



## STABILITY CONDITION OF THE TRACTOR TRAILER VIBRATIONS

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### Abstract

An important problem connected with the dynamics of vehicles are transverse vibrations of transport aggregates such as tractor, truck-trailer. This is of big importance as the discussed units are achieving higher and higher speeds, thereby posing hazard to traffic safety, besides, it is worth knowing the nature of these vibration, that is, whether they are forced or self-excited. The above mentioned issue was given much attention, in the former Soviet Union countries, and also later, [1], [2], [4]. In work [5], vibrations of a trailer whose connection to the truck or tractor by a rotary joint with one degree of freedom, was studied. However, it was a certain simplification of the real system, therefore, in this work a connection with two degrees of freedom has been introduced. The equation of motion have been derived and then the influence of the system parameters on the trailer vibration have been analyzed. The investigation showed that stability of the system is determined mostly by the damping.

**Keywords:** transfers vibration, tractor trailer, stability, damping

### 1. Motion equation model

The system presented in Fig.1 will be studied.

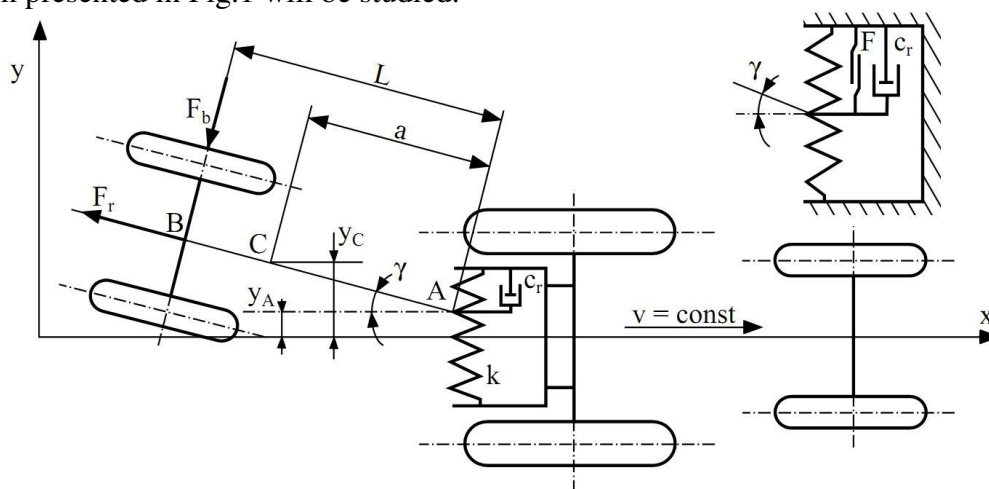


Fig. 1. Two degree of freedom model of a trailer

In order to examine all the effects, which occur during the motion, it was assumed that the unit is moving along a straight and horizontal road with the constant speed  $v$ .

Assuming  $\sin\gamma \approx \gamma$ ,  $\cos\gamma = 1$   $F = f_r \cos\gamma$ ,  $F_r = fG_k$ , the equation of motion has the form;

$$\begin{aligned} m \left( \ddot{y} + a \ddot{\gamma} \right) &= -Cr_o \dot{y}_A - ky_A - f_r F_f + F_f \gamma - F_b; \\ am \ddot{y} + J_A \ddot{\gamma} &= -LF_b, \end{aligned} \quad (1)$$

where:

- $m$  [kg] - body mass,
- $J$  [kgm<sup>2</sup>] - moment of inertia,
- $k$  [Nm<sup>-1</sup>] - spring stiffness,
- $c_r$  [Nsm<sup>-1</sup>] - damper rate,
- $F$  [N] - force of dry friction,
- $F_r$  - coefficient of dry friction,
- $F_b$  [N] - side force,
- $F_f$  [N] - force of rolling friction,
- $f$  - coefficient of friction,
- $G_k$  [N] - gravity force of wheel section.

The unknown side force can be obtained from the form [1],

$$F_b = k_y \Psi, \quad \text{and } \Psi = \gamma + \frac{y_A + L \dot{\gamma}}{v} \quad (2)$$

where:  $k_y$  N deg<sup>-1</sup> - resisting coefficient.

Now the equation (1) can be written as:

$$\begin{aligned} m \ddot{y}_A + \left( c_r + \frac{k_y}{v} \right) \dot{y}_A + ky_A + am \ddot{\gamma} + \frac{k_y L}{v} \dot{\gamma} + (k_y - F_f) \gamma &= -f_r F_f, \\ am \ddot{y}_A + \frac{k_y L}{v} \dot{y}_A + J_A \ddot{\gamma} + \frac{k_y L}{v} \dot{\gamma} + k_y L \gamma &= 0. \end{aligned} \quad (3)$$

A stability condition can be investigated from the characteristic equation

$$\begin{vmatrix} ms^2 + \left( c_r + \frac{k_y}{v} \right) \lambda + k & am s^2 + \frac{k_y L}{v} \lambda + (k_y - F_f) \\ am s^2 + \frac{k_y L}{v} \lambda & J_A s^2 + \frac{k_y L^2}{v} \lambda + k_y L \end{vmatrix} = 0 \quad (4)$$

and from above, equation (4) can be written as: a

$$a_o \lambda^4 + a_1 \lambda^3 + a_2 \lambda^2 + a_3 \lambda + a_4 = 0 \quad (5)$$

where :

$$a_o = mJ_A;$$

$$a_1 = \left( k_r + \frac{k_y}{v} \right) J_A + (L - 2a) \frac{mk_y L}{v},$$

$$a_2 = (L - a)mk_y + amF_f + cJ_A + k_r \frac{k_y L^2}{v},$$

$$a_3 = \left( k_r v + cL + F_f \right) \frac{k_y L}{v},$$

$$a_4 = ck_y L.$$

The equation (4) has a stable solution if:

$$a_i > 0 \text{ and} \quad (6a)$$

$$a_3 (a_1 a_2 - a_3 a_0) - a_1^2 a_4 > 0. \quad (6b)$$

Since the coefficient of dry friction is a not component of coefficient  $a_1$  and characteristic equation, it can be concluded that it does not affect on the stability system. Let us analyze the influence of the system parameters on the trailer vibration stability.

Coefficient  $a_0$  always positive;

Coefficient  $a_1 > 0$  if  $v > 0$  hence we receive:

$$a < \left( k_r + \frac{k_y}{v} \right) \frac{J_{Av}}{2mk_y L} + 0,5L. \quad (7)$$

For the trailer speed  $v > 0$ , the distance of mass center from the center of connection should be :

$$a < 0,5L + \frac{J_A}{2mL}. \quad (8)$$

If this condition is not fulfilled there occur conditions for transverse vibrations.

If  $a_1 > 0$ , then the remaining coefficients ( $a_2, a_3, a_4$ ) with  $v > 0$  have also positive values which can not be said about inequality (6b). With positive values of all coefficients, the above mentioned inequality can be negative.

## 2. Example

For determination of the negative influence of vibrations on the trailer stability, an example for the following data:  $m=3522$  kg,  $J_c=3453$  kg m<sup>2</sup>,  $L=2,5$ m,  $a=2,15$ m,  $k_y=80000$  N deg<sup>-1</sup>,  $k=5886$  N/m,  $c_t=2452$  N sm<sup>-1</sup>,  $v=8,6$  m/s will be analyzed.

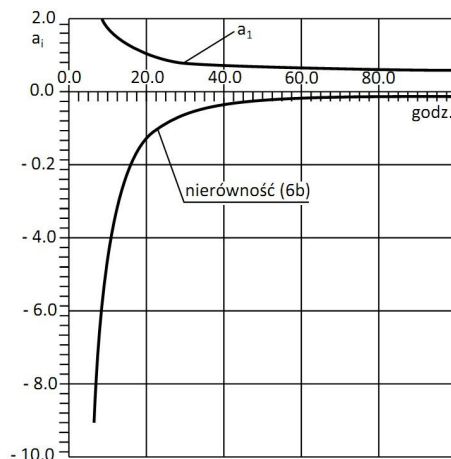


Fig. 2. Dependence of coefficient  $a_1$  and inequality (6b) on the speed

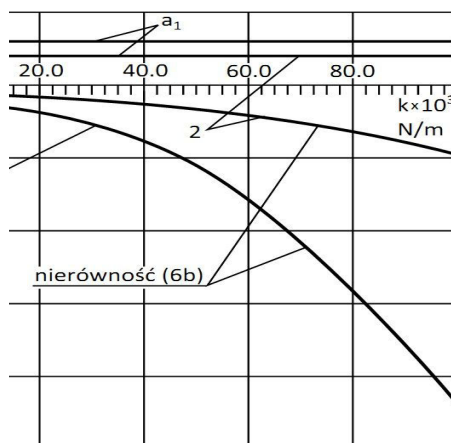


Fig. 3. Dependence of coefficient  $a_1$  and inequality (6b) on stiffness  $k$ : 1 - asphalt,  $v=30\text{km/h}$ , 2 - unpaved road,  $v=14\text{km/h}$

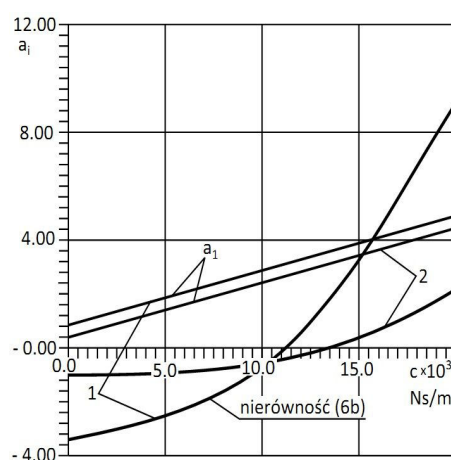


Fig. 4. Dependence of coefficient  $a_1$  and inequality (6b) on damping  $c$ , 1- $v=14\text{km/h}$ , 2- $v=30\text{km/h}$

In figure 2, there has been presented a dependence of  $a_1$  coefficient and inequality (6b) on the trailer speed. The figure shows that for all speeds  $a_1 > 0$ , whereas, inequality (6b) is negative. However, for speeds higher than  $40\text{km/h}$ , inequality (6b) and vibration amplitudes are insignificant.

Fig.3 shows a dependence of  $a_1$  coefficient and (6b) inequality on the connection stiffness for two speeds and different road surfaces. For all the values of coefficient  $a_1 > 0$ , inequality (6b) is negative. However, it can be noted that the absolute value of inequality (6b) increases along with  $k$  stiffness value rise.

Fig.4 demonstrates the dependence of coefficient  $a_1$  and inequality (6b) on the connection damping. From the chart it can be seen that with  $c > 10 \cdot 10^3$ , both curves are positive and the vibrations decrease rapidly.

### 3. Conclusions

1. Increasing the trailer connection makes the vibrations become unstable.
2. Damping in the connection has the largest influence on the trailer vibrations.
3. Being familiar with the impact of stiffness and damping is of great importance for constructors.

## References

1. Guskov V. V.: Tractor trains. Mashinostrojenie, 1982.
2. Guskov V. V.: Tractors. Theory. Mashinostrojenie, 1988.
3. Borkowski W.: Dynamika maszyn roboczych. Warszawa, WNT, 1996.
4. Holka H.: Obciążenie i drgania agregatów traktorowych. Praca habilitacyjna, Mińsk, Białoruś, 1998.
5. Holka H., Peszyński K.: Modelling and analysing of transfers vibration of the tractor trailer. Engineering Mechanics 2001, Svratka, Czech Republic, 2001.

