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FATIGUE CRITERION BASED ON THE HUBER-VON MISES-HENCKY CRITERION FOR NON-PROPORTIONAL LOADINGS

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Abstract

In the present paper to determine the calculated fatigue limit, under non-proportional loads, there has been proposed a modification of the Huber-von Mises-Hencky criterion. To do so the Novoshilov interpretation of that criterion was applied. As an 'effective amplitude', there was assumed the maximum value of shear stress in the non-proportional load cycle. There was also proposed the weight function showing preference of the directions of easy slip in network A2. Verification calculations were performed for literature data. The data included experimental fatigue limits reported under biaxial loads, sinusoidally variable from phase shift. The present results were compared with the Huber-von Mises-Hencky criterion. The analysis allowed for determining that the solutions proposed demonstrate greater accuracy, especially under loadings of a high degree of load non-proportionality.

Keywords: *multiaxial fatigue, fatigue criteria, integral approach*

1. Introduction

The application of the Huber-von Mises-Hencky criterion (HMH) to calculate the fatigue life and fatigue strength for non-proportional loads is problematic [1]. The criterion can be provided with varied physical interpretations: specific distortion strain energy, octahedral shear stress, root-mean-square value of principle shear stresses or, in a form proposed by Novoshilov, root-mean-square value with shear stresses on all the planes crossing the point in question [1], namely:

$$\sigma_{equ} = \sqrt{\frac{1}{4\pi}\int_{\gamma=0}^{\pi}\int_{\varphi=0}^{2\pi}\left(\tau_{\gamma\varphi}\right)^2 \sin\gamma d\gamma d\varphi}\,, \tag{1}$$

where:

$\tau_{\gamma\varphi}$ – vector of shear stress, defined in plane Δ,
φ, γ – angles describing the location of plane Δ (Fig. 1).

The last mentioned interpretation has become the springboard for a group of solutions referred to as integral approach. It is claimed that such approach makes it possible to use the HMH criterion to describe non-proportional states of loadings. The proposals in that group of criteria differ in their definition of ‘effective amplitude’ of shear stress $\tau_{\gamma\varphi,a}$. The most common are those proposed by Simbürger [after 2], Zenner [after 2] and Papadopoulos [3].

In Simbürger’s proposal the criterion has the form of:

$$A = \sqrt{\frac{15}{8\pi}\int_{\gamma=0}^{\pi}\int_{\varphi=0}^{2\pi} A_n^2 \sin\gamma d\gamma d\varphi} \leq 1 \quad (2)$$

where effective amplitude A_n is expressed as the relationship:

$$A_n = \frac{\sigma_{v,a} - m\sigma_{v,m}}{Z_{go}} \quad (3)$$

$$\sigma_{v,a} = a\sigma_{n,a} + b\tau_{n,a}, \sigma_{v,m} = a\sigma_{n,m} + b\tau_{n,m} \quad (4)$$

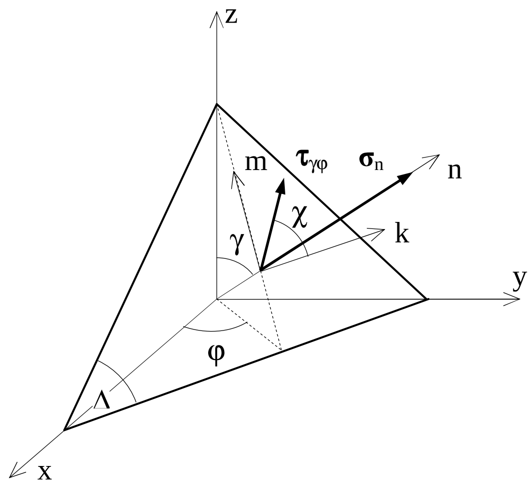


Fig. 1. Vector of shear stress $\tau_{\gamma\varphi}$ on the plane defined with angles φ and γ

Amplitudes and mean values of normal and shear stresses in (4) are determined based on the longest view method [4]. Zenner proposed a similar solution:

$$\sigma_{equ,a} = \sqrt{\frac{15}{8\pi}\int_{\gamma=0}^{\pi}\int_{\varphi=0}^{2\pi}\begin{bmatrix} a\tau_{\gamma\varphi,a}^2\left(1 + m\tau_{\gamma\varphi,m}^2\right) + \\ +a\sigma_{\gamma\varphi,a}^2\left(1 + m\sigma_{\gamma\varphi,m}^2\right)\end{bmatrix} \sin\gamma d\gamma d\varphi} \leq Z_{go} \quad (5)$$

The Papadopoulos criterion is a sum of the greatest in all the planes Δ (Fig. 1) generalized amplitude of shear stress and maximum stress hydrostatic:

$$\tau_{eq,a} = max_{\varphi,\gamma}(T_a) + a\sigma_{H,max} \leq Z_{so} \quad (6)$$

Generalized amplitude of shear stress for each plane Δ is determined from:

$$T_a(\varphi,\gamma) = \sqrt{\frac{1}{\pi}\int_{\chi=0}^{2\pi}\tau_a^2(\varphi,\gamma,\chi)d\chi} \quad (7)$$

where:

$$\tau_a(\varphi,\gamma,\chi) = \frac{1}{2}\left[max_t\tau(\varphi,\gamma,\chi,t) - max_t\tau(\varphi,\gamma,\chi,t)\right] \quad (8)$$

2. Criterion proposal

The present paper analyses the simplest solution assuming in (1), for each direction n, maximum value of shear stress $\tau_{\gamma\varphi} = max_t(\tau_{\gamma\varphi})$ throughout the cycle. The criterion assumed the form of:

$$\sigma_{eq,a} = \sqrt{\frac{1}{4\pi}\int_{\gamma=0}^{\pi}\int_{\varphi=0}^{2\pi}\left(max_t(\tau_{\gamma\varphi})\right)^2 \sin\gamma d\gamma d\varphi} \leq Z_{go} \tag{9}$$

For the states of stress of the components defined in line 6 table 1, the evolution of instantaneous values of stresses $\tau_{\gamma\varphi}$ for $\omega t = 0, 30, 60$ and $90°$, has been presented respectively in Fig. 2.a, b, c and d. Hodograph for maximum values $max_t(\tau_{\gamma\varphi})$ of that state of stress is visible in Fig. 2.e.

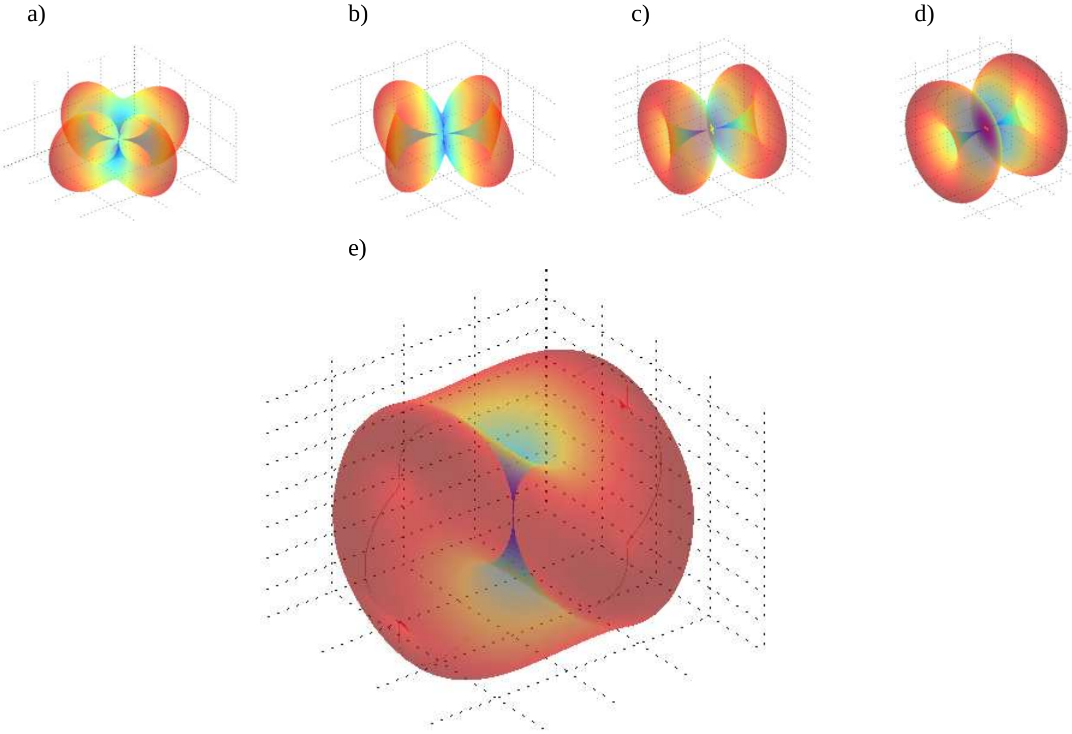


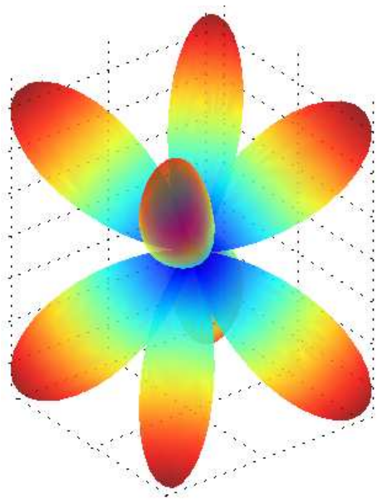
Fig. 2. Geometric form of weight function w (inside) against the respective maximum values of shear stresses throughout the loading cycle (outside)

Besides, the present paper proposes the solution with weight function w demonstrating the preference of the directions of easy slip <111> in crystallographic network A2. The weight function has been assumed in a form of:

$$w = w_m + w_a \cdot \sin\left(2(\gamma - \pi/2)\right)^2 \cdot \sin\left(2(\varphi - \pi/2)\right)^2 \tag{10}$$

Its geometric form for value $w_m = 0, w_a = 1$ has been presented in Fig. 3.

The weight function must get oriented in such a way as to make its location against to the hodograph of the loading correspond to the situation of the least favourable location of the crystalline structure of the grain. The function must be rotated in a way as to make the direction of one of its maxima coincide with the direction of the maximum value of the loading throughout the cycle. The geometric form of the rotated weight function against the maximum shear stresses is presented in Fig. 4.



Rys. 3. Geometric form of weight function w

The value of shear stress in equation (1) is determined, thus:

$$\tau_{\gamma\varphi} = max_t(\tau_{\gamma\varphi}) \cdot w \tag{11}$$

Finally, the assumed value of the fatigue limit is obtained from:

$$\sigma_{eq,a} = \sqrt{\frac{1}{4\pi} \int_{\gamma=0}^{\pi} \int_{\varphi=0}^{2\pi} \left(max_t(\tau_{\gamma\varphi}) \cdot w\right)^2 \sin\gamma d\gamma d\varphi} \leq Z_{go} \tag{12}$$

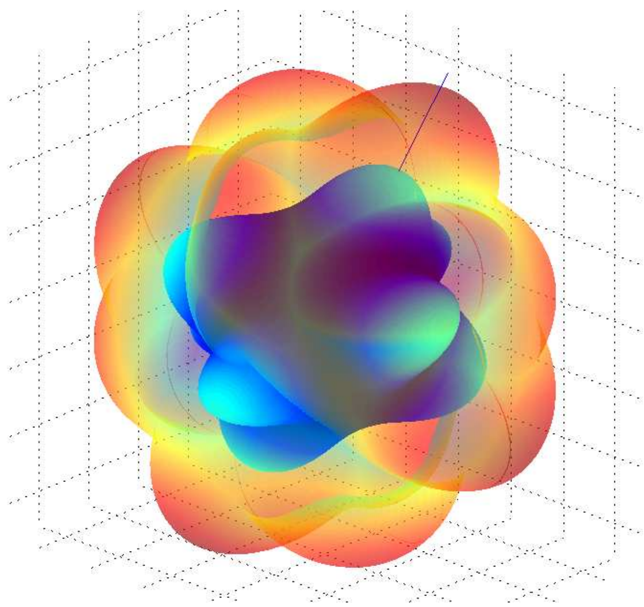


Fig. 4. Geometric form of the weight function w (inside) against the respective maximum values of shear stresses in the loading cycle (outside)

3. Results

The calculations were made for literature data [5], see breakdowns in columns 1, 2 and 3 Table 1. Those are the amplitudes of normal and shear stresses obtained from bending and torsion with phase-shift. Lines from 1 to 8 concern material 'mild steel' demonstrating properties Z_{so} = 137.3 MPa, Z_{go} = 235.4 MPa as well as Z_{so}/Z_{go} = 0.58, and lines from 9 to 18 concern material 'hard steel' demonstrating properties Z_{so} = 196.2 MPa, Z_{go} = 313.9 MPa as well as Z_{so}/Z_{go} = 0.63. For the purpose of the calculation of the values of coefficients in the equation of the weight function (10), there was assumed $w_m = 0{,}95, w_a = 0{,}05$.

The results of the calculations have been broken down in Table 1. In columns 4, 5 and 6 there have been noted the assumed values of fatigue limits, respectively, for the fatigue limits, respectively, for HMH criterion and both proposals. In columns 7, 8 and 9 there was noted a relative error of the calculated fatigue limit against the fatigue limit obtained experimentally.

One can note that for the proportional loadings (the angle of the phase shift equal 0) the results obtained with HMH criterion and the solution proposed (9) are identical. However, for non-proportional loadings the HMH criterion gives worse results.

The present results have been described with the mean value and standard deviation. For the HMH criterion the mean error value is 4.7% and standard deviation – 4.2%. For the criterion in integral form without weight function the results are better. There were obtained, respectively, 1.7% of the mean error value and 2.2% of the standard deviation. As for the criterion in an integral form with the weight function, the present results demonstrate the lowest mean error value – 0.3% and standard deviation of 2.2%.

Table 1. Results of calculations of the fatigue limit estimate

Item	Nisihary data [3]			Calculated fatigue limit			Relative terror of the calculated fatigue limit		
	$\sigma_{x,a}$	$\tau_{xy,a}$	ϕ	HMH	$\max(\tau_{\gamma\varphi})$	$\max(\tau_{\gamma\varphi}) \cdot w$	HMH	$\max(\tau_{\gamma\varphi})$	$\max(\tau_{\gamma\varphi}) \cdot w$
	MPa	MPa	°	MPa	MPa	MPa	%	%	%
	1	2	3	4	5	6	7	8	9
1	99.9	120.9	0	232.0	232	227	-1.4	-1.4	-3.6
2	103.6	125.4	60	240.6	231.6	226.9	2.2	-1.6	-3.6
3	108.9	131.8	90	252.9	239.7	234.9	7.4	1.8	-0.2
4	180.3	90.2	0	238.6	238.6	234	1.3	1.3	-0.6
5	191.4	95.7	60	253.2	240.4	235.9	7.6	2.1	0.2
6	201.1	100.6	90	266.1	247	242.4	13.0	4.9	3.0
7	213.2	44.8	0	226.9	226.9	222.9	-3.6	-3.6	-5.3
8	230.2	48.3	90	244.9	239	234.9	4.0	1.5	-0.2
9	138.1	167.1	0	320.7	320.7	314.3	2.2	2.2	0.1
10	140.4	169.9	30	326.1	321.9	315.5	3.9	2.5	0.5
11	145.7	176.3	60	338.3	325.6	319	7.8	3.7	1.6
12	150.2	181.7	90	348.7	330.5	323.8	11.1	5.3	3.2
13	245.3	122.7	0	324.6	324.6	318.6	3.4	3.4	1.5
14	249.7	124.9	30	330.4	324.7	318.8	5.2	3.4	1.6
15	252.4	126.2	60	333.9	317	311.2	6.4	1.0	-0.9
16	258	129	90	341.3	316.9	311	8.7	1.0	-0.9
17	299.1	62.8	0	318.3	318.3	312.8	1.4	1.4	-0.4
18	304.5	63.9	90	324.0	316.2	310.8	3.2	0.7	-1.0

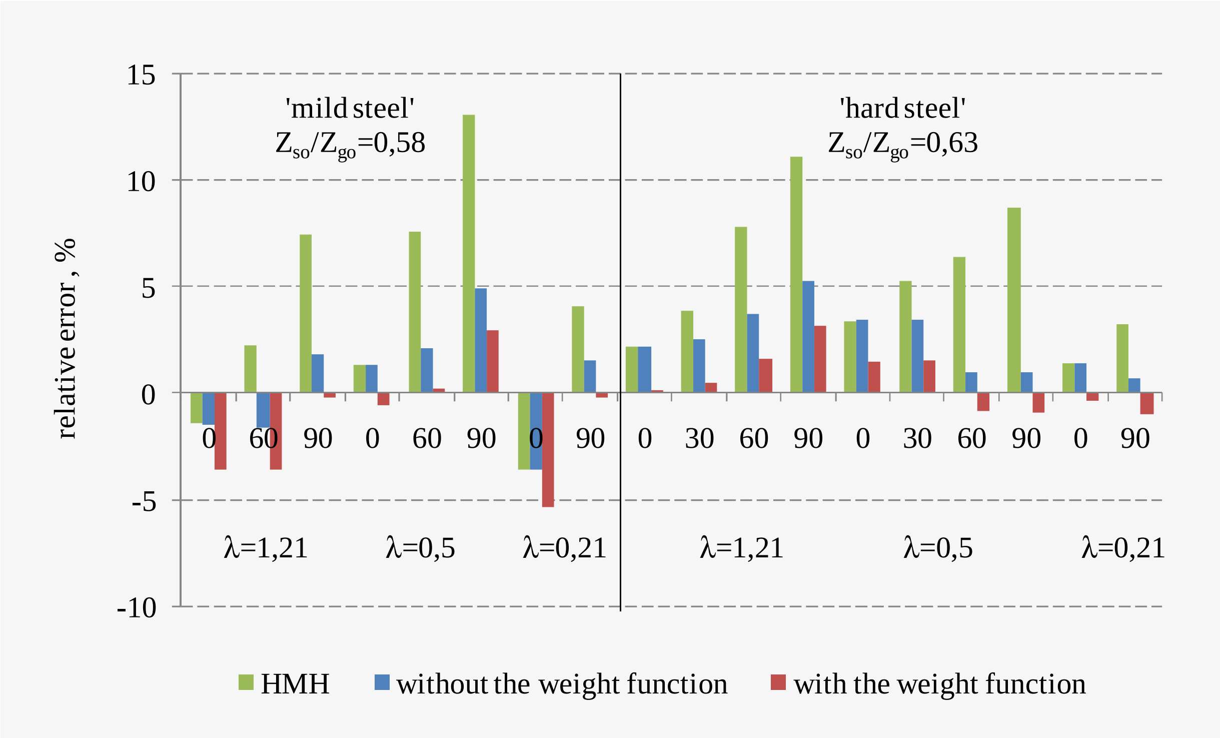


Fig. 5. Geometric form of weight function **w** (inside) against the respective maximum values of shear stresses throughout the loading cycle (outside)

4. Conclutions

1. The modification of the Novoshilov criterion involving the assumption the 'effective amplitude' in a form of the greatest value of shear stress in the cycle, enhanced the results of estimating the fatigue limit as compared with the Huber-von Mises-Hencky criterion, especially in the case of the loadings of a maximum degrees of non-proportionality of the loading.
2. The application of the weight function demonstrating the preference for the directions of easy slip in network A2 caused a further decrease in error of the estimated assumed value of the fatigue limit. Unfortunately, for the proportional loadings when the phase shift angle equals 0, there was recorded a slight deterioration of the results.

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