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CONSIDERATION IN DIAGNOSTICS OF THE GREY SYSTEM THEORY

Bogdan Żółtowski

University of Technology and Life Sciences in Bydgoszcz
Faculty of Mechanical Engineering
ul. Prof. Kaliskiego 7 85-789 Bydgoszcz, Poland
tel.: +48 52 340 82 40 fax: +48 52 340 82 45
e mail: bogzol@utp.edu.pl

Abstract

The publication describes Grey System Theory (GST) and takes into account the Grey differential model (GM) and Grey Generating Space (GS). Grey System Theory shows to which extent the vibration signals (deriving from tested objects) influence the evaluation and analysis of condition of the machine. The above-mentioned theory applies under the existence of fix, non-negative and monotonic data correlated with insufficient and uncertain data' sources. In relation to these circumstances, the Forecasting (rolling window) method seems to be appropriate solution, which remains the main subject of this paper. The present research use vibration methods to recognize the technical state of the machines and SVD method (Singular Value Decomposition) as well as GST (Grey System Theory) was used for results validation.

Keywords: *grey systems, vibration diagnostics, forecasting, modeling of condition*

1. INTRODUCTION

The Grey System Theory (GST) was established in 1982 by Chinese scientist known as J.- L. Deng. At the beginning, despite the opportunity for widespread application of GST, the theory did not attract the attention of scientists and researchers base in western countries. In the early nineties of twenty century the circumstances has changed that is why we can observe the wide popularity of the GST which allows easy forecasting the state of the entities, machines respectively. The theory is commonly used across other related areas such as social and natural sciences, demography, hydrology and economy (such as anticipation of the market condition bases on particular data sources). Furthermore the idea of GST offers practical solutions available not only for scientists but also for engineers and entrepreneurs in the way of appropriate decision-making process [1,8,9,10,11].

One of the most adequate tool, which guide the management of algorithm sequence is MATLAB program with an important Toolbox (Statistic) function. The knowledge and the ability of practical usage of MATLAB program is required to understand the algorithm sequence described in following section [1,2,3,4,5].

The algorithm described below is not so complex, that is why the combination of its deep revision and appropriate MATLAB' skills force the correct implementation of the algorithm [6,7,12].

2. THE GREY SYSTEM ALGORITHM TOWARDS CONDITION FORECASTING

Most of the technical algorithms, including Grey System algorithm, can be expressed through the mathematical shape. First of all, it seems necessary to admit that Grey Model (GM) describes the system' behaviors in relation to particular symptom defined as ($x^{(0)}(t)$) in which t means the following obtained symptom, for instance ($t \in <1,2,3,\ldots,\infty$), maybe presented under differential quotation at the bases on k simultaneously forcing e (the same foundation) which can also be shown as GM (k, e). The principle states that k concern the factor which force the changes under differential quotation and e takes the below-presented mathematical forms [2,4,5]:

$$\sum_{i=0}^{e} a_i \frac{d^{n-i}x^{(1)}}{dt^{n-1}} = \sum_{j=1}^{k-1} b_j y_j^{(1)} \qquad (1)$$

in which:

$$x^{(1)}(k) = \sum_{i=1}^{t} x^{(0)}(i) \text{ is variable factor of base object} \qquad (2)$$

y_j – independent behavior allows correct interpretation of reviewing object
a_i, b_i – polynomials rate estimating form time line $x^{(0)}(t)$, $t = 1,2,3\ldots,\infty$

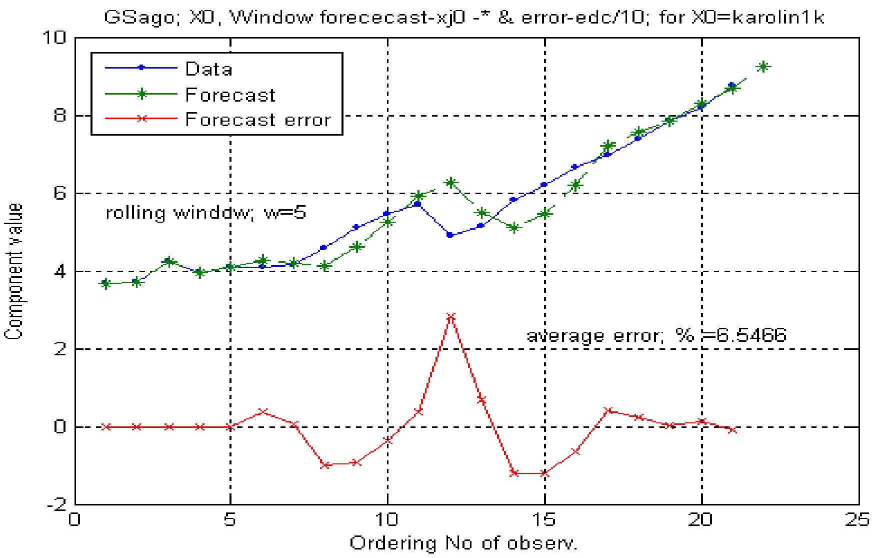

Fig.1. Model GM of monotonic data vibration signal

In the most of circumstances the starting point is obtained as GM (1,1) equation which means the differential quotation with only one forcing factor. The solution of this problem can be guided by following mathematical process:

Stage 1

Determination of observation' vector:

$$x^{(0)} = [x^{(0)}(1), x^{(0)}(2), x^{(0)}(3), \ldots, x^{(0)}(n)], \qquad (3)$$

requirements: $n \geq 4$,

in which n means number of observations.

Stage 2
Formation of Accumulating Generating Operation (AGO):

$$x^{(1)}(t) = \sum_{i=1}^{t} x^{(0)}(i),\ t = 1,2,3,\ldots n \qquad (4)$$

which clearly defines the vector' monotonicity and growth:

$$x^{(1)} = [x^{(1)}(1), x^{(1)}(2), x^{(1)}(3), \ldots, x^{(1)}(n)] \qquad (5)$$

requirements: $x^{(1)}(1) = x^{(0)}(1)$.

Stage 3
Basing on above-defined AGO vector, it is appropriate to describe the Grey differential model (GM) in relation to starting position GM (1,1):

$$\frac{dx^{(1)}(t)}{dt} + ax^{(1)}(t) = u \qquad (6)$$

in which:
a – growth index,
u – controllable variable factor,
t – uncontrollable factor (e.g. time, asset depreciation).

Stage 4
The solution of the above-presented differential quotation with constant growing variable *t*, as following:

$$\dot{x}^{(1)}(k+1) = [x^{(0)}(1) - u/a] \exp(-ak) + u/a, \qquad (7)$$

in which $\dot{x}^{(1)}$ describes the potential forecast of Accumulating Generating Operation

Stage 5
The replacement of the differential complete accretion (Stage 3) relates to *t=1* and composition of the precedent and progressive equations.

- precedent equation: $x^{(1)}(k+1) - x^{(1)}(k) + ax^{(1)}(k) = u$
- progressive equation: $x^{(1)}(k+1) - x^{(1)}(k) + ax^{(1)}(k+1) = u$

The combination of the equations gives us the model of:

$$x^{(1)}(k+1) - x^{(1)}(k) = -a/2[x^{(1)}(k) + x^{(1)}(k+1)] + u \qquad (8)$$

k = 1,2,3,…,n.

Stage 6
The conversion of above-mentioned model relates to the following *k* values (basing on previous obtained observation vector - $x^{(0)}$) to estimate the unknown extra differential quotation ratios [*a*, *u*]. These estimation is indicated via selection of 'smaller squares' to final attainment of matrix solution:

$$[a,u]^T = (B^TB)^{-1} B^TY$$

in which: $Y = [x^{(0)}(2), x^{(0)}(3), x^{(0)}(4), \ldots, x^{(0)}(n)]^T \qquad (9)$

$$B = \begin{bmatrix} -[x^{(1)}(1)+x^{(1)}(2)] & 1 \\ -[x^{(1)}(2)+x^{(1)}(3)] & 1 \\ \dots\dots\dots\dots\dots & \dots \\ -[x^{(1)}(n-1)+x^{(1)}(n)] & 1 \end{bmatrix} \quad (10)$$

Stage 7

The inverse transformation of AGO presents the potential forecast (basing on AGO' vector) obtaining:

$$\dot{x}^{(0)}(k+1) = \dot{x}^{(1)}(k+1) - \dot{x}^{(1)}(k), \quad (11)$$

furthermore the conversation with previous-presented progressive and precedent equation (Stage 5) allows to determine the final forecast in relation to starting GM (1,1) model:

$$\dot{x}^{(0)}(k+1) = [x^{(0)}(1) - u/a](e^{-ak} - e^{-a(k-1)}) \quad (12)$$

$$k = 2,3,4,\dots,n$$

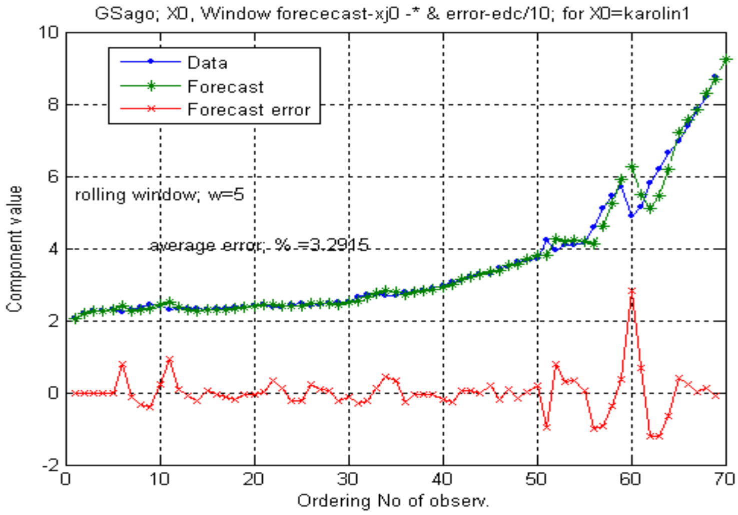

Fig.2. Forecast error in prognosis signal GST

The final stage of these algorithm shows the general principle of the Grey System Theory (GST). With accordance to *Stage 2* it is easily to suggest that the method can fully describe the monotonic and expanding process relates to depreciation of particular asset' or machine' elements. That is why the above-describes method can be used to properly describe and forecast the condition of the machine basing mainly on vibration' symptoms.

What is more important the sufficient forecast can be obtained after few observations, at the same time, the increasing number of observations is positively correlated with growth the possibility of gathering wrong research data – decrease the efficiency of the observation.

The Forecasting (small-size rolling window) method seems to be appropriate solution for short term prediction which simultaneously covers and analyse huge number of data. The determination of appropriate and final forecast (basing on previous-described numerical method) can be possible by cooperation with MATLAB program that is why the adequate researcher skills are highly important factor which definitely determines the forecast' success.

3. ONE – DIMENSIONAL FORECASTING ACCORDING TO GREY SYSTEM METHOD

As it was mentioned before, the already reviewed algorithm can be easily implement across practical basis. This stage of the report describes the influence of the Grey System method on the process of diagnosis and analysis of machines. The excellent example of this phenomenon contains the review of airplane' turbine bearing.

As a results of assets depreciation we can observe the rise of oscillations. The number of observations regarding oscillations' features – velocity of vibration (mm/s) across equal time interval (each 20 hours) – are presented below:

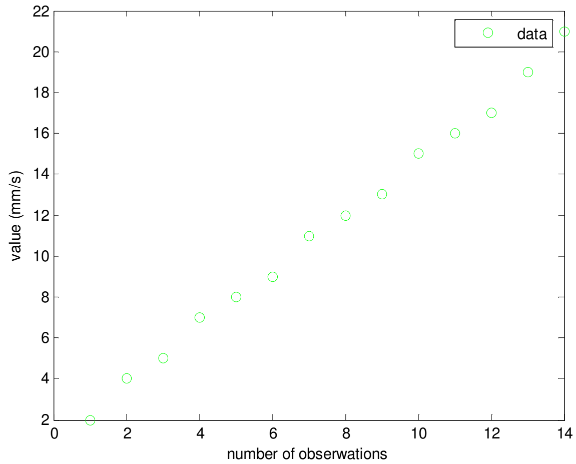

Fig.3. Value of observation signal

The observation' results shows the monotonic and expanding line that is why it seems appropriate to adopts the Grey System Theory (GST). Furthermore, we implement these algorithm at the MATLAB program and upload already gathered data which helps us to obtain the following forecast:

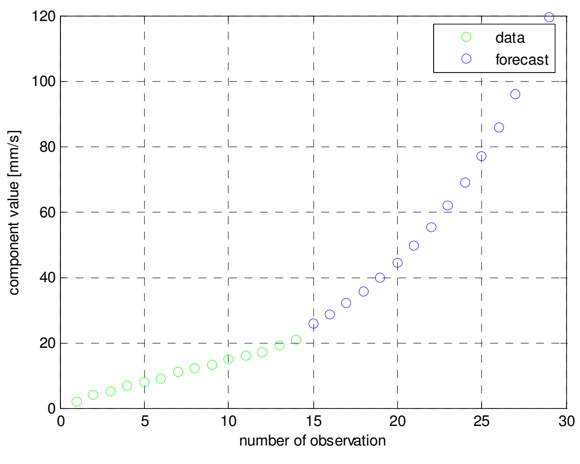

Fig.4. Monotonic line forecasting

Part of the evolution of the airplane' turbine bearing basing on 14 observations and the Forecasting (rolling window) method (15 continual inspections – 300 working hours)

According to above-mentioned graph, the obtained forecast contains several similarities towards quadratic function graph. This phenomenon is fully adequate to practical reality which means that particular unit depreciation is irregular – the higher depreciation ratio drives the velocity of depreciation process. The above-mentioned forecast has visional similarities that allows me to state that the forecast is fully suitable. After measurement of forecast failure' ratio presents the value slightly more than 5%. Basing on this result we can state that forecast fulfill the credibility requirement (the forecast can be accepted with around 5% of failure).

Moreover, it seems also necessary to admit that this method is mainly dedicated for short-term forecast. What is more important, the implementation of measurement should base on continual observations across equal periods of time.

4. MULTIDIMENSIONAL CONDITION' FORECAST

Currently the multidimensional condition forecast is at the stage of early development so that the implementation of related analysis methods is very difficult. Generally, the multidimensional condition forecast bases on the other necessary tools, connected with the artificial intelligence called – neural network, data fusion respectively. Moreover, the Singular Value Decomposition (SVD) is also closely related with the above-mentioned forecast. However, the deep description of that tools and methods are specified in the other reports because of theirs discrepancies with the Grey System algorithm basis.

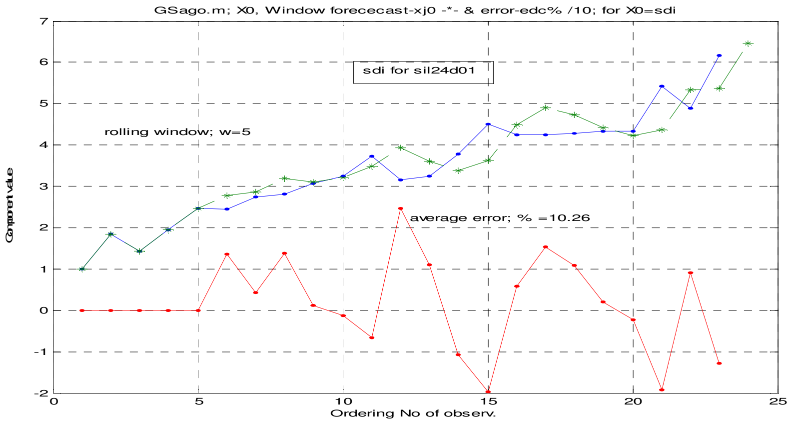

Fig.5. Forecast state of SVD method

The fundamental knowledge of the multidimensional condition forecast is required to become fully capable to interpret processes related with vibration diagnostics' studies.

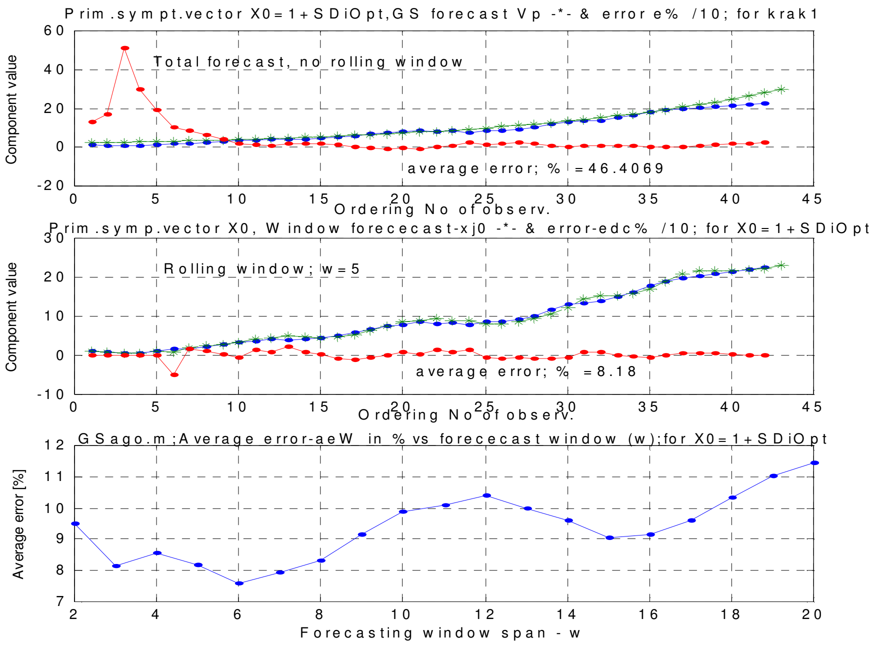

Fig.6. Multidimensional condition forecast GST

5. SUMMARY

This report clearly states the Grey System Theory is especially adequate to forecast one-dimensional unit' condition. The wide opportunity to application of the theory can be utilized across statistical purposes (estimation of revenue or trends) as well as in assets' vibration diagnostics.

The well and suitable performance of Forecasting (rolling window) method allows to decrease the possibility of forecasting failure that is why the appropriate interpretation of Grey System algorithm and increase the knowledge drives the regularity and conformity of the research.

Finally, the report reminds that the proper utilization of above method relates to depreciation of machines' units, with the monotonic and expanding continual values.

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