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GERALIZATION OF GOODMAN'S DIAGRAM ON THE HIGH-CYCLE FATIGUE RANGE

Bogdan Ligaj

University of Technology and Life Sciences in Bydgoszcz
ul. Prof. S. Kaliskiego 7, 85-789 Bydgoszcz, Poland
tel.: +48 52 3408253, fax: +48 52 3408271
e-mail: bogdanj@utp.edu.pl

Abstract

The calculations of fatigue life of the structural components in random load conditions are connected with many problems. One of them is taking into consideration the impact of the cycles of variable value of R coefficient, which is included in service load composition, on estimated value. This involves application of calculations of two-parametric fatigue characteristics in system S_m-S_a in time. There are many models of two-parametric characteristics. One of them is characteristics being a development of the Goodman's conception, the biggest value of which is undoubtedly a simple description requiring the knowledge of following properties: R_m, R_{-1} and $m_{o.}$ In order to evaluate the characteristics mentioned above, it was compared to two-parametric fatigue characteristics determined in experimental way. The results of the analysis have been presented in form of diagram, which reveals the differences between characteristics depending on the number of cycles and the cycle asymmetry coefficient.

Keywords: *fatigue life, two-parametric characteristics, S355J0 steel*

Nomenclature

A_5 – elongation [%],
N – cycle number - general notation (fatigue life),
N_o – cycle number – fatigue life corresponding with fatigue limit,
$R = S_{min}/ S_{max}$ – cycle asymmetry ratio,
R_e – material yield point [MPa],
R_m – material tensile strength [MPa],
R_{-1} – fatigue limit under oscillating load (R = -1) for N_0 cycle number, [MPa],
S – specimen stress – general notation, [MPa],
$S_a = 0,5(S_{max} - S_{min})$ – sinusoidal cycle stress amplitude [MPa],
$S_m = 0,5(S_{max} + S_{min})$ – mean sinusoidal cycle stress [MPa],
S_{max} – maximum sinusoidal cycle stress [MPa],
S_{min} – minimum sinusoidal cycle stress [MPa],
Z – contraction [%],
m_0 – exponent in formula describing Wöhler fatigue diagram for oscillating load (R = -1),
ψ_N – factor of material sensitivity to cycle asymmetry, for $N \neq N_0$.

1. Introduction

One of the known forms of diagrams of maximum pressure is Haigh diagram. In the figure 1 the first quarter of co-ordinate system [1] has been depicted. There the diagram mentioned above has been marked with a full line (defined with points A and B). Point B in the figure corresponds to material tensile strength (R_m), whereas point A corresponds to fatigue limit under oscillating load (R_{-1}). The area marked with the points AB0 corresponds to unlimited fatigue limit. In the discussed figure the lines, along which the cycles of specified cycle asymmetry coefficient R value are distributed, have been placed. The point of intersection of the lines mentioned with the maximum pressure diagram determinates the fatigue limit for the optional value of the R coefficient.

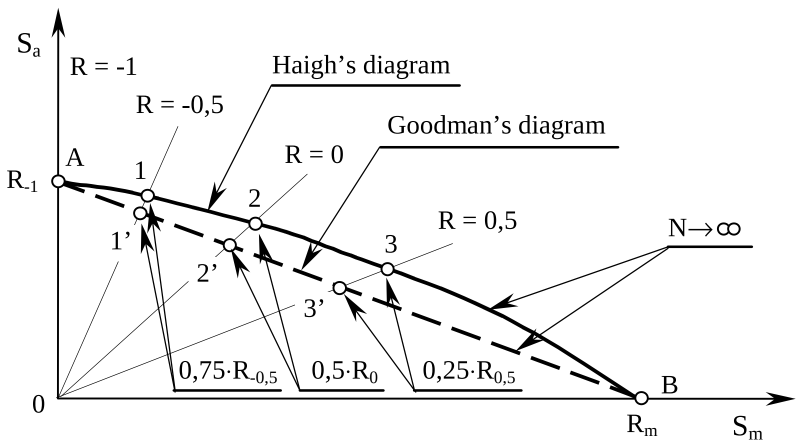

Fig. 1. Diagrammatic depiction of the maximum pressure diagram of Haigh and Goodman [1]

The simplification of the Haigh's diagram proposed by Goodman consists in replacing the curve with a straight line connecting points A and B. It influences the decrease of the fatigue limit for loads characterized by the cycle asymmetry coefficient from the range $-1 < R < 1$. The Goodman's diagram described with relation (1) has application in fatigue calculations within fatigue limit range, the result of which is the evaluation of factor of safety.

$$\frac{S_a}{R_{-1}} + \frac{S_m}{R_m} = 1{,}0 \tag{1}$$

In case of calculations of fatigue life in range of high-cycle fatigue the two-parametric fatigue characteristics, the example of which is known in the literature [1] Heywood's diagram, have application.

The development of the Goodman's conception is two-parametric fatigue characteristics proposed and published in work [2, 3, 5]. In the figure 2 the graphical form of the characteristics mentioned above, including the first and second quarter of co-ordinate system, has been presented. The simplification proposed by Goodman has been widened to the area of dominating compressive stress (second quarter). Point A in the figure concerns the fatigue limit under oscillating load (R_{-1}), whereas points B, C, K mean material tensile strength (R_m). The line JAEB divides the diagram into two areas: unlimited (described by JAEBKJ points) and limited fatigue life. By leading a line which crosses points B and M, where M is amplitude of fatigue limit under oscillating load R =-1 from the range of high-cycle fatigue, a contour line characterized by specified value of number of cycles N_j is determined. The lead contour line enables us to determine amplitude S_{aj} and average value S_{mj} of load characterized by specified value of cycle asymmetry coefficient R.

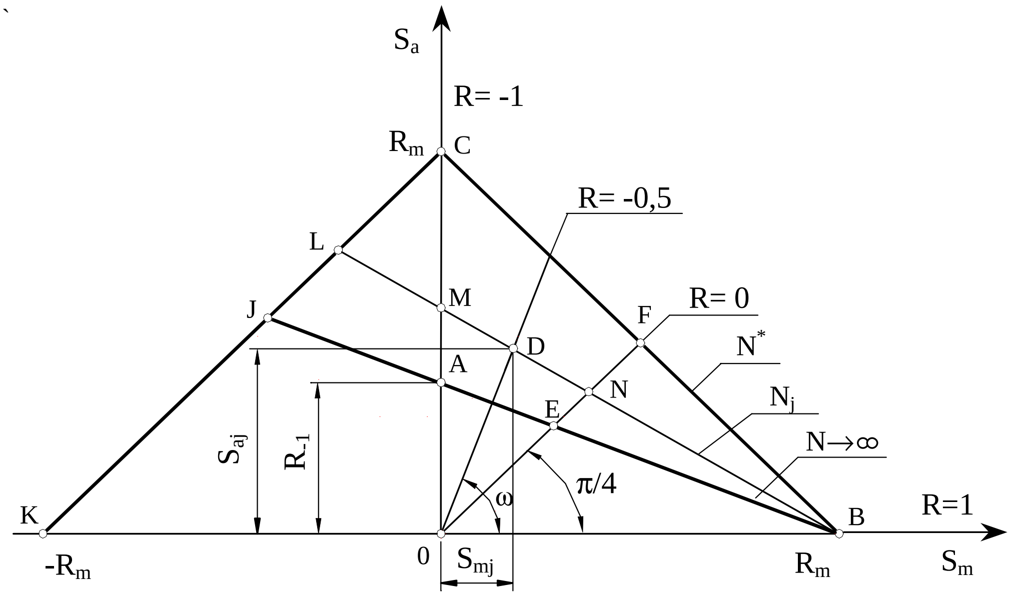

Fig. 2. Two-parametric fatigue characteristics [5]

The discussed fatigue characteristics is based on the use of relation describing the S-N curve for cycle asymmetry coefficient $R = S_{min}/S_{max}$=-1,0 in form:

$$S_a^{m_o} \cdot N = R_{-1}^{m_o} \cdot N_o = C_o \tag{2}$$

and formula (1). After appropriate transformations, depending on need, the following relations occur:

$$N = N_o \left[\frac{R_{-1}}{S_a} \left(1 - \frac{S_m}{R_m} \right) \right]^{m_o}, \tag{3}$$

lub

$$\frac{S_a}{R_m} = \frac{R_{-1}}{R_m} \left(\frac{N_o}{N} \right)^{\frac{1}{m_o}} \left(1 - \frac{S_m}{R_m} \right), \tag{4}$$

lub

$$N = \frac{2^{m_o} \cdot N_o \cdot R_{-1}^{m_o}}{\left[(1+\psi_N) \cdot S_{max} - (1-\psi_N) \cdot S_{min} \right]}. \tag{5}$$

The aim of this work is to compare two-parametric fatigue characteristics for S355J0 steel determined experimentally with two-parametric model of fatigue properties based on the conception of Goodman.

The scope of the work includes presentation of the empirical examination results for S355J0 steel in conditions of constant amplitude sinusoidal load of variable R coefficient value and elaboration on their base on two-parametric fatigue characteristics. The properties of the steel, which have been determined experimentally, enable us to elaborate on two-parametric fatigue characteristics described by formula (3). Then the specified characteristics are to be compared and the whole is to be ended by a summary.

2. Results of the empirical examinations of S355J0 steel

The research on static properties in conditions of tensile stress have been carried out on normalized fivefold cylindrical samples of circular intersection (fig. 3a) made according to PN-EN 10002-1 +AC1 standard, whereas in research on fatigue life in conditions of constant amplitude sinusoidal load the cylindrical samples of circular intersection (fig.3b) made according to PN-74/H-04327 standards have been used.

a) b)

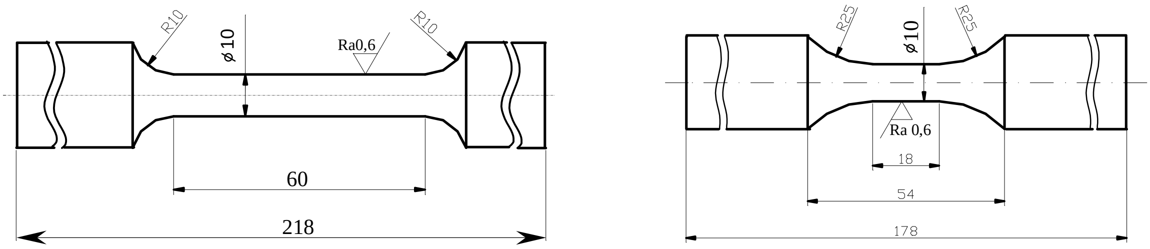

Fig. 3. Shape and measurements of samples for: a – statistical examinations, b – fatigue examinations

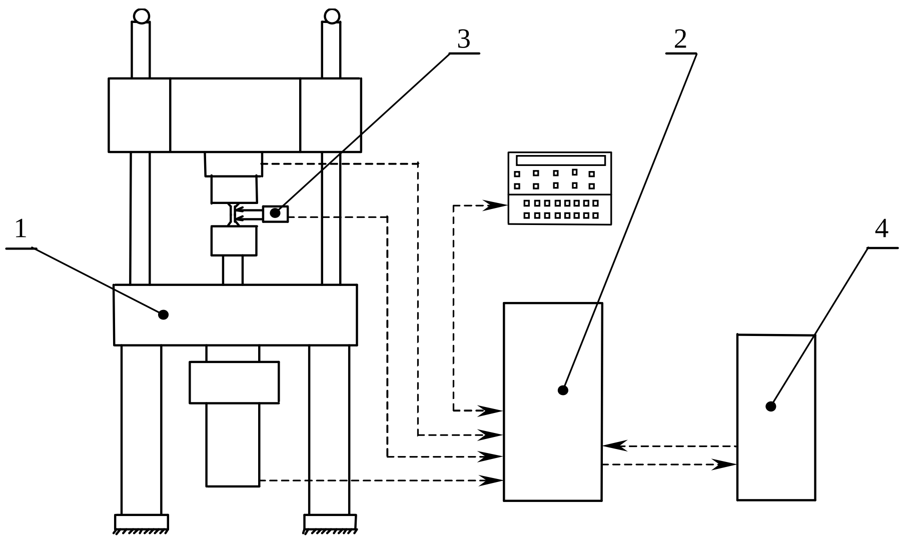

Fig. 4. Diagrammatic presentation of the construction of the test stand: 1, 2 – INSTRON 8501 testing machine with control and measurement system, 3 – extensometer, 4 – PC

The examinations have been carried out on test stand, the construction of which has been illustrated by figure 4. Our instrumentalism included: INSTRON 8501 testing machine with control and measurement system, extensometer and PC with software. During the examinations of properties in conditions of static loads extensometer of 50mm basis has been used, whereas in fatigue examinations extensometer of 10mm basis has been used.

The examinations of the static properties of steel have been carried out according to PN-EN 10002-1 +AC1 standard. The achieved results enabled us to determine the yield point, tensile strength, Young module, deformation and reduction of the sample. In table 1 the average values of the specified parameters have been depicted as well as standard deviation value has been given.

Tab. 1. Static strength properties of S355J0 steel [4]

	Static properties of S355J0 steel				
	R_e	R_m	E	A_5	Z
	MPa	MPa	MPa	%	%
Average values	499,9	678,0	208159	17,2	59,8
Standard deviation value	8,4	7,1	1306	0,99	0,9

Then the examinations in conditions of the constant amplitude sinusoidal load characterized by different cycle asymmetry coefficient value R have been carried out. The experiment has been carried out for 5 values of the coefficient R: 0, -0.5, -1.0, -1.25 and -2. For each load type the examination has been made on five levels of amplitude of nominal stresses, realizing three repetitions on each level. The acquired results enabled us to determine the fatigue life diagrams, which have been described by formula (6). The values of the slope and y-intercept for individual load types have been put together in table 2.

$$\log S_a = a \log N + b \tag{6}$$

Tab. 2. Values of the slope a and y-intercept b in formulas describing S-N curve of assumed values of cycle asymmetry coefficient R [4]

	Cycle asymmetry coefficient R				
	0	-0,5	-1,0	-1,25	-2,0
	1	2	3	4	5
a	-0,0628	-0,0528	-0,0811	-0,0709	-0,0592
b	2,7630	2,7810	2,9247	2,8894	2,8233

The empirical examinations which have been carried out enabled us to elaborate on two-parametric fatigue characteristics depicted in the figure 5. In co-ordinate system S_m/R_m–S_a/R_m the contour lines corresponding to specified number of cycles in range from 10^2 to 10^7 for assumed in examinations cycle asymmetry coefficient R have been marked by separate lines.

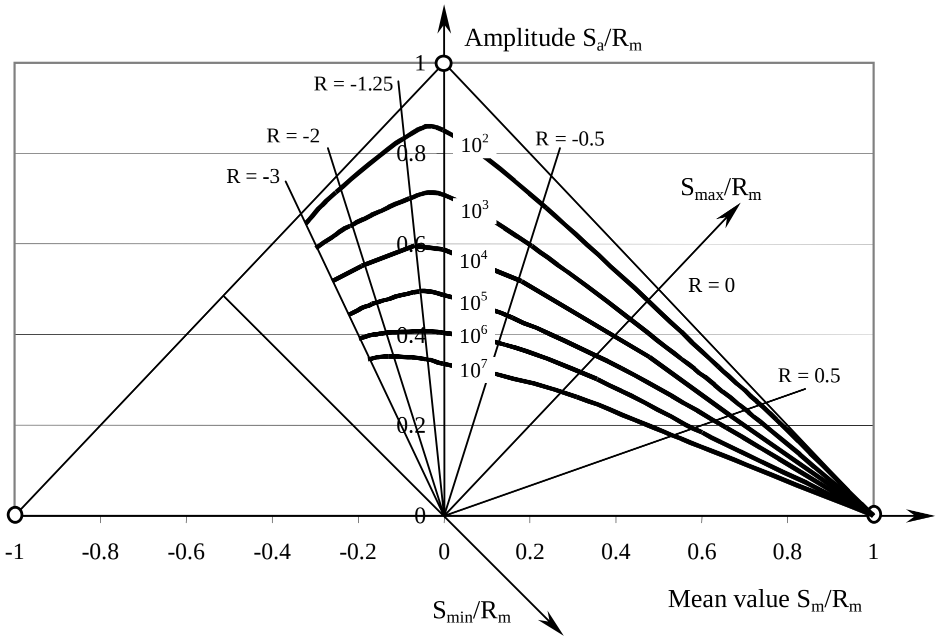

Fig. 5. Two-parametric fatigue characteristics determined experimentally [4]

3. Two-parametric fatigue characteristics

The experiment, conducted in static and variable load conditions, enabled us to determine parameters used in equation describing two-parametric characteristics. On the base of the slope in equation describing fatigue life diagram for R = -1 the value of the index exponent m_0 = 12,33 has been determined. The fatigue limit has been accepted as R_{-1} = 274 MPa for the number of cycles $N_0 = 10^6$. Material tensile strength R_m has been assumed according to the table 1.

The data mentioned enabled us to elaborate on two-parametric fatigue characteristics (fig.6) illustrated in the co-ordinate system S_m/R_m–S_a/R_m, similarly with characteristics determined experimentally.

Contour lines corresponding to specified number of cycles in range from 10^2 to 10^7 have been described by equation (7)

$$\frac{S_a}{678} = \frac{274}{678}\left(\frac{10^6}{N}\right)^{\frac{1}{12,33}}\left(1 - \frac{S_m}{678}\right). \tag{7}$$

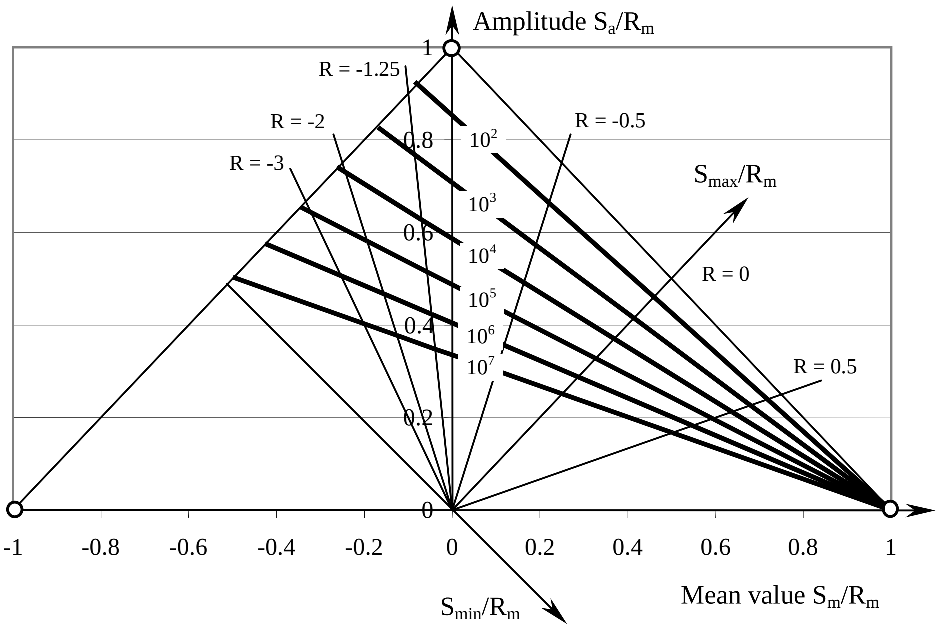

Fig. 6. Two-parametric fatigue characteristics elaborated on the base of the conception of Goodman

Two-parametric characteristics illustrated in the figures 5 and 6 differ from each other, especially in the range for negative average value. The Values of the differences are to be shown in the next point.

4. Comparison between research results

The conducted empirical examinations as well as calculation results enabled us to determine two-parametric models of fatigue properties of S355J0 steel. Characteristics illustrated by the figures 5 and 6 include contour lines corresponding to the defined number of cycles. Knowing the location of the contour lines, one can read the value of amplitude and average value which characterizes the point of intersection between the contour line and the line for cycles of stable value of R coefficient. Reading the amplitude value for examination results $S_{a\ ex}$ and calculation results S_{ac}, the relative values of the differences δ can be calculated according to the formula (8)

$$\delta = \frac{S_{ac} - S_{a\ ex}}{S_{a\ ex}} \cdot 100\% \tag{8}$$

The calculated values of differences have been depicted on the bar chart (fig. 7).

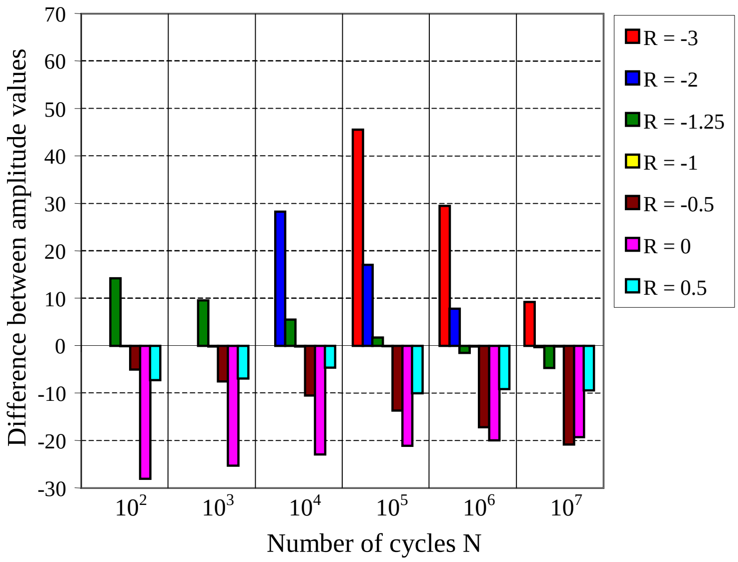

Fig. 7. Bar chart depicting the difference between amplitude values

For the range of the variability of coefficient R > -1 to R = 1 and the number of cycles from 10^2 to 10^7 the values of the amplitude which characterize the empirical examination results, were higher than the results determined on the base of two-parametric characteristics. For the cycle asymmetry coefficient R = 0,5 values of the differences range from –4,5% to -9,4%. For the coefficient R = 0 divergences from the range –19,2% to -28% have been achieved. In case of coefficient R = -0,5 the values of the differences were increasing with the change of the number of cycles from –4,9% to –20,7%.

In case of cycle asymmetry coefficient R = -1 the results of experiments and calculations are identical. It is connected to the fact that the elaborated characteristics is based on the fatigue properties determined in conditions of the oscillating loads.

In case of coefficient R = -1,25 for the number of cycles from 10^2 to 10^5 the results of calculations were higher than the examination results. The value of the differences for the mentioned range has been changing from 14,2% to 1,8%. For the number of cycles 10^6 and 10^7 respectively the values –1,5% and –4,6% have been gained.

For the coefficient R = -2 for the number of cycles from 10^4 to 10^7 the gradual decline of the value of the differences between amplitudes from 28,3% to –0,2% has been achieved. In case of the number of cycles 10^2 and 10^3 the maximal load value was higher than the material tensile strength ($S_{max} > R_m$).

Achieved research results point out that the biggest differences have been achieved for the coefficient R = -3. On account of the condition $S_{max} > R_m$ the analyzed range of the number of cycles has been limited to values 10^5, 10^6 and 10^7. For the range of the number of cycles mentioned above the following values of the amplitude differences have been achieved: 45,5%, 29,6% and 9,2%.

5. Summary

The comparison between the empirical examination results and the calculation results points out a decline of the amplitude values in the first quarter of the co-ordinate system (for the range of the variability of the cycle asymmetry coefficient $-1 < R < 1$) for all numerals from 10^2 to 10^7. In the second quarter for the mentioned values of the number of cycles the distinct increase of the amplitudes occurred, especially for coefficient $R = -2$ and $R = -3$. The occurring differences are connected to the shape of the contour lines on the analyzed diagrams, which result from the applied simplification in the two-parametric fatigue characteristics.

Application of the two-parametric fatigue characteristics based on the conception of Goodman during initial calculations, e.g. in machine design process, can be explained by the lack of precise data about cyclic properties of the material and about service load.

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