to the experimental curve in the high-cycle range, where elastic component of the total deformation prevails.

However, in the low-cycle range, where plastic strain prevails, the curves are characterized by quite a different shape. The above is also confirmed by the analysis of coefficients in the relationship (1) which were determined using abovementioned methods.

Figure 2a shows the value of the fatigue strength coefficient σ_f' depending on the method used to determine the fatigue curve. Values of that coefficient are within the range ±20% of the value achieved based on experimental tests. Values obtained using the method proposed by Bäumel and Seeger as well as using the modified Mitchell's Method correspond to the largest extent to experimental results.

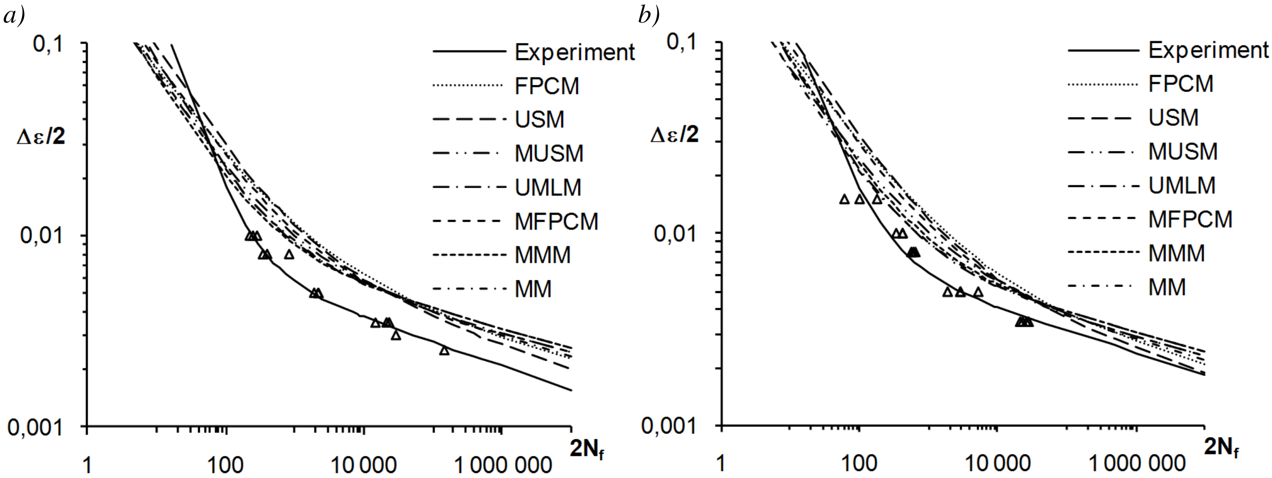


Fig. 1. Approximate fatigue curves determined according to abovementioned methods compared to experimental curve for aluminium alloy grades 2024-T3 (a) and D16CzATW (b)

Tab. 1. Monotonic and fatigue properties of aluminium alloys for aircraft purposes

Aluminium alloy	Monotonic properties				Fatigue properties			
	E	S_u	σ_f	RA	σ_f'	b	ε_f'	c
2024-T3	67 560 MPa	488 MPa	616 MPa	0,233	777 MPa	-0,1234	2,002	-1,1203
D16CzATW	68 402 MPa	460 MPa	613 MPa	0,287	791 MPa	-0,1142	1,456	-1,0718

Table 2. List of coefficients for approximate fatigue curves

Fatigue curve parameters	2024-T3				D16CzATW			
	σ_f', MPa	b	ε_f'	c	σ_f', MPa	b	ε_f'	c
FPCM	721,6	-0,0974	0,2153	-0,5141	717,1	-0,1018	0,2690	-0,5272
USM	928,6	-0,12	0,3416	-0,6	874,8	-0,12	0,3955	-0,6
MUSM	696,4	-0,09	0,2176	-0,56	664,0	-0,09	0,2348	-0,56
UMLM	815,5	-0,095	0,35	-0,69	768,2	-0,095	0,35	-0,69
MFPCM	617,6	-0,0817	0,2649	-0,5631	615,6	-0,0846	0,3383	-0,5832
MMM	823,3	-0,0963	0,2649	-0,664	795,0	-0,0980	0,3383	-0,664
MM	927,8	-0,11	0,28	-0,66	874,0	-0,11	0,28	-0,66

Whereas for the fatigue strength exponent b, calculated values are higher than the value achieved experimentally (fig. 2b). The value calculated using the universal slope method best corresponds to the experimental curve in that case.

There are significant differences between calculated values and values determined experimentally for both materials, either in case of fatigue ductility coefficient ε_f' (fig 2c), as well as fatigue ductility exponent c (fig 2d). The biggest differences occur for ε_f': i.e. values calculated for the alloy grade 2024-T3 are 6 times lower than the experimental ones (obtained using Bäumel and Seeger's Method) and 9 times lower (achieved using Four-Point-Correlation Method and Modified Universal Slopes Method), whereas for the alloy grade D16CzATW they are 4 times lower (for Universal Slopes Method) and 6 times lower (for Modified Universal Slopes Method). The differences between calculated and experimental values are much lower for *c*: i.e. values calculated for the alloy grade 2024-T3 are about 1,6 times lower than the experimental ones (obtained using Bäumel and Seeger's Method) and almost 2,2 times lower (achieved using Four-Point-Correlation Method), whereas for the alloy grade D16CzATW they are 1,6 times lower (for Bäumel and Seeger's method) and 2 times lower (for Four-Point-Correlation Method).

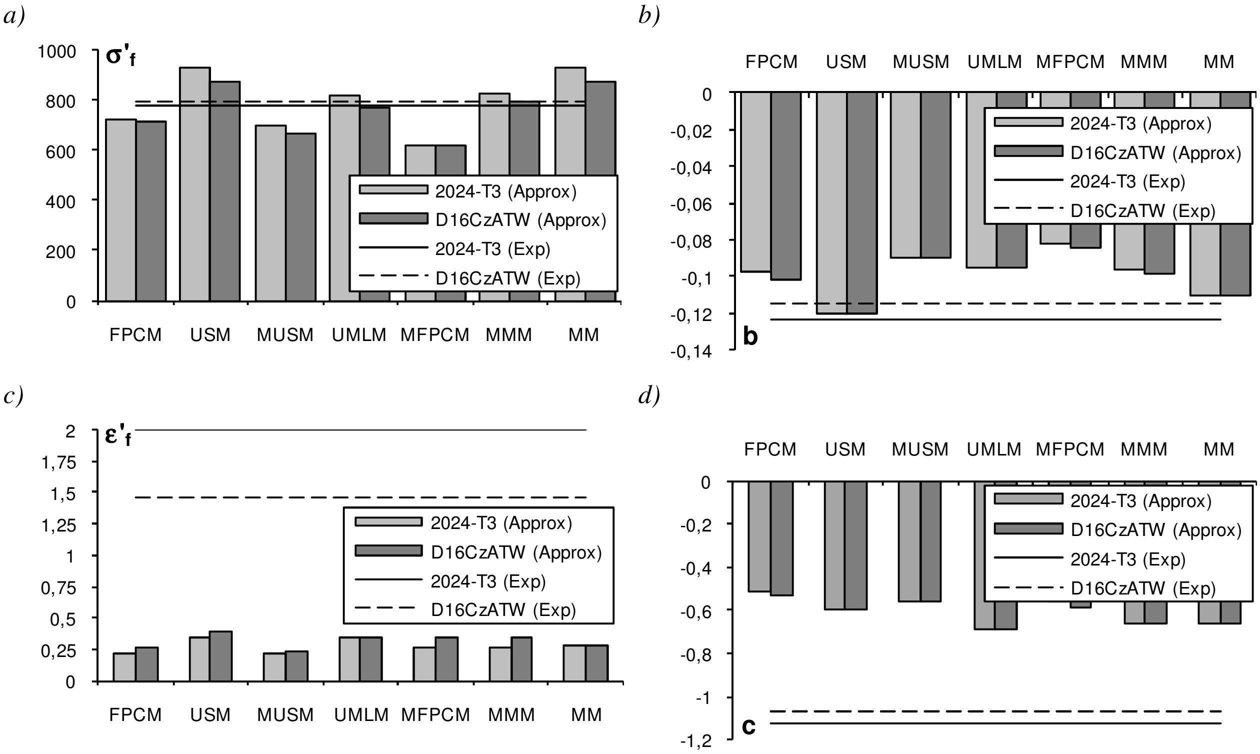


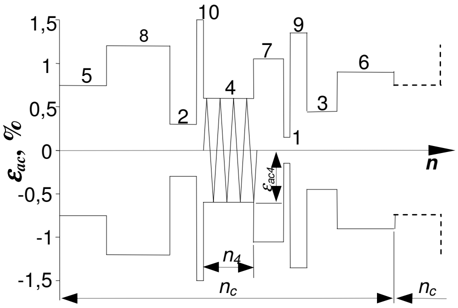
Fig. 2. The value of the fatigue strength coefficient σ'_f, the fatigue strength exponent b, the fatigue ductility coefficient ε'_f as well as the fatigue ductility exponent c depending on the method used to determine the fatigue curve

Relatively high conformity between the fatigue ductility coefficient and exponent determines similar shape of the curves in the high-cycle range. Whereas too small values of the fatigue ductility coefficient and exponent cause insufficient inclination of the curves in the low-cycle range.

It should be noted that in most methods the coefficient ε'_f and the exponent *c* are associated with the necking of *RA* sample and thus the measurement error for this quantity, which is particularly difficult to determine for sheet samples, significantly influences the form of approximate fatigue characteristics.

5. Application of approximate curves to fatigue life analysis

Taking into account the abovementioned differences between the curve determined based on experimental data and approximate curves, authors evaluated suitability of individual approximate curves for determination of fatigue life. This evaluation was performed on the basis of results of fatigue life calculations for programmed load, the example of which is provided in the figure 3.



ε_{acmax}, %	ζ	*Other*
1,5		
1,0	0,34	n_c=100
0,8	0,56	k=10
0,5	0,77	
0,35		

Fig. 3. Load program

This load is characterised by the value of maximum amplitude of the total strain ε_{acmax} as well as the spectrum modulation coefficient ζ referred to the amplitude of the total strain as follows:

$$\zeta = \sum_{j=1}^{k} \frac{\varepsilon_{acj}}{\varepsilon_{ac\max}} \cdot \frac{n_j}{n_c}, \tag{22}$$

where j is the number of the load level, k –number of levels in the load block, n_c – total number of cycles in the load block. All cycles of the programmed load block are alternating cycles (R=-1). The fatigue life determined by calculations based on the experimental curve was assumed as the reference level. The calculations were performed using the Palmgren-Miner linear cumulative damage rule.

Figure 4 presents the difference between the values of fatigue life N_{cal} calculated by means of approximate fatigue curves determined using different methods. The analytical fatigue life N_{exp} determined based on experimental curve was assumed as the reference level.

As it can be noted, the lower the ε_{acmax} value in the load block, the higher the errors of the fatigue life approximation. This results from the fact that approximate curves are shifted to the right in relation to the experimental curve in that range (fig. 1). In most cases, the lower the ζ value the higher the calculation error. This results from the fact that load block of lower ζ value is characterised by more cycles with low strain amplitude. The calculation error decreases as the ε_{acmax} value grows, whereas the fatigue life determined based on approximate fatigue curves N_{cal} assumes lower value than the fatigue life determined on the basis of the experimental curve N_{exp}. The experimental curve matches the curve determined using the Four-Point-Correlation Method worst of all. Approximate curves better match the experimental curve for the alloy grade D16CzATW than 2024-T3.

6. Summary

The fatigue strength coefficient σ'_f and the fatigue strength exponent b of either approximate and experimental curves sufficiently conform to each other for both analysed alloys. This conformity determines similar shape of curves in the range where elastic strain prevails. Significant difference between the value of fatigue ductility coefficient ε'_f and the fatigue ductility exponent c determines different shapes of curves in the range of high plastic strain. However, the elastic part of the equation (1), especially exponent b, shall be responsible for a very large error in the fatigue life approximation results in the range of low strain values because of the shift of approximate curves towards higher fatigue life values (fig. 5).

If there is no experimental data, approximate curves provide some information on fatigue properties of a given material. However, use of approximate curves can lead to very big errors, particularly in fatigue life calculations for variable amplitude load.

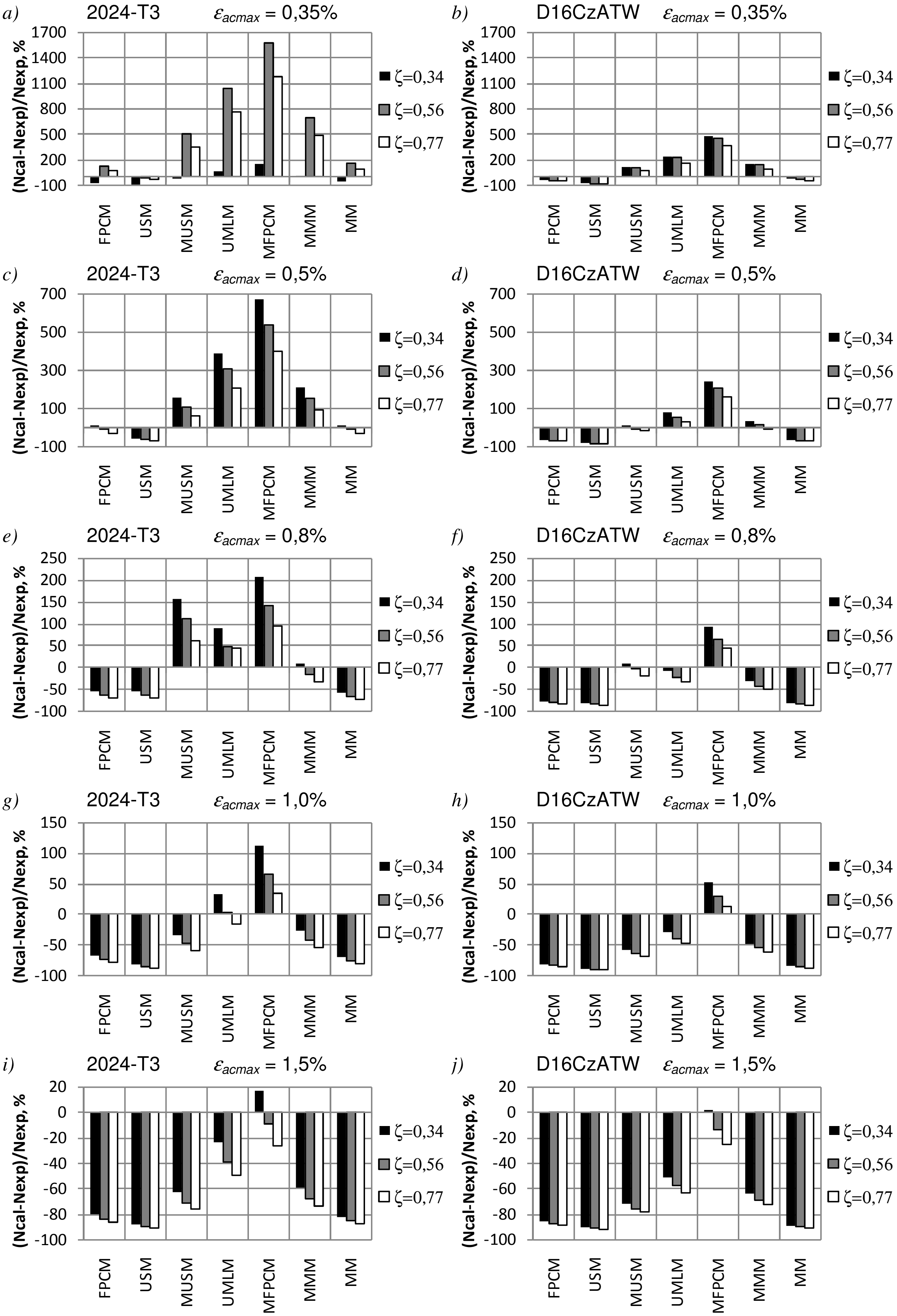


Fig. 4. Differences between values of fatigue life N_{cal} calculated using approximate fatigue curves determined by means of different methods as compared with the analytical fatigue life N_{exp} determined on the basis of the experimental curve

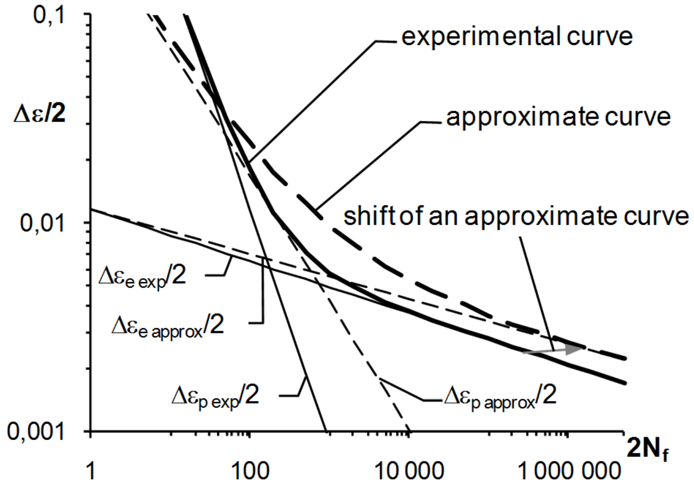


Fig. 5. Shift of approximate curve as a source of calculation error

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