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CHANGING THE SHIP PROPULSION SYSTEM PERFORMANCES INDUCED BY VARIATION IN REACTION DEGREE OF TURBOCHARGER TURBINE

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Abstract

This paper presents a detailed mathematical model of ship propulsion system including slow speed diesel engine, screw propeller and governor. This mathematical model is then employed to investigate the system performances both in steady state and unsteady states. The simulation results are checked and the model is verified using experimental data. Next an original method for the system sensitivity analysis against changing of parameters is given. This method is finally applied for investigating the influence of degree of reaction of turbine of turbocharger on the system behaviour.

Keywords: *diesel engine, ship propulsion, sensitivity, turbocharger, degree of reaction*

Symbols

a	constant, parameter
A	area, auxiliary function
b	constant, parameter
c	specific heat, constant, parameter
C	coefficient, constant
d	constant, parameter
D	diameter, draught
e	constant, parameter
f	fuel-to-air ratio, frequency
F	equivalence ratio
h	specific enthalpy, calorific value, fuel rack
J	mass moment of inertia, performance index, advance number
K	parameter, coefficient
l	length
L	auxiliary function
m	mass, number
M	torque, moment
n	number, polytropic exponent
p	parameter
P	pressure, set of parameters
Q	heat, heat transfer, propeller torque
r	number
R	gas constant, resistance, weighting matrix
Re	Reynolds number
S	sensitivity
t	time
T	temperature, time constant, thrust
u	specific internal energy, speed, input, control
V	volume, speed, function
x	direction coordinator
y	direction coordinator
z	number, direction coordinator, input, disturbance

Greek letters

α	coefficient, heat transfer coefficient
Δ	generalized linear matrix, displacement, average
ε	cooler effectiveness, insensitivity
η	efficiency, acceleration
κ	adiabatic exponent, thermal conductivity
λ	excess air coefficient (factor), canonical variable, Lagrange multiplier
ν	blade speed ratio, dynamic viscosity, power angle, sensitivity coefficient

π	pressure ratio
ρ	reaction degree of turbine, density
φ	crank angle, function
ψ	flow function
ω	angular velocity

Subscripts

0	initial value/condition
amb	ambient
am	air inlet manifold (receiver)
av	air inlet valve (or port)
blr	auxiliary blower
B	brake power
c	compressor
clr	cooler
com	combustion
cyl	cylinder
cr.	critical
d	discharge
dyn	dynamic
e	engine
eq	equivalent
f	fuel, final value/condition
g	exhaust gas
gm	exhaust gas manifold (receiver)
gv	exhaust gas valve (or port)
G	governor
hr	heat release
hr0	start of heat release
H	pitch
i	index of cylinder number, integration
j	index of cylinder number, integration
l	mechanical losses
max.	maximum
min.	minimum
mix	mixing
mot	motored
nom.	nominal
opt	optimum
P	propeller
pis	piston
red	reduced, corrected
ref	reference
res	residual gas
s	index of cylinder number, integration
sto	stoichiometric
sur	surrounding, surface
sw	swept
t	time, turbine, theoretical
tc	turbocharger
W	water

Abbreviations

CPP	Controllable Pitch Propeller
FB	Fuel Burnt
FBR	Fuel Burnt Rate
FPP	Fixed Pitch Propeller
MCR	Maximum Continuous Rating
MEP	Mean Effective Pressure
MMEP	Mechanical losses in the form of Mean Effective Pressure
TDF	Thrust Deduction Factor

1. Introduction

To design and analyze a marine slow speed diesel engine and its control system it is necessary to know which parameters have considerable influence the system performance. Additionally, it should be cleared how these performances are affected by the system parameters. In other words *sensitivity* of the system should be analyzed. To interpret sensitivity of the system, one needs a clear criterion or set of criteria. Let's consider only the related control system. In practice there are some defined limitations in Classification Society Rules. However, most of them take into consideration only the response of the system, which in the case of marine diesel engines control system is angular velocity of propeller or engine shaft, with no particular attention on other state variables. Perhaps the main reason is that the most known models for control system of ship propulsion system could not deliver all dynamic events of the system. Here an *Instantaneous Value Model of Ship Propulsion System*, which gives the main dynamic events of the system has been presented. By simulating the system, accuracy and satisfactory of the model was confirmed, when the results are compared with experimental data. Based on these results, author has found that the level of dependency of engine performances to reaction degree of turbine of turbocharger is very high. In this paper it is tried to answer how this parameter influence the system characteristic. Different criteria for parameter sensitivity are discussed and a new one is delivered.

Sensitivity question arises whenever we attempt to construct a physical system from a set of mathematical specifications. One of the goals of sensitivity analysis is to assign accuracy requirements for system parameters consistent with sensitivity significance in the system model. The utilization of sensitivity in control system design was extended in the late 50's and early 60's by many authors who contributed during this period by relating stability and other system characteristics to system sensitivity functions. Thus, sensitivity became an important part of control system design and synthesis. It is also shown that sensitivity analysis could be a useful tool for optimum control system design.

2. Diesel engine model

The model is a zero-dimensional instantaneous quasi-steady one for a turbocharged two-stroke diesel engine. It comprises the compressor, the charge air cooler, auxiliary blowers, the scavenging air receiver, the inlet air port or valve, the cylinders, the exhaust port or valve, the exhaust receiver and the turbine.

All gases follow the law of the ideal gas. The basis of the model is to consider the multi-cylinder engine as a series of thermodynamic control volumes, linked by mass or work transfer. These control volumes are compressor, scavenging air receiver, cylinders, exhaust receiver and turbine. Heat, work and mass transfer across the boundaries of these control volumes are then calculated. The subsystems are treated as quasi-steady open thermodynamic systems. The model is that due to [1, 3, 5, 8] which themselves are developments of other studies like [7].

2.1. Compressor and inlet air system

The compressor ratio is given as the ratio of the total pressures to ambient pressure:

$$\pi_c = \left(P_c / P_{amb}\right), \tag{1}$$

in which

$$P_c = P_{am} - \Delta P_{blr} + \Delta P_{clr} + P_{dyn}, \tag{2}$$

$$\Delta P_{blr} = P_{blr0} - a_{blr}(P_{am} - P_{amb})^2, \tag{3}$$

$$P_{dyn} = a_{dyn}(P_{am} - P_{amb}), \tag{4}$$

where P indicates pressure, superscripts *am*, *blr*, *clr*, *dyn* and *amb* express air inlet manifold (receiver), auxiliary blowers, charge air cooler, dynamic and ambient, respectively; and a_{blr}, a_{dyn} and P_{blr0} are constants that have to be expressed based on experimental tests or characteristic of the auxiliary blowers and the compressor. Constant a_{blr} is taken to give zero pressure rise at approximately between 40% and 50% power. At higher power levels the auxiliary blowers do not operate. The constant a_{dyn} is taken to give a typical dynamic pressure at the compressor outlet.

Charge Air Cooler: A linear relationship between pressure drop in the cooler and pressure of the scavenging air receiver has been considered:

$$\Delta P_{clr} = a_{clr}(P_{am} - P_{amb}), \tag{5}$$

where a_{clr} is a constant. Mass accumulation in the charge air cooler is modelled through the scavenging air receiver.

Compressor: The power of compressor, N_c, is

$$N_c = \dot{m}_c h_c - \dot{m}_c h_{amb}, \tag{6}$$

where $\dot{m}_c$ is the air flow rate after compressor and ambient air enthalpy, h_{amb}, can be calculated for fresh air at ambient temperature and enthalpy of air passing the compressor, h_c, should be calculated for fresh air at temperature of outlet air from the compressor:

$$T_c = T_{amb} + \frac{T_{amb}}{\eta_{ca}} \left(\pi_c^{\left(\frac{k_{amb}-1}{k_{amb}}\right)} - 1 \right). \tag{7}$$

The air mass flow after compressor and compressor efficiency are functions of angular velocity of turbocharger shaft, ω_{tc}, pressure ratio, π_c, ambient air temperature, T_{amb} and ambient air pressure, P_{amb}, can be illustrated using the static characteristic of the compressor.

Finally, total efficiency of the compressor η_c, when the mechanical efficiency, η_{cm}, is assumed independent from ω_{tc} and π_c (and therefore constant), can be obtained:

$$\eta_c = \eta_{ca} \cdot \eta_{cm} . \tag{8}$$

2.2. Flow through valves and ports

A simple one-dimensional model for flow through a valve (or port) using the analogy of an orifice and having an equivalent flow area or scaled real flow area is used:

$$\dot{m}_v = C_d A_v \psi_v \frac{P_1}{\sqrt{R_1 T_1}} , \tag{9}$$

where $\dot{m}_v$ is mass flow rate through the valve (or port), A_v is flow area, C_d is discharge constant coefficient, ψ_v is flow function, T is temperature, R is gas constant and subscript 1 denotes the condition of the flow at upper point.

$$\psi_v = \begin{cases} \left[\frac{2k}{k-1} \left(\pi^{\frac{2}{k}} - \pi^{\frac{k+1}{k}} \right) \right]^{1/2} & ; \quad \pi < \pi_{cr.} \\ \left[k \left(\frac{2}{k+1} \right)^{\frac{k+1}{k-1}} \right]^{1/2} & ; \quad \pi \geq \pi_{cr.} \end{cases} , \tag{10}$$

where

$$\pi = \left(P_2 / P_1 \right) \ , \ \pi_{cr.} = \left(\frac{2}{k+1} \right)^{\frac{k}{k+1}} . \tag{11}$$

k is adiabatic exponent and subscript 2 denotes the condition of the flow at lower point.

The appropriate forms of equation (10) for inlet air and exhaust gas valves (or ports) are considered. Both normal and reverse flow, i.e. from scavenging air receiver to the cylinder or reverse flow, and from the cylinder to the exhaust receiver or reverse flow have been considered in the model.

2.3. Control volumes: receivers and cylinders

The scavenging air receiver, exhaust receiver and cylinders are modelled through the mass balance and the energy balance equations. Both normal flow and reverse flow are taken into account, but reverse flow through the compressor is unlikely to occur and auxiliary blowers prevent such a regime in low speeds. The possibility of reverse flow at the turbine is ignored, too.

The total volume of scavenging air receiver is modelled as the total of the volumes of charge air cooler and inlet air boxes. Exhaust pulses from the cylinders are conducted to the exhaust receiver through exhaust diffusers. Two flow regimes can be considered: normal flow from cylinders to the receiver and from the receiver to turbine, and reverse flow from any cylinder. Heat transfer from exhaust gas to the receiver surface area must be accounted, unless the receiver is very well lagged. It is assumed that the exhaust gas from cylinders is to be perfectly mixed with the contents of the exhaust receiver. The equivalence ratio in the receiver will not be constant if gas is entering from a cylinder during its scavenging or overlap period.

The basic periods occurring in the cylinders can be divided as: 1) closed cycle period, 2) blow-down period and 3) scavenging period. These processes are considered separately, and subdivided according to flow direction as normal and reverse.

Closed cycle period includes combustion process. The one-zone model for combustion is applied here. For scavenging period a two-zone model is taken into account so that the cylinder charge comprises a fresh air zone and a residual gas zone. It is considered that a part of the incoming fresh inlet air is mixed with the residual gas. The model is a compromise between pure displacement and perfect mixing. For other periods a one-zone model is applied which is closed to reality.

General equations of mass, m, and energy balance are as follows, respectively:

$$\frac{dm}{dt} = \sum_{i=1}^{n} m_i \,, \tag{12}$$

$$\frac{dT}{dt} = \left[\frac{1}{m} \left(\sum \frac{dQ}{dt} + \sum h\, \dot{m} \; - P \frac{dV}{dt} - u \frac{dm}{dt} \right) - \frac{\partial u}{\partial F} \frac{dF}{dt} \right] \cdot \frac{1}{\partial u / \partial T} \,, \tag{13}$$

where u is internal energy and V is volume. Moreover, the fuel-to-air ratio f, is related to equivalence ratio of exhaust gas F, as $F=f/f_{sto}$, and f_{sto} is the stoichiometric fuel-to-air ratio.

The derivative of the equivalence ratio, F, is:

$$\begin{cases} \dfrac{dF}{dt} = \dfrac{F1}{m} \left(\dfrac{F1}{f_{sto}} \dfrac{dm_{fb}}{dt} - F \dfrac{dm}{dt} \right), \\ F1 = 1 + F\, f_{sto} \end{cases} \tag{14}$$

where

$$\frac{\partial u}{\partial T} = c_v = c_p - R \,, \tag{15}$$

$$\frac{\partial u}{\partial F} = u - u_{air} \,, \tag{16}$$

hence c_v and c_p are specific heat in constant volume and constant pressure, respectively.

Therefore, the general equation of energy balance of two-zone control volume can be given as in (17) in which the subscript 1 and 2 are used for the two distinct zones. The mass of zones are none-zero.

$$\frac{dT_1}{dt} = \frac{\dfrac{\left(L_1 + L_2 - P\dfrac{dV}{dt}\right)}{m_2 T_2 \dfrac{\partial u_2}{\partial T_2}} + \dfrac{L_1}{P}\left(\dfrac{1}{V_1} + \dfrac{1}{V_2}\right) - \dfrac{1}{V_2}\dfrac{dV}{dt} - A_1}{\dfrac{1}{T_1} + m_1 \dfrac{\partial u_1}{\partial T_1}\left(\dfrac{1}{m_2 T_2 \dfrac{\partial u_2}{\partial T_2}} + \dfrac{1}{PV_1} + \dfrac{1}{PV_2}\right)}, \quad (17)$$

where

$$A_1 = \frac{1}{m_1}\frac{dm_1}{dt} + \frac{1}{R_1}\frac{\partial R_1}{\partial F_1}\frac{dF_1}{dt} - \frac{1}{m_2}\frac{dm_2}{dt} - \frac{1}{R_2}\frac{\partial R_2}{\partial F_2}\frac{dF_2}{dt}, \quad (18)$$

$$L_i = -u_i \frac{dm_i}{dt} - m_i \frac{\partial u_i}{\partial F_i}\frac{dF_i}{dt} + \sum \frac{dQ_i}{dt} + \sum h_i \dot{m}_i. \quad (19)$$

A corresponding equation is derived for zone 2.

2.4. Combustion and heat release

Heat release rate can be calculated based on Wiebe's function:

$$FBR = K_2(1+K_1)\left(\frac{t - t_{hr0}}{\Delta t_{com}}\right)^{K1} \cdot \exp\left[-K_2\left(\frac{t - t_{hr0}}{\Delta t_{com}}\right)^{(1+K_1)}\right], \quad (20)$$

where *FBR* indicates non-dimensional fuel burning, K_1 is the shape factor, K_2 is the combustion efficiency coefficient, t_{hr0} illustrates time in which combustion starts, and Δt_{com} refers to the duration of combustion. According to [5]:

$$K_1 = K_{1ref}\left(\frac{ID_{ref}}{ID}\right)^{0.5}\left(\frac{m_{z,cyl}}{m_{z,cyl\,ref}}\right)\left(\frac{\omega_{ref}}{\omega}\right)^{0.3}, \quad (21)$$

$$\Delta t_{com} = \Delta t_{com\,ref}\left(\frac{F_{cyl}}{F_{cyl\,ref}}\right)^{0.6}\left(\frac{\omega}{\omega_{ref}}\right)^{0.5}, \quad (22)$$

where *ID* is ignition delay and the subscript *ref*, *z* and *cyl* indicate reference point, beginning of compression and cylinder in question, respectively.

Now, the rate of heat release can be easily found:

$$\frac{dQ_f}{dt} = FBR \cdot \frac{Q_f}{\Delta t_{com}}, \quad (23)$$

$$Q_f = \eta_{com} m_f h_{for.}, \quad (24)$$

where h_{for} is the enthalpy of formation, Δt_{com} is duration of combustion, m_f is fuel mass and

η_{com} is the energy conversion rate denoting the efficiency of combustion as follows [2]:

$$\eta_{com} = \begin{cases} 1.0 & ; \quad \lambda \geq \lambda_{cr.} \\ a_\eta \lambda \exp(c_\eta \lambda) - b_\eta & ; \quad 1.0 \leq \lambda < \lambda_{cr.} \\ 0.95\lambda + d_\eta & ; \quad \lambda < 1.0 \end{cases}, \tag{25}$$

in which λ is the excess air factor, $\lambda = 1/F$. a_η, b_η, c_η and d_η are constants which depend on the critical smoke limit of the excess air coefficient of combustion $\lambda_{cr.}$. $\lambda_{cr.}$ is a constant around 1.4.

The mass of fuel injected can be defined as a function of relative fuel rack position:

$$m_f = m_{f\,MCR} X_f\,, \tag{26}$$

where $m_{f\,MCR}$ is the mass of the fuel injected when the fuel rack position is 1.0 and X_f is fuel index.

2.5. Heat transfer

Hear transfer coefficient in cylinders, α_{cyl}, is:

$$\alpha_{cyl} = \frac{K_4 P_{cyl}^{\;0.8}}{D_{cyl}^{\;0.2} T_{cyl}^{\;0.53}} \cdot \left[K_5 \bar{u}_{pis} + K_6 \frac{V_{sw} T_{ref}}{P_{ref} V_{ref}} \left(P_{cyl} - P_{cyl\,mot} \right) \right]^{0.8}, \tag{27}$$

in which K_4 is selected as 0.0832. during compression and expansion and K_5 and K_6 are equal 2.28 and 3.24e-3, respectively. They are equal 6.18 and zero for charge renewal period, respectively. V_{sw} is the swept volume by the piston. Cylinder pressure of a motored engine, $P_{cyl\ mot}$ is the cylinder pressure with no combustion and $\bar{u}_{pis}$ is the average speed of cylinder piston. Then the convective heat transfer (gas-to-wall) can be calculated:

$$\frac{dQ_{cyl}}{dt} = \alpha_{cyl} A_{cyl} \left(T_{cyl} - T_{cyl\,sur} \right), \tag{28}$$

the subscript $cyl\ _{sur}$ denotes the surface of the cylinder and the total surface area for heat transfer calculation A_{cyl}, comprises cylinder liner, cylinder cover, exhaust valve and piston crown surface areas.

Heat transfer in exhaust gas receiver is

$$\left(\frac{dQ_{gas-sur}}{dt} \right)_i = \alpha_{g\,i} A_{gm} \left[\frac{T_{gm} + T_{cyl\,i}}{2} - T_{gm\,sur} \right], \tag{29}$$

where $dQ_{gas\text{-}sur}/dt$ refers to the heat transfer from exhaust gas coming from cylinder i to the exhaust receiver surface or inversely. α_g is the heat transfer coefficient and subscript gm denotes gas receiver. The temperature of the exhaust gas in the model is taken as the mean value of instantaneous exhaust gas temperature from the cylinder and the average exhaust gas temperature in the receiver. The temperature of exhaust pulse can be defined as:

$$T_{cyl\,i} = \begin{cases} \left(T_{cyl\,fre} \right)_i ; \left(dm_{cyl\,res} / dt \right)_i = 0 \\ \left(T_{cyl\,res} \right)_i ; \left(dm_{cyl\,res} / dt \right)_i \neq 0 \end{cases}. \tag{30}$$

The area of heat transfer can be determined according to the main dimensions of the receiver. The convective heat transfer coefficient of the receiver, α_g is [4]:

$$\alpha_g = \alpha_{g\,ref} \left(\frac{\left(T_{gm} + T_{cyl\,i}\right)/2}{T_{g\,ref}} \right)^{0.502} \left(\frac{\dot{m}_{gv_i}}{\dot{m}_{gv_{ref}}} \right)^{0.4} , \tag{31}$$

where subscript *gv* denotes the exhaust gas valve. It is considered that the receiver is insulated and no heat loss from the receiver surface to the surrounding is included in the model. The receiver surface is calculated separately for each cylinder. Temperature of the receiver body can be derived from energy balance equation in the following form:

$$\frac{dT_{gm_{sur}}}{dt} = \frac{1}{c_{gm_{sur}} m_{gm_{sur}}} \sum_{i=1}^{n_{cyl}} \left(\frac{dQ_{gas-sur}}{dt} \right)_i , \tag{32}$$

where $c_{gm\ sur}$ is the specific heat of the applied metal in the receiver construction and $m_{gm\ sur}$ is the mass of exhaust receiver body.

2.6. Turbine and turbocharger shaft speed

The turbine mass flow is

$$\dot{m}_t = A_{t\,eq.} \psi_t \frac{P_{gm}}{\sqrt{R_{gm} T_{gm}}} , \tag{33}$$

where subscript *gm* denotes exhaust gas manifold (receiver), A_{teq} is the equivalent nozzle area and has been substituted in mass flow rate equation through a nozzle instead of $C_d A$. ψ_t is the flow function of turbine and can be calculated in the same way as was presented for valves with the pressure ratio:

$$\pi_t = \frac{P_{exit}}{P_{gm}} . \tag{34}$$

Turbine Pressure Ratio: The turbine pressure ratio π_t, is the total-to-static pressure ratio and it is given as the ratio of static pressure after the turbine and the exhaust gas pressure in the exhaust receiver. The turbine inlet total pressure is assumed to be equal the exhaust receiver pressure:

$$P_{exit} = P_{amb} + P_{back} , \tag{35}$$

$$P_{back} = a_{back} \left(P_{gm} - P_{amb}\right), \tag{36}$$

where P_{back} is the back pressure.

Equivalent nozzle area of turbine: It is selected by fitting the equivalent nozzle area with experimental data derived from the whole engine test and then interpolated for each time step.

Turbine Efficiency: In transient conditions turbine efficiency, η_t, is not only a function of pressure ratio but also turbine blade speed ratio, hence the blade speed varies from optimum value. Therefore, in general case:

$$\bar{\eta}_t = 1 - a_t \left(\bar{\nu}_t - 1 \right)^2 , \tag{37}$$

$$\bar{\eta}_t = \frac{\eta_t}{\eta_{t\,opt}} \;,\; \bar{\nu}_t = \frac{\nu_t}{\nu_{t\,opt}} \;\text{ and }\; \eta_{t\,opt} = \frac{\eta_{tc}}{\eta_{ca}} . \tag{38}$$

Subscript t refers to turbine and the bar sign indicates non-dimensional value and subscript opt refers to values of parameters at optimum turbine blade speed. The blade speed ratio is:

$$\nu = \frac{u_t}{C_{g_t}} , \tag{39}$$

$$C_{g_t} = 2\sqrt{h_t (1 - \rho_t)} . \tag{40}$$

ρ_t is the reaction degree of turbine (the main parameter for present analysis) and h_t is theoretical enthalpy drop in the turbine:

$$h_t = \frac{k_{gm}}{k_{gm} - 1} R_{gm} T_{gm} \left[1 - \pi_t^{\left(\frac{k_{gm} - 1}{k_{gm}} \right)} \right] . \tag{41}$$

Angular velocity of turbocharger shaft: The power of turbine is defined as follows:

$$N_t = \dot{m}_t h_{exit} - \dot{m}_t h_{gm} . \tag{42}$$

The enthalpy of outflow gases, h_{exit}, can be calculated for exhaust gas at temperature of exhaust receiver, with equivalence ratio of fresh air and h_{gm} should be evaluated for exhaust receiver condition. Temperature of exhaust gas at outlet from turbine is:

$$T_{exit} = T_{gm} - T_{gm} \eta_t \left[1 - \pi_t^{\left(\frac{k_t - 1}{k_t} \right)} \right] . \tag{43}$$

Finally, when power of compressor and turbine are defined, the derivative of angular velocity of turbocharger shaft, ω_{tc}, can be calculated:

$$J_{tc} \frac{d\omega_{tc}}{dt} = \frac{(N_t - N_c)}{\omega_{tc}} , \tag{44}$$

where J_{tc} is mass moment of inertia of turbocharger shaft. The mechanical losses of turbocharger, because of high speed of the shaft, are neglected.

2.7. Mechanical efficiency

The mechanical losses in the form of mean effective pressure $MMEP$, is as follows:

$$MMEP = a_l + b_l P_{peak}^{\;c_l} + d_l (\omega - \omega_{\min})^{e_l} , \tag{45}$$

where P_{peak} is the peak pressure of the cylinder, ω_{min} is the minimum angular velocity of engine and a_l, b_l, c_l, d_l, e_l are constants specified based on the experimental results.

3. Propeller and power transmission system

Generally, in order to specify the dynamic torque of propeller, Q, it is necessary to have propeller angular velocity, $\omega_P(t)$, propeller pitch $H(t)$ and ship's velocity $v(t)$. The influence of changing in ship's speed on the other elements of system is neglected. In order to calculate Q, usually total non-dimensional torque coefficient, K_Q, is used:

$$Q = K_Q \, \rho_W \, D^5 \, n_p \left| n_p \right|, \tag{46}$$

$$K_Q = f\left({}^{H}\!/_{D}, J_v, \mathrm{Re}, {}^{A_E}\!/_{A_0}, {}^{d_P}\!/_{D} \right), \tag{47}$$

$$J_V = \frac{V\,(1-w)}{D\,n_p}, \tag{48}$$

where ρ_W is sea water density, D is propeller diameter, n_P is rate of revolution of the propeller in rps, H/D is pitch ratio, J_v is advance number, A_E/A_0 is area ratio of the propeller, d_P is diameter of propeller hub, and w is wake fraction coefficient.

Non-dimensional ideal torque coefficient (with no losses) for FPP (Fixed Pitch Propeller) is as follows, [6]:

$$K_Q{}^* = \sum_{i=1}^{L} \left[C_Q(i) \cdot J_v{}^{s(i)} \cdot {}^{H}\!/_{D}{}^{t(i)} \cdot {}^{A_E}\!/_{A_0}{}^{u(i)} \cdot Z_P{}^{v(i)} \right], \tag{49}$$

where C_Q is a constant expressed based on model tests, Z_P is number of the propeller blades, and s, t ,u and v are exponents of advance number, pitch ratio, area ratio and number of blades, respectively. The characteristics of CPP (Controllable Pitch Propeller) are generally different than FPP and variation of ${}^{d_P}\!/_{D}$ should be also taken into account. This is included in the model by decreasing the propeller open water efficiency by 2% due to existing of propeller hub.

After taking into consideration the effect of interaction between hull and propeller by the hull efficiency, η_H, the relative rotative efficiency, η_R and the losses due to friction in bearings, it is possible to calculate the propeller torque, Q, for the real case.

The shafting system is modeled as a rigid body. The derivative of propeller shaft angular velocity, w, is

$$\frac{d\omega}{dt} = \frac{M - Q}{J_E + J_P}, \tag{50}$$

where M is engine torque and J_E and J_P are engine and propeller mass moment of inertias.

The propeller pitch adjusting mechanism is modeled as a pure I-action linear element. Due to hydraulic system aspects, difference between desired propeller pitch and its dynamic value is limited. In addition, the value of pitch can not exceed a specified permissible value which has different values in ahead and astern running. As a usual practice, maximum pitch ratio can be selected as 15% more than nominal pitch ratio for both ahead and astern motion with negative sign for astern running.

4. Ship dynamics

In the case of calm water, the ship motion equations will reduce to one equation for *surge*:

$$\Delta \cdot \frac{dv}{dt} = T \cdot (1 - TDF) - R, \tag{51}$$

where Δ is the total mass of vessel, T is propeller thrust force, TDF is thrust deduction factor and R is total ship's resistance. TDF can be calculated for the design conditions:

$$TDF = \left(\frac{T - R}{T} \right)_{design}. \tag{52}$$

5. Model of governor

Indeed, the time constant of the governor is several times shorter than the time constant of the diesel engine, especially in comparison with turbocharger. Therefore, in many delivered models, governor has been considered as an inertialess element. Thus, the static characteristic of the governor has been taken as the dynamic one. The input signals for the governor are actual angular and desired angular velocity as a set point. The output signal is the displacement of fuel rack. The governor of this study is modelled as a PI controller. A scavenging air pressure fuel limiter or a torque limiter can be included in the governor model.

6. Simulation

For simulation purpose a bulk carrier built by Mitsui Eng. Co. & Shipbuilding directly driven by a two stroke test engine MAN-B&W L70MC-type equipped with a MAN-B&W NA 57TO turbocharger and propelled by a controllable pitch propeller B-Wageningen type have been selected. The vessel's deadweight is 34600 t for speed of 15.5 Kn. and the required power is equal to 6730 kW. The engine rate of revolution is 111 rpm. The propeller is a 6145 mm diameter, 5 blades with nominal pitch ratio of 0.71 and its area ratio is equal 0.6.

The model is coded in state space form using MATLAB-SIMULINK 7.0. The state variables of the engine model, except the cylinder variables are as follows:

- the mass of scavenging air in the scavenging air receiver,
- the temperature of scavenging air in the scavenging air receiver,
- the mass of exhaust gas in the exhaust gas receiver,
- the mass of stoichiometric exhaust gas in the
- exhaust gas receiver,
- the temperature of exhaust gas in the exhaust gas receiver,
- the temperature of exhaust gas receiver wall,
- the start of the heat release and
- the turbocharger shaft angular velocity.

The state variables of each cylinder in the engine model are:

- the mass of fresh air zone,
- the temperature of the fresh air zone,
- the mass of the residual gas zone,
- the mass of stoichiometric exhaust gas in the residual gas zone and
- the temperature of the residual gas zone.

The other state variables are:

- rate of revolution of propeller shaft
- pitch ratio, and
- ship's speed.

Comparison of the simulated steady-state performance of the engine to measured data was carried out using measured engine speed and measured fuel rack position as input data of the engine simulation model. For steady state simulation, dynamic model is applied when time is enough long to derive steady state values.

Comparison of the simulated transient response to the measured data was made in a similar way, i.e. using the measured transient fuel rack position and the measured transient engine speed as input data of the engine model. The measured engine power and measured brake mean effective pressure were derived from the measured water brake load, the measured engine speed and the rate of change of the engine speed.

In a test run with stepwise fuel rack position adjustments at high power levels, the fuel rack position was decreased from 1.01 to 0.78 and restored to 1.01, see Fig. 1. The engine speed fluctuated between 92 [rpm] and 111 [rpm] and the engine power varied between 63% to 101%. Fig. 2 illustrates a very good agreement between the simulated and the measured response.

Generally, the presented transient responses confirm validity, enough accuracy and compatibility of the model and permit to apply the model for different aims.

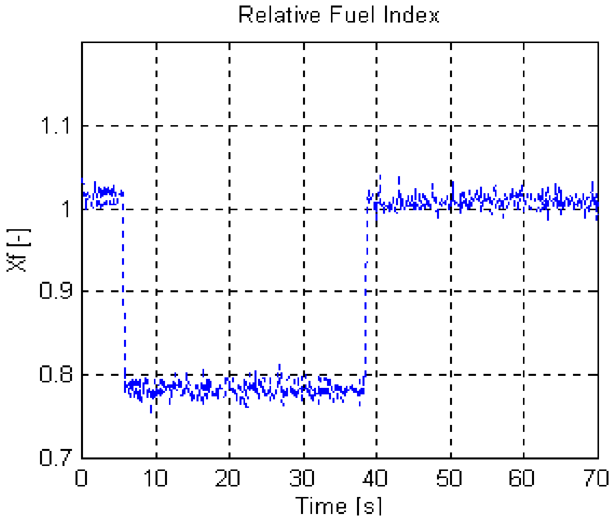

Figure 1. Measured relative fuel index

7. Sensitivity analysis method

Consider the system

$$\dot{\mathbf{x}} = f[\mathbf{x}(t), \mathbf{p}, t], \qquad \mathbf{x}(t_0) = \mathbf{x_0} \tag{53}$$

where $\mathbf{x}(t)$ is an n-dimensional state variable vector and $\mathbf{p}$ is an r-dimensional parameter vector. Assume that $\mathbf{x_{nom.}}$ represents state variable matrix with nominal values of $\mathbf{p}$, i.e., $\mathbf{p_{nom.}}$ which will be called *nominal state variables vector*,

$$\dot{\mathbf{x}}_{\mathbf{nom.}} = f[\mathbf{x}_{\mathbf{nom.}}(t), \mathbf{p}_{\mathbf{nom.}}, t], \qquad \mathbf{x}_{\mathbf{nom.}}(t_0) = \mathbf{x}_{\mathbf{0\,nom.}} \tag{54}$$

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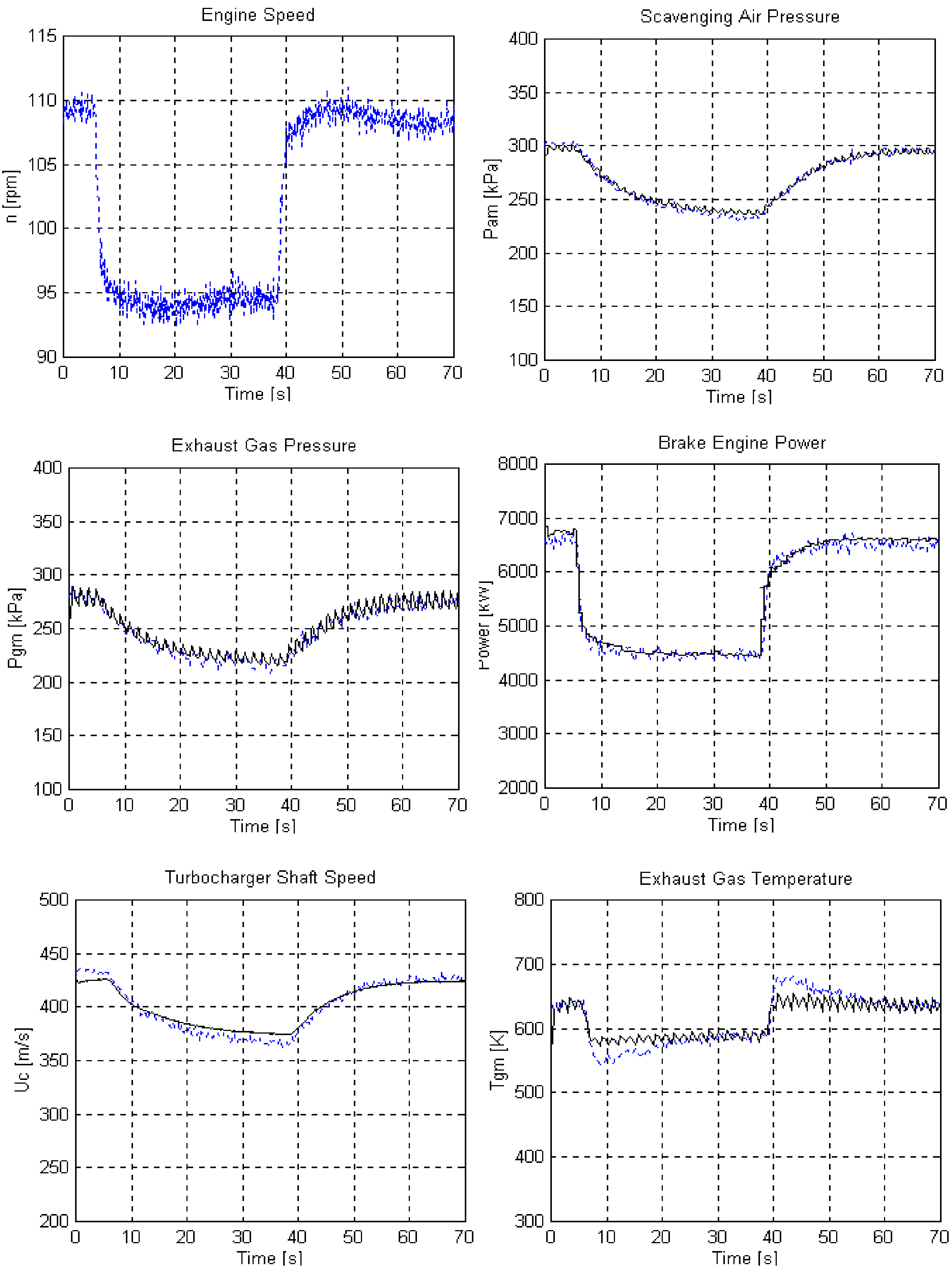

Figure 2. Transient response of engine and comparison of results for measured values (dashed blue) and simulation results (solid black)

When all inputs and parameters, except parameter p_j are remained without any changing, and only p_j is set to its maximum possible value, $\mathbf{x}(t)$ changes to $\mathbf{x}^j_{\mathbf{max.}}$. This can be called *maximum state variables vector due to changing the j^{th} parameter.* In the same way when p_j is set to its minimum possible value, *minimum state variable vector due to changing the j^{th} parameter,* $\mathbf{x}^j_{\mathbf{min.}}$ will be shaped:

$$\dot{\mathbf{x}}^j_{\mathbf{max.}} = f[\mathbf{x}^j_{\mathbf{max.}}(t), \mathbf{p}^j_{\mathbf{max.}}, t], \qquad \mathbf{x}^j_{\mathbf{max.}}(t_0) = \mathbf{x}^j_{\mathbf{0\,max.}}, \tag{56}$$

$$\dot{\mathbf{x}}^j_{\mathbf{min.}} = f[\mathbf{x}^j_{\mathbf{min.}}(t), \mathbf{p}^j_{\mathbf{min.}}, t], \qquad \mathbf{x}^j_{\mathbf{min.}}(t_0) = \mathbf{x}^j_{\mathbf{0\,min.}}, \tag{57}$$

where i illustrates the state variable index and j shows the parameter index.

Two auxiliary parameters are defined:

$$\delta 1^j_i(t) = \left| \frac{x_{i\ nom.}(t) - x^j_{i\ max.}(t)}{x_{i\ nom.}(t)} \right| \cdot 100, \tag{58}$$

$$\delta 2^j_i(t) = \left| \frac{x_{i\ nom.}(t) - x^j_{i\ min.}(t)}{x_{i\ nom.}(t)} \right| \cdot 100, \tag{59}$$

where $\delta 1^j_i(t)$ and $\delta 2^j_i(t)$ are *relative partial differences* at the time in question, in percent.

Time average difference for i^{th} state variable and j^{th} parameter, Δ^j_i , which expresses mean deviation from normal condition in percent is defined as follows:

$$\Delta^j_i = \frac{\int_0^{t_{si}} \left(\frac{\delta 1^j_i(t) + \delta 2^j_i(t)}{2} \right) d\tau}{\int_0^{t_{si}} d\tau}, \tag{60}$$

where $t_{s\,i}$ is *arising time (solution time)* for state variable in question and the model is simulated for the widest range of changing of operation point, as much as possible.

By *weighting* the effects or priority of considered state or state dependent variables, e.g., from 0 to 100, it is possible to obtain *sensitivity index, S_j* , for j^{th} parameter.

$$S_j = \frac{\sum_{i=1}^{n} w_i \cdot \Delta^j_i}{\sum_{i=1}^{n} w_i} \tag{61}$$

If sensitivity index exceeds *sensitivity certain value, Λ,* system should be called *sensitive* to the considered parameter, if not the parameter in question can be named *neutral*. Selection of weights and sensitivity certain value depend directly on the system behaviour and designer experience.

Beside of these dynamic concepts, steady state aspects must be taken into consideration. It is well known in Classification Society Rules that steady state error of the system is to be limited to several percent. Usually it is around 2%. Therefore, sensitivity criterion should be completed by this consideration, i.e., all above mentioned procedure must be repeated for steady state conditions. Finally, the sensitivity criterion can be defined as follows: *„The system, is sensitive to parameter p_j, if sensitivity index S_j is greater than both dynamic sensitivity certain value Λ_D, and steady state* *sensitivity certain value Λ_S.".*

8. Sensitivity estimation

For the present case, among of different parameters, only one parameter, i.e. reaction degree of turbine (applied in turbocharger) is selected and sensitivity of the system against its changing is examined. The selected parameter and its considered nominal, maximum and minimum values are 0.5, 0.1 and 1.0, respectively. The most important variables of the system are as follows:

1. angular velocity of propeller shaft,
2. relative fuel index,
3. air pressure in the scavenging air receiver
4. engine power,
5. brake mean effective pressure,
6. compressor tip speed,
7. gas pressure in the exhaust gas receiver, and
8. gas temperature in the exhaust gas receiver.

Discussion about priority and importance of these variables is out of scope of this study. The system is simulated for a wide range of changing in operating point of the system, where this operating point is varied from 110% to 70% of nominal load. The *sensitivity certain value* for both dynamic and steady state sensitivity is selected equal 2. Tab. 1 gives *weights* of each of the variables for sensitivity analysis. Pressure of exhaust gas in the exhaust gas receiver is weighted as zero, because it is an indirect function of other considered variables. Steady state (static) and dynamic values of *average differences* are given in Tab. 2.

Table 1. The considered variables and their weights for sensitivity analysis

N0.	Variable	Weight (out of 100)
1	Angular velocity of propeller shaft	100
2	Relative fuel index	100
3	Air pressure in the scavenging air receiver	80
4	Engine Power	80
5	Brake mean effective pressure	50
6	Compressor tip speed	20
7	Gas temperature in the exhaust gas receiver	10
8	Gas pressure in the exhaust gas receiver	0

Table 2. Analyzing the effect of changing in "Reaction Degree of Turbine"

Variable	Dynamic Mean Diff.	Static Diff.
Angular velocity of propeller shaft (ω)	0.002	0.024
Fuel index (X_f)	0.234	8.577
Engine power (N)	0.020	0.113
Break Mean Effective Pressure (BMEP)	0.166	0.267
Compressor tip speed (Uc)	0.326	0.418
Air pressure in the scavenging air receiver (Pam)	0.361	0.304
gas pressure in the exhaust gas receiver (Pgm)	0.343	0.304
gas temperature in the exhaust gas receiver (Tgm)	18.783	64.202

9. Conclusion

The considered system includes a ship propulsion plant with its control system. For the system in question diesel engine and propulsion performances have been modeled and simulated. Next, based on the simulation results the sensitivity of these performances against changing of degree of reaction of turbine of turbocharger is examined. According to the calculated results it is possible to explain how much the whole system is sensitive to the changing of considered parameter. The results show that *dynamic sensitivity index* $(S_j)_{Dynamic}$ for the mentioned conditions is 58.4 and *steady state sensitivity index* $(S_j)_{Static}$ is 354.0. When they are compared to the defined *sensitivity certain value*, one can conclude that they are significantly higher than this reference level (29 and 177 times more). This confirms that ship propulsion system is very sensitive to the changing of degree of reaction of turbine. Just for comparison, it should be mentioned that sensitivity index for other parameters of the system is significantly lower than the values calculated here and usually is between 0 and 50. It is a confirmation of well-known practical engineering rule for diesel engine that turbocharger characteristics have a great influence on engine operation. However, the results and delivered method indicate how this influence can be quantified and expressed in early design stages.

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