



## THE SVD METHOD APPLICABILITY IN COMBUSTION ENGINE DIAGNOSTICS INVESTIGATION

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### **Abstract**

*Conducted researches of combustion engine depended on delimitations of vibroacoustics measures for fit engine and comparison of this measures, with measures appointed for damaged engine (e.g. damaged injector) and accomplishment the assessment of received results influence on engine state. The present research use vibration methods to recognize the technical state of the engine and SVD method (Singular Value Decomposition) was used for results validation.*

**Keywords:** *diagnostic inference, singular value decomposition, combustion engine*

### **1. Introduction**

Combustion engines technical state diagnostic investigations with use of vibration are very difficult and only few proposed methods could have wider technical use in diagnostics. The paper contains application of operational modal analysis and SVD methods focused to a combustion engine, identify the technical state. The combustion engine No. 138C.2.048 with 1.4l. swept capacity, power 55 kW / 75 KM, generally applied to Fiat Uno 75i.e., is the investigation object. The engine is situated in the investigative laboratory of combustion engines in UTP Bydgoszcz. It makes possible to introduce generated vibration signals as well as the investigation of his adjustment influence on the combustion engine vibration signals change.

New approach to investigation of combustion engine technical state is vibroacoustics as a diagnostic tool. The main idea of vibroacoustics investigation is following the changes of vibration estimators as a result of engine maladjustment, waste, damages or its failure is the main idea of operational modal analysis. The present research use vibration methods as Operational Modal Analysis and SVD to recognized the technical state of the combustion engine. Operational modal is the name for the technique to do modal analysis on operational data - cases where we do not excite the structure artificially but just allow the natural operating loads to excite the structure.

As a validation of investigation results in this paper is shown presentation of Singular Value Decomposition (SVD) method. The SVD method is the appropriate tool for analysing a mapping from one vector space into another vector space, possibly with a different dimension.

### **2. Model of diagnostics signal generation**

The investigations object was a combustion engine no. 138C.2.048 applied to the Fiat and Lancia cars. Basis on this system during investigations was created model of diagnostics signal generation [2,3,5]. The proposed model of combustion engine diagnostic signal generation is shown on Figure 1.

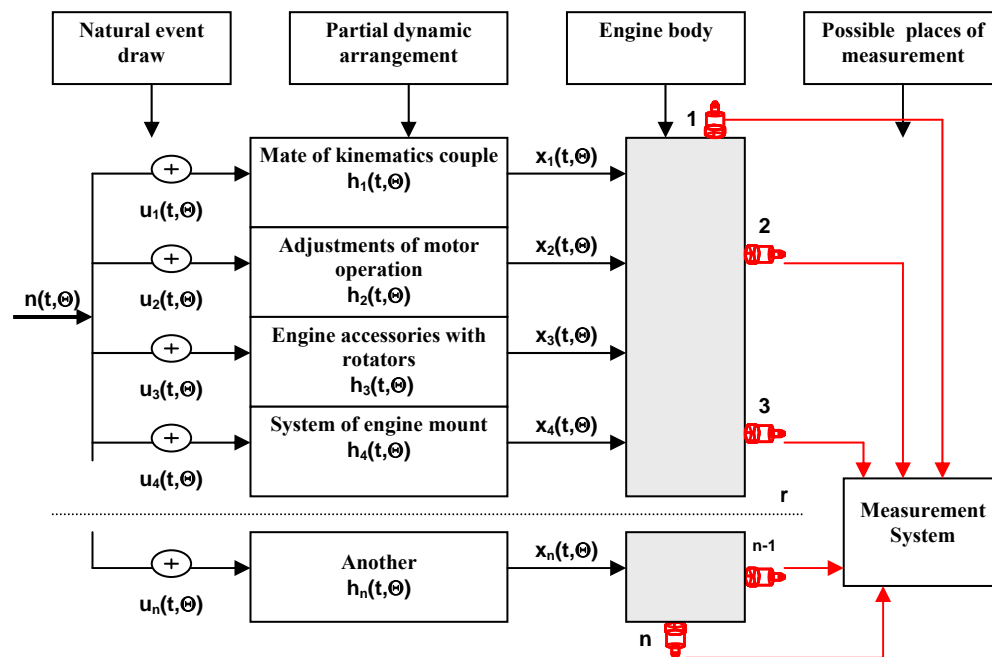


Fig.1. Combustion engine diagnostics signal generation model[2,3,5,7]

The received signals in the any point of engine body are the sum of the answer at all elementary events  $u_n(t, \Theta)$ , outputs in individual partial dynamic arrangements with the pulse function of input  $h_n(t, \Theta)$ . These influences after passing by proper dynamic arrangements are sum up on the engine body, on chosen points was measured by the vibration transducers. As a result of conducted measurements output signals was used to estimation. By  $n(t, \Theta)$  was marked accidental influence stepping out from presence of dynamic micro effects such as friction [2,3,4,5].

Conducted investigations of combustion engine depended on delimitations of vibroacoustics measures for fit engine and comparison them with measures appointed for damaged engine (eg. damaged injector) and accomplishment the assessment of received results influence on engine state by operational modal analysis methods.

### 3. Operational modal analysis and vibroacoustics methods investigations results

In this paper Least Squares Complex Exponential method was used to determine the modal model parameters, by which the correlation function is approximated by the sum of exponentially decaying harmonic functions. This method, applied to impulse response of system is a well-known method in modal analysis yielding global estimators of system poles – the root of the transfer function denominator. The modal model of combustion engine was created for put dynamic states on the basis of received measuring results. During investigations have been done vibroacoustics measures for fit engine and for engine with damaged injector and spark plug for each cylinder [2,3,4]. As a results of engine modal tests was created the stabilisation diagrams for each technical state. Basis on the stabilisation diagrams was created the modal model includes modal order, natural frequency and damping [2,3,4,7]. Figure 2 display the window with the stabilization diagram of engine in fit state.

Table 1 present the results of modal investigations – modal model for put engine technical states. Basis on modal model parameters and estimators of vibroacoustics signal received during investigations in table 2 was shown the main observation matrix for engine performance. The final observation matrix of engine performance described 13 symptoms. The matrix have six modal symptoms ( $\omega_1$  - first natural frequency, rząd1 - modal order of first natural frequency,  $\xi_1$  - modal damping coefficient of first natural frequency,  $\omega_2$  - second natural frequency, rząd2 - modal order of second natural frequency,  $\xi_2$  - modal damping coefficient of second natural frequency) and the last seven symptoms are vibration process ( $H(f)$  – real part of transfer function,  $H(f)L$  – imagine part of transfer function,  $\gamma_{xy}^2$  – coherence function,  $A_{RMS(t)}$  – Root Mean Square in time domain,  $\beta_{kurt}$  – Kurtosis,  $C_s$  - Crest factor, I - Impulse factor). [4]

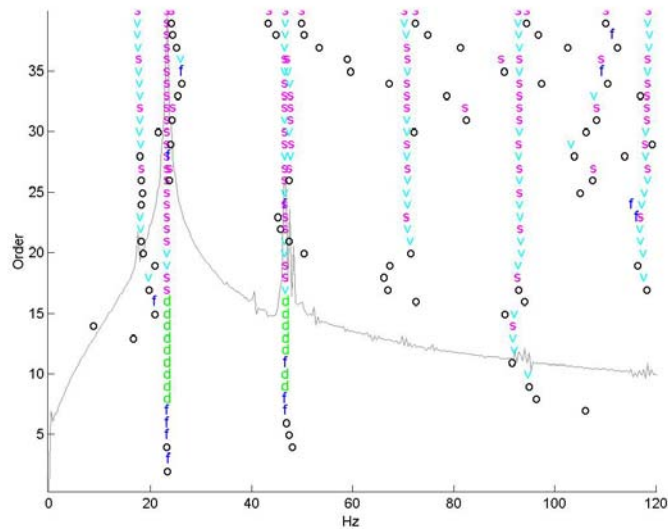


Fig. 2. Operational Modal Analysis stabilization diagram of investigated engine in fit technical state: *s* – stable pole, *v* – the frequency of vibration and modal vector is stabilized, *d* – the frequency of vibration and the stifling is stable, *f* – only the frequency of the vibration is stable, *o* – the pole is unstable [6]

Tab. 1. Parameters of modal model received during investigations for put of 9 technical states of combustion engine:  $\omega$  – is the free vibration frequency, Order – order of the model,  $\xi$  – is the modal damping coefficient

Technical state	Parameters of modal model						
1- fit engine	$\omega$ (Hz)	23.27	46.96				
	Order	18	17				
	$\xi$ (%)	0.67	1.34				
2 - damaged injector on 4 <sup>th</sup> cylinder	$\omega$ (Hz)	16.62	21.82	38.09			
	Order	20	19	20			
	$\xi$ (%)	4.08	0.68	4.33			
3 - damaged injector on 3 <sup>th</sup> cylinder	$\omega$ (Hz)	17.81	22.57	27.94	39.74		
	Order	19	17	28	18		
	$\xi$ (%)	4.81	1.47	4.82	2.00		
4 - damaged injector on 2 <sup>th</sup> cylinder	$\omega$ (Hz)	16.33	22.13	27.99	38.59	49.13	
	Order	29	18	31	17	23	
	$\xi$ (%)	7.05	3.11	7.13	4.09	5.51	
5 - damaged injector on 1 <sup>th</sup> cylinder	$\omega$ (Hz)	17.36	22.82	29.24	40.03	50.87	91.64
	Order	23	19	34	27	25	16
	$\xi$ (%)	6.69	1.18	6.17	3.21	2.90	2.24
6 - damaged spark plug on 4 <sup>th</sup> cylinder	$\omega$ (Hz)	20.13	22.05	39.08	49.60		
	Order	18	24	23	23		
	$\xi$ (%)	1.93	7.93	6.98	4.80		
7 - damaged spark plug on 3 <sup>th</sup> cylinder	$\omega$ (Hz)	16.52	20.70	25.51	41.43	47.43	
	Order	19	17	29	27	26	
	$\xi$ (%)	10.11	2.48	6.73	4.61	4.07	

8 - damaged spark plug on 2 <sup>th</sup> cylinder	$\omega$ (Hz)	16.50	21.89	37.74	46.34		
	Order	23	18	24	20		
	$\xi$ (%)	11.47	1.21	6.33	1.78		
9 - damaged spark plug on 1 <sup>th</sup> cylinder	$\omega$ (Hz)	17.59	23.58	45.93			
	Order	25	17	18			
	$\xi$ (%)	3.83	0.71	1.27			

Tab. 2. The main observation matrix for engine performance

State	$\omega_1$	rząd 1	$\xi_1$	$\omega_2$	rząd 2	$\xi_2$	H(f)	H(f)L	$\gamma^2_{xy}$	$A_{RMS(t)}$	$\beta_{kurt}$	$C_s$	I
1	23,27	18	0,67	46,96	17	1,34	68,56	-2,18	108,18	0,2177	1,5567	1,7239	1,9268
2	21,82	19	0,68	38,09	20	4,33	47,08	30,59	100,22	0,1392	1,8989	2,1204	2,4456
3	22,57	17	1,47	39,74	18	2,00	36,42	8,84	104,40	0,2040	1,7532	1,8656	2,1198
4	22,13	18	3,11	38,59	17	4,09	31,34	-15,28	91,11	0,1769	1,9245	2,0762	2,3992
5	22,82	19	1,18	40,03	27	3,21	46,16	-75,94	101,15	0,2312	1,7148	2,0982	2,3673
6	20,13	18	1,93	39,08	23	6,98	42,24	-8,50	83,73	0,1702	2,5205	2,8157	3,3986
7	20,70	17	2,48	41,43	27	4,61	38,76	22,77	82,34	0,1363	2,2943	2,2926	2,7564
8	21,89	18	1,28	46,34	20	1,78	40,51	-19,29	83,29	0,1726	1,7401	2,0176	2,2929
9	23,58	17	0,71	45,93	18	1,27	19,45	-23,34	99,63	0,1904	1,6144	1,8260	2,0527

#### 4. Results validation

As a results of investigation in this paper is shown presentation of singular value decomposition (SVD) method usage for combustion engine technical state results validation. Figure 3 display the window with the SVD module, that was used for analysis.

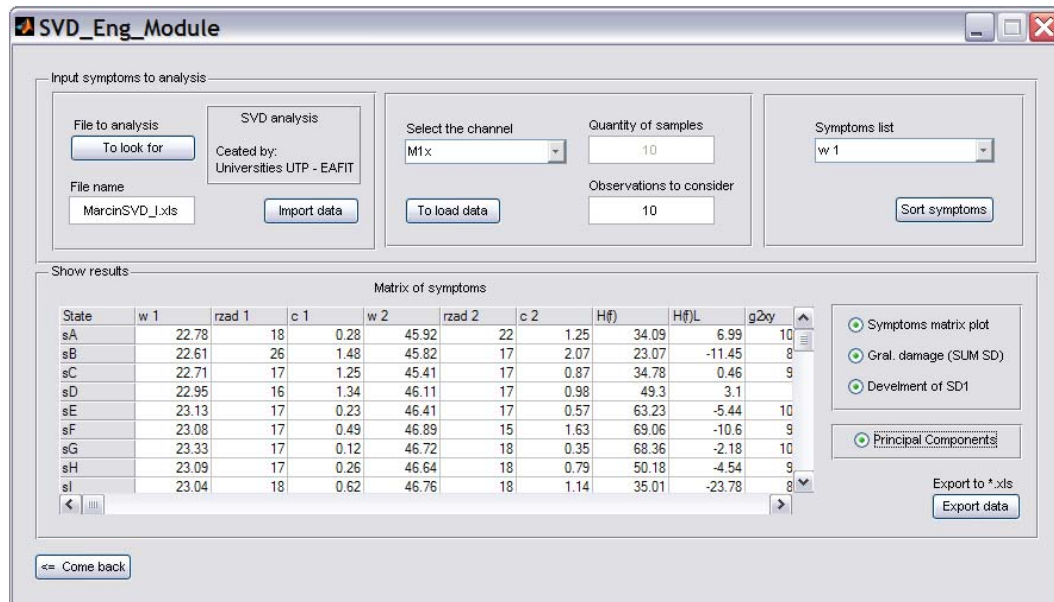


Fig. 3. SVD module window

The SVD method is the appropriate tool for analyzing a mapping from one vector space into another vector space, possibly with a different dimension [1]. The first step of SVD procedure is to centre and normalization all symptoms given in table two relative to the initial value of symptom vector. The observation matrix of transformate symptoms relative to the initial value is shown on Figure 4.

The second step of SVD procedure is to calculate the first generalized damage and evolution of damage [1]. Graphical interpretation of this calculations is given in Figure 5.

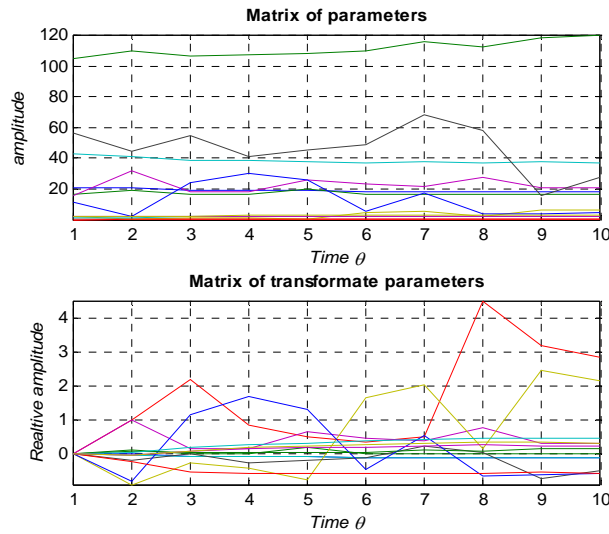


Fig.4. Matrix of symptoms before and after transformation

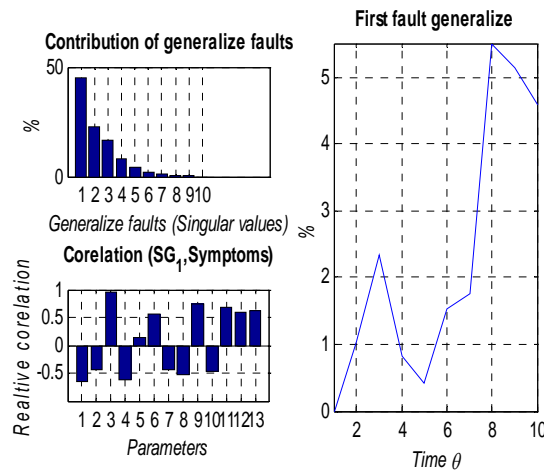


Fig.5. Graphical interpretation of first generalized damage and evolution of damage

The graphical results of SVD methods for put engine technical states are given in Figure 6,7 and Figure 8.

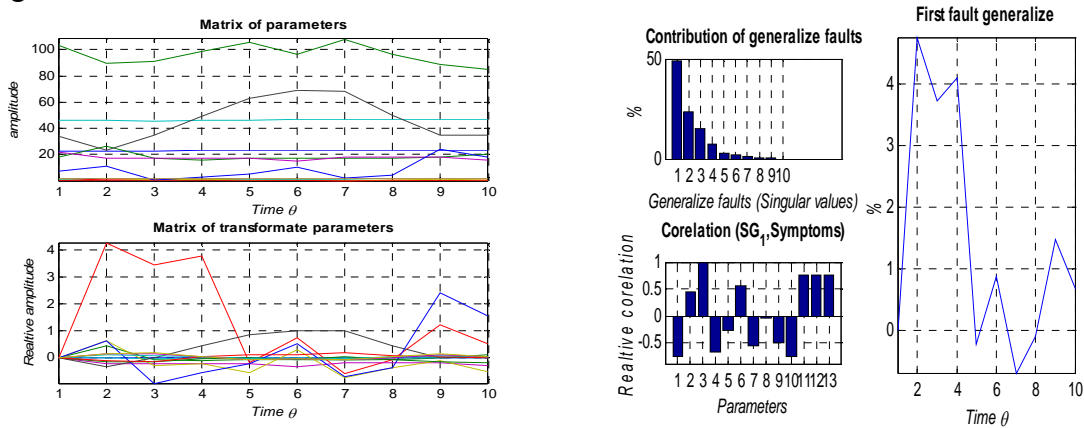


Fig.6. Matrix of symptoms before and after transformation and graphical interpretation of first generalized damage and evolution of damage for fit engine

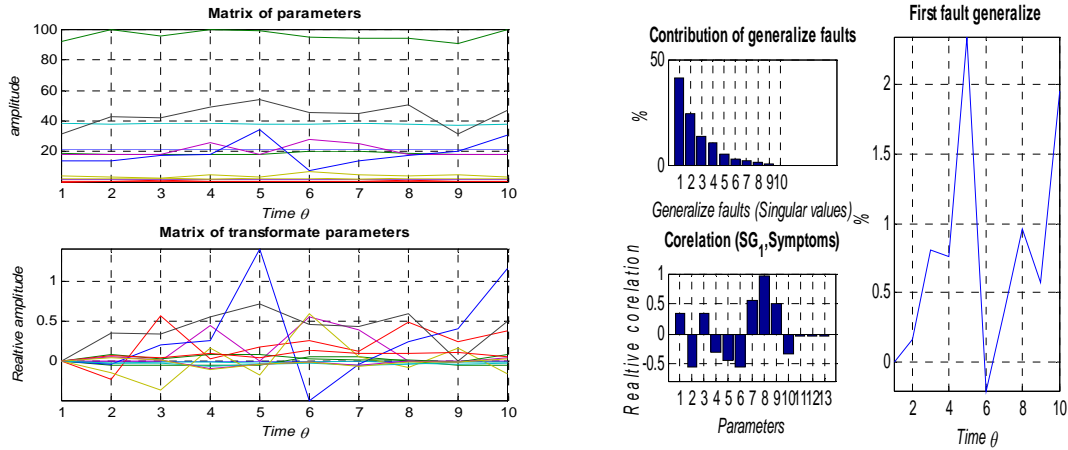


Fig.7. Matrix of symptoms before and after transformation and graphical interpretation of first generalized damage and evolution of damage for engine with damaged injector on 4<sup>th</sup> cylinder

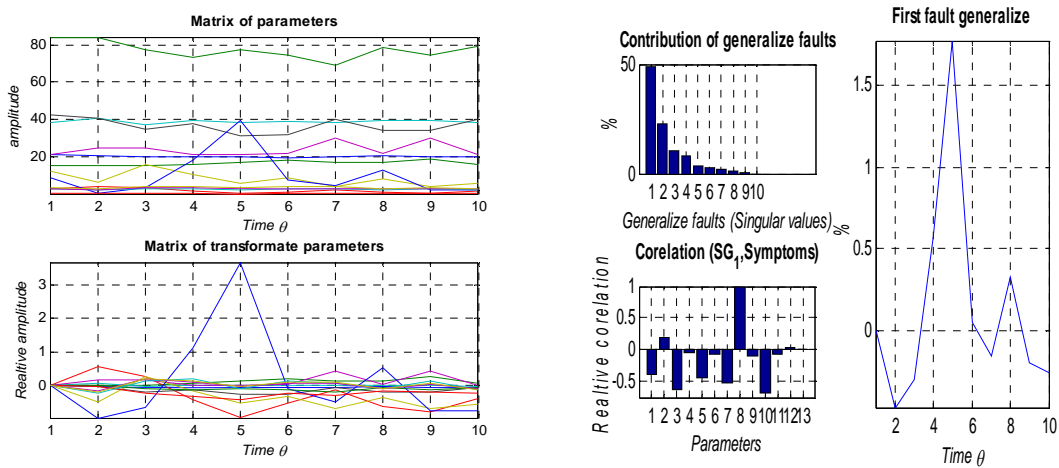


Fig.8. Matrix of symptoms before and after transformation and graphical interpretation of first generalized damage and evolution of damage for engine with damaged spark plug on 4<sup>th</sup> cylinder

In SVD procedure as a result we got a line up of five best symptoms given in Table 3 that are most important in description of set technical state of combustion engine.

Tab. 3. Results of SVD method with five best symptoms for set of engine technical state

State	1 symptom	2 symptom	3 symptom	4 symptom	5 symptom
1	$\xi_1$	$\omega_1$	$A_{RMS(t)}$	$\beta_{kurt}$	$C_s$
2	H(f)L	rzqd 1	$\xi_2$	H(f)	$\gamma_{xy}^2$
3	$\xi_1$	H(f)L	rzqd <sub>1</sub>	$\omega_1$	H(f)
4	rzqd <sub>2</sub>	H(f)L	$\omega_2$	$\xi_2$	$\xi_1$
5	$\xi_1$	rzqd <sub>1</sub>	$\beta_{kurt}$	$A_{RMS(t)}$	$\gamma_{xy}^2$
6	H(f)L	$\xi_1$	$A_{RMS(t)}$	H(f)	rzqd <sub>2</sub>
7	$\xi_1$	$C_s$	I	rzqd <sub>1</sub>	$\gamma_{xy}^2$
8	H(f)L	$\xi_1$	$\xi_2$	$\gamma_{xy}^2$	$\omega_1$
9	H(f)L	$\xi_1$	$\beta_{kurt}$	$C_s$	I

Making data analysis in SVD method as a result we got the lineup of symptoms together with the proportional description of given individual symptom of combustion engine technical state. At the end of SVD procedure at Figure 9 is displayed the contribution of the third Principal

Components (PC\*). The PC<sub>1</sub> is the first principal component of analyzed data, it described the direction of fault in the system and it take 56,44 % of importance degree of the symptoms. Thanks to SVD methods we could decide which symptom given in observation matrix is the best to recognize a set of combustion engine technical state [1].

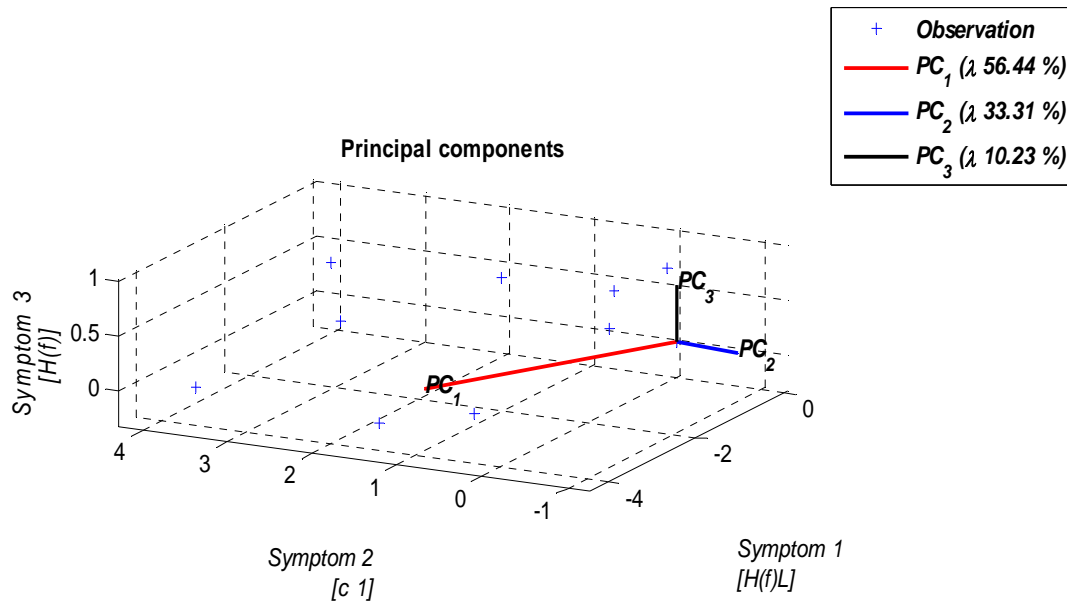


Fig.9. Contribution of the Principal Components

Relationships cause - consecutive expressing quantitative relation between studied variable symptoms results in this work were qualified using the function of the multiple regression. Basis on SVD results as a best symptoms in multiple regression were given:  $\omega_1$  – first natural frequency,  $\xi_1$  - modal damping coefficient of first natural frequency,  $\xi_2$  - modal damping coefficient of second natural frequency,  $H(f)L$  – imagine part of transfer function,  $\gamma^2_{xy}$  – coherence function. The equation of multiple regression is obtained in the form:

$$y = -1,44923\omega_1 - 0,61558\xi_1 - 0,35989\xi_2 - 0,06520H(f)L + 0,14424\gamma^2_{xy} + 35,9994, \quad (1)$$

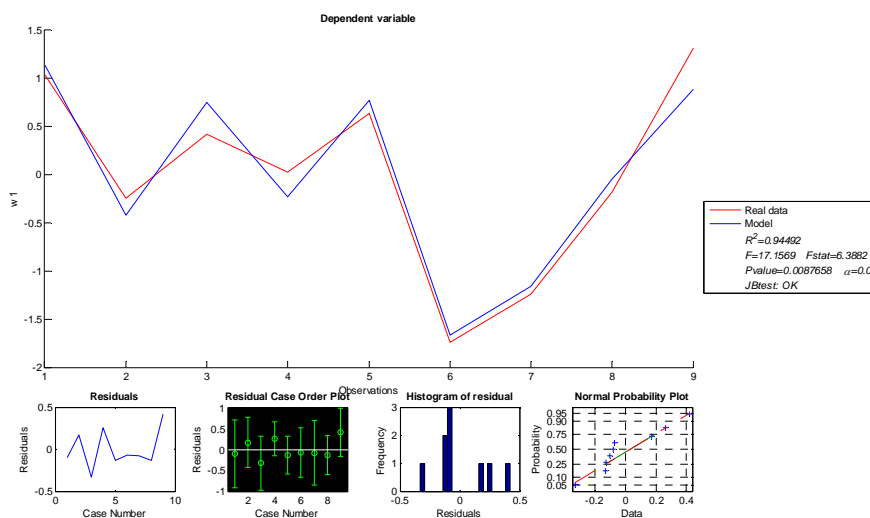


Fig. 10. Graphical interpretation of multiple regression for first dependent variable  $\omega_1$



Graphical interpretation of this calculations for first dependent variable  $\omega_1$  is given in figure 10. The red line present real data received during investigations, the blue line – estimated model for dependent variable.

## 5. Conclusion

Received in the experiment modal parameters and numerical estimators of vibroacoustics signal unambiguously show that the previously assumed conditions of the combustion engine's state reflect themselves in modal as well as other parameters characterising the vibrations and they are possible to be identified.

The use of the operational modal analysis in diagnostic investigations finds its use as one of many methods of marking the actual technical state of studied object. To complete the analysis process a SVD method and multiple regression were used. SVD methods marked most important symptom in description of engine technical state. On the basis of the results, it is possible to determine the actual technical state of an object of the same type by means of comparison of the achieved results with the model ones and assigning them to the particular model's state, which answers to a particular damage, or its loss, in the object.

The introduced in paper results of investigations are the part of realized investigative project and they do not describe wholes of the investigative question, only chosen aspects.

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