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INFLUENCE OF MESH MORPHOLOGY NEAR THE NOTCH ON PRECISION OF SCF DETERMINATION

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Abstract

The researches considered the problem of mesh forming for numerical analyses of notched elements. Stress concentration factor K_t was assumed as the characteristic value determined during the calculations. The calculations were performed for flat bars with opposite U-shaped notches and for round bars with V-shaped notches. Both analysed the notchs generally assumed as shallow notch $K_t \approx 1.7$ and sharp notch $K_t \approx 2.8$. Two-dimensional FEM linear elastic analyses were performed in the ANSYS software environment. For the purpose of the analyses, free and mapped meshes for coarse geometry and mapped meshes for modified geometry with one and two subareas were assumed. It has been revealed that precision of numerical calculations for stress concentration factor depends on morpholology of the mesh located near the notch. It has been revealed that free mesh enables to obtain a satisfying precision of the calculations. Introduction of division in formed geometry of the notch for sub-areas followed by their division according to the standard did not improve precision.

Keywords: *notch, local approach, stress concentration factor, finite element mesh*

1. Introduction

The procedure of determination of fatigue life for mesh elements with a notch assumes, in the first stage, estimation of damage level, assuming strain, stress or the value corresponding to the energy in the notch as a parameter. Damage level estimation can be performed locally and non- locally. The local approach determines damage at one point. In most cases, the stress on the notch root is the parameter. Its value is usually determined according to net nominal stress taking into account the stress concentration factor K_t. The non-local approach assumes that the selected parameter is determined in some sort of area near the notch root [3].

The simplicity of the description as well as wide experimental verification of the local approach in fatigue calculations of notched elements make the approach commonly used. Among the group of formulas representing the manner of description, the highest respect [8] was gained by Neuber theory (1) and strain energy density theory (2):

$$\frac{\sigma^2}{E} + \sigma\left(\frac{\sigma}{K'}\right)^{1/n'} = \frac{(K_t S)^2}{E}, \tag{1}$$

$$\frac{\sigma^2}{E} + 2\frac{1}{1+n'}\sigma\left(\frac{\sigma}{K'}\right)^{1/n'} = \frac{(K_t S)^2}{E}, \quad (2)$$

where:
σ - local stresses in the notch,
S – net nominal stresses,
E, n', K' – material parameters .

Experimental verification of the Neuber theory indicates a conservative character of the results gained with the use of the theory [1]. Juxtaposition of the researches results and calculations based on strain energy density theory indicates that the theory might underestimate stress values on the notch root [4]. In order to eliminate the above mentioned limitations, various modifications of notation are proposed (1) and (2). Example of this approach is introduction of power density of stresses parameter [4] on the member of an equation describing local stresses, maintaining unchanged form of unit based on nominal stresses. Other method, to decreases conservatism of Neuber theory, is modification of its notation on the site of nominal stresses via use of the notch factor K_f in it. The factor includes minor plastic changes in the notch also occurring when nominal stresses S are much lower than yield strength Sy [9]. The approach, in most cases, does not result in elimination of factor K_t from calculations. Apart from comfortable cases when the value of the factor K_f was determined directly during researches, its value is usually set indirectly depending on Peterson [7] or Neuber [8] theories based on K_t.

Having the above in mind, a major importance of stress concentration factor K_t for fatigue calculations of elements with notch performed on the local approach basis is revealed. Determination of the value of stress concentration factor K_t for various notch geometries and load types is performed via experiment and calculations with the use of analytic and numerical methods [1]. For the purposes of calculations, for stress concentration factor K_t regarding various types and sizes of notches, they were made as diagrams or described as simplified dependences [7]. Assumed, due to their ease in calculation, simplifications result that the value of K_t factor might involve even 10% error [5]. It provides a significant influence on the precision of the entire fatigue calculations as K_t factor occurs in second power in notations (1), (2) and derivatives.

Some sort of solution to the problem of precision is provided by determination of K_t factor value based on approximate dependencies [5]. Limitations of the approach results from significant complexity of notations depended on the type or sizes of notches, moreover, due to the fact that it indicates various precision for individual methods of notch load [6]. The use of numerical methods not bearing such limitations, for engineering calculations as well as for scientific researches, seems to be a natural solution. Even though, based on FEM commercial software introduces more efficient algorithms for mesh generation and much stronger solvers, control over precision of calculations dependent on dicretization error still seems to be an essential issue. First of all, the error depends on the size and morphology of the notch and the size and finite element order. Basic features of mesh morphology is its density near the notch and its rarefaction away from the notch. Such structure of the notch enables to control the size of computer resources need for analyses and to control the time for calculations. Other essential feature of mesh morphology is the shape of finite elements located very near to the notch root and its influence on precision of determining of stresses on the notch roots. Presence of triangle and irregular elements in the area, created via automatic division of notch geometry, might lower precision of calculations.

2. Calculations and its conditions

The researches considered the problem of mesh forming for numerical analyses of elements with notch. Researches were performed in plane strain state for, assumed according to the work [2], round bars with V-shaped notches (fig. 1a) and in plane stress state for flat bars with opposite

U-shaped notches (fig. 1b). Both analysed the notch generally assumed as shallow notch K_t≈1.7 and sharp notch K_t≈2. Geometrical dimensions of both bars geometry presented on fig. 1 presented in the table 1.

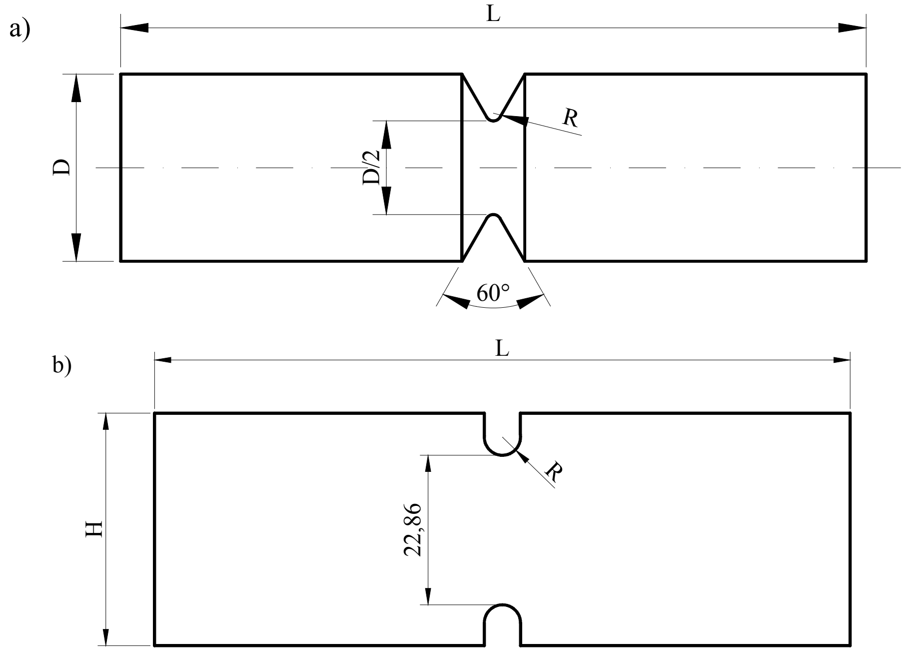


Fig. 1. Bar samples for analyses; a) round bar with V-shaped notches, b) flat bar with opposite U-shaped notches.

Due to significant length of the round bar, on the stage of preliminary calculations, speculations considering what length of the bar could be omitted during analyses has been made. The analyses enabled to reduce the model and quickening of calculations without any loss of precision. A characteristic, transverse dimension of the bar has been assumed as a reference length: *D* diameter for round bar or height *H* for the flat bar (Fig. 1).

Tab. 1. Nominal dimensions of analysed bars

	V-shaped notch			U-shaped notch		
	L [mm]	D [mm]	R [mm]	L [mm]	H [mm]	R [mm]
shallow notch, $K_t \approx 1.7$	107.95	12.7	0.529	78	41.12	9.128
sharp notch, $K_t \approx 2.8$	107.95	12.7	1.588	78	35.56	2.778

FEM linear elastic analyses were made in the environment of ANSYS software. They were of two-dimensional character, what in the case of round bars lead to analysis of axisymmetric issue. The analyses used tetragonal finite elements with the second order shape function. Due to dual symmetry of both, geometrical shape and boundary conditions, the analysis assumed the quarter of the bar sample. Rejection of the part of the bar sample located on the second side of the longitudinal surface and transverse axis or symmetry surface which depended on bar sample geometry, has been adequately taken into account properly defining symmetric boundary conditions on the division edges.

Changes in precision of calculations depended on the length of the bar normalised with transverse characteristic dimension presented on Fig. 2. Arrangement of calculation error depended mainly on bar length and whether analyses were performed in plane stress state or plane strain state. It only slightly depended on stress concentration factor. In plane strain state (for round bars) it was four times lower than in flat stress state (for round bars). In both cases, the error value decreased practically to zero when the length being formed exceeded twice the value of transverse characteristic dimension of the bar. On the above grounds, for the analyses of the round bar, according to designations on Fig. 2, the total length L=25.4mm was used and for the flat bar with shallow notch L=78mm, and with sharp notch L=71.2mm.

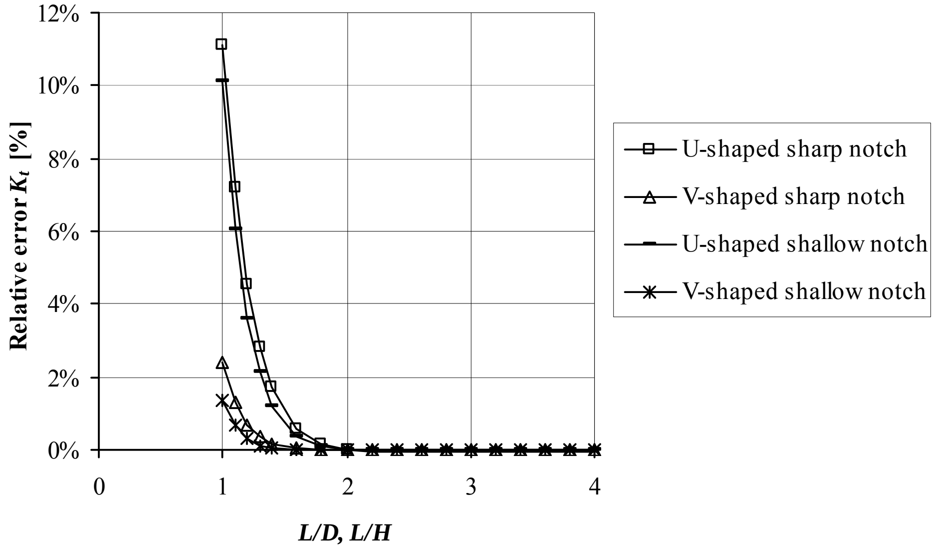


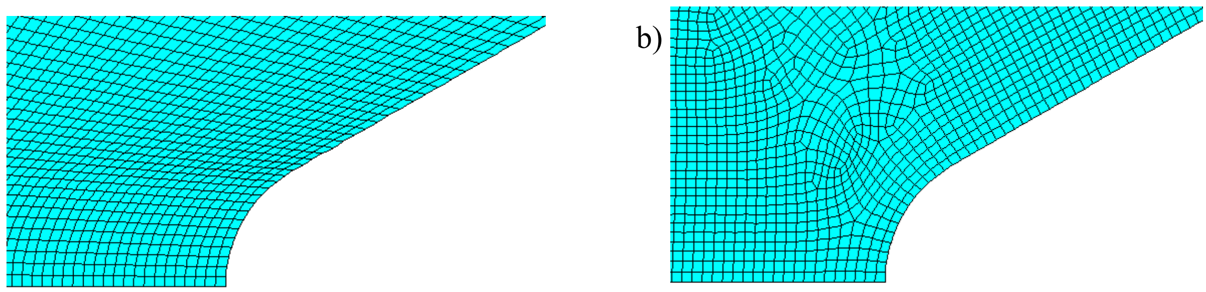
Fig. 2.Change of calculation precision K_t depended on the length L of the bar.

Taking into consideration the guidelines of the analysis of which results are presented on fig. 2 and the assumption to perform calculations for the quarter of the bar, calculation length of L_o=L/2 has been assumed as characteristic dimension for that length. For the analyses of the round bar calculation length of L_o=12.74mm was used and for flat bar with shallow notch L_o=39mm; with sharp notch L_o=35.6mm.

3. Calculations

Calculations have been performed for sharp notches $K_t \approx 2.8$ as for both types of bars and with the use meshes formed through four methods. Examples of creations of such meshes for V-shaped notch with the angle of β=60° are presented for the coarse geometry on Fig. 3a and b, and for geometry with sub-areas indicated near notch on Fig. 3c and d.

a)



b)

c) d)

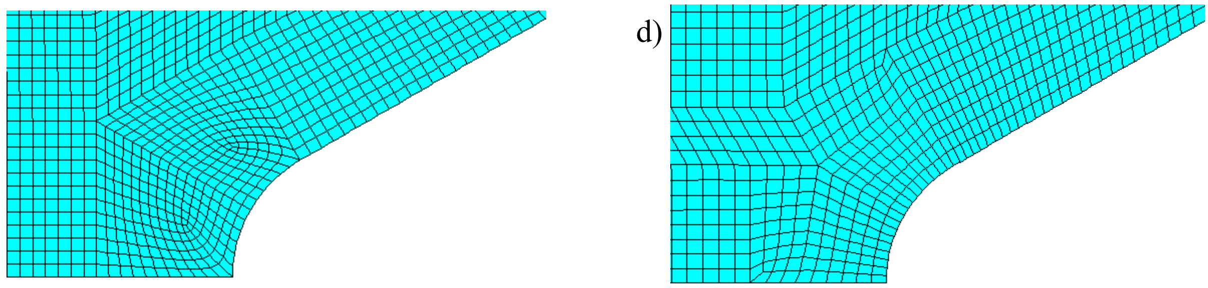


Fig. 3. Meshes for analyses differentiated with respect to the way of forming: a) mapped mesh, b) free mesh, c) mapped mesh with one sub-area, d) mapped mesh with two sub-areas.

Meshes for analyses were created of finite elements with second order shape function, tetragonal 8-node and triangular 6-node. Mapped mesh Fig. 3a has been assumed as a reference. Due to specific character of its creation, degeneration of the shape of the finite element comes along with decrease of the notch angle. For the 0° of the notch angle it is impossible to create mesh with the use of the mapped method. The above mentioned limitations do not concern free mesh (Fig. 3b). The method enables to create regular elements on the edges of the areas describing the notch but in some distance from the notch, the algorithm of automatic division generates elements of disunified size and shape. In order to employ the advantages of the mapped method for meshes creation and to avoid its limitation, a division on sub-area of formed geometry near the notch was proposed. Two versions of such solution have been agreed. The first includes one sub-area (Fig. 3c) in the way that its side comprise arch described via notch root radius. The second version includes two sub-areas (Fig. 3d) so each comprises half on the arch described via notch root radius.

4. Results

Calculation results for coarse geometry are presented on Fig. 4, and Fig. 5 for modified. Calculations were performed for various sizes of finite elements. Along with the decrease of size of the elements, the number of node increased which illustrates in increase of DOF for the problem. Stresses determined during the analyses were calculated into values of stress concentration factor K_t.

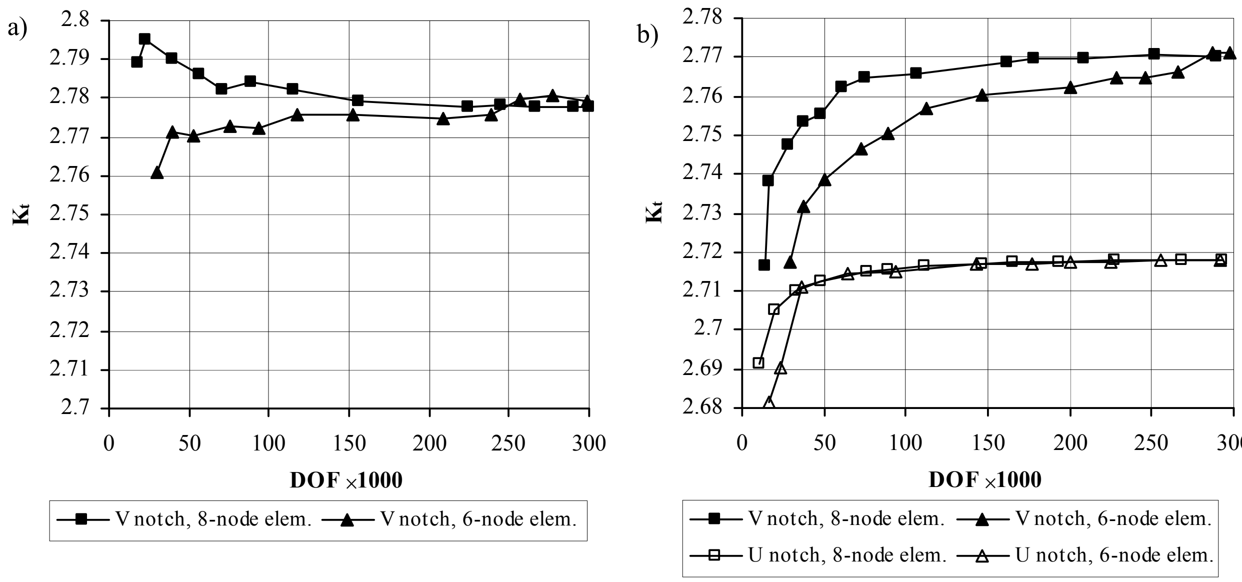


Fig. 4. Result of analyses for: a) mapped meshes, b) free meshes

Results analyses for coarse geometry (Fig. 4b) indicates that the number of nodes in finite element influences more on the precision in plane strain state than in plane stress state. In plane stress state both types of elements allow to obtain a satisfying precision even at approx. 100k DOF. For V-shaped sharp notches mapped meshes enabled to obtain *Kt*=2.78 at approx. 150k DOF, for free meshes a slightly lower value has been obtained *Kt*=2.77 at 170k DOF. Introduction of sub-areas near the notch resulted in decreased precision. In case of one sub-area, the result of *Kt* below 2.76 at 250k DOF has been obtained. For two sub-areas, the value of *Kt* approached 2.77 but only at approx. 300k DOF has been obtained.

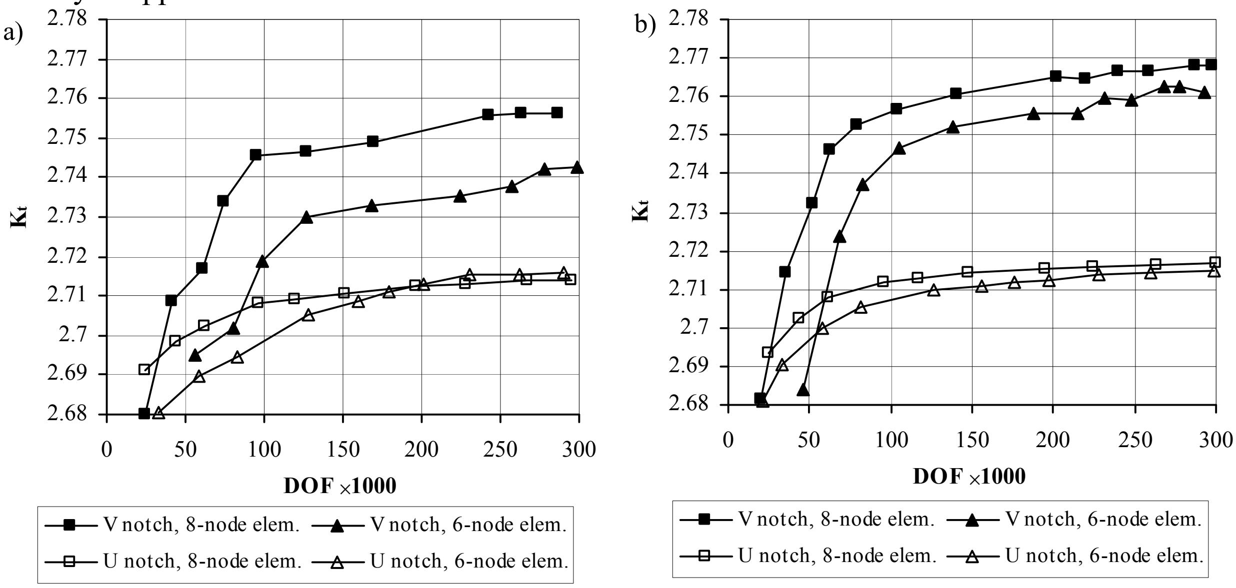


Fig. 5. Result of analyses for mapped meshes: a) with one sub-area; b) with two sub-areas

5. Conlusions

The precision of numerical calculations of stress concentration factor depends on the morphology of mesh near the notch. Mapped mesh enables to obtain the highest level of precision. Slightly lower level, but with the higher number of DOF, can be obtained with the use of free mesh. Introduction of notch geometry division on sub-areas followed by mapped meshing did not improve the precision of calculations. 8-node elements allow to obtain higher precision of stress concentration factor with lower number of DOF. The result of the work enable to indicate not only the optimal structure of mesh but also to indicate the size of finite element allowing to obtain high precision of calculations with controlled number of DOF.

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