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ANALYSIS OF FRAME STABILITY AS SAFETY REQUIREMENT

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Abstract

This work is an analysis of an accident that occurred in a warehouse during loading of a new multi-level storing frame. The frame was designed in a professional design office with aid of computer program. It is of great importance to carry out checking procedures at various steps of the computerized design process. In this article two different methods were applied in order to calculate the critical buckling force. Then the results were compared. The Euler's and the Rayleigh's method yielded convergent results. The both methods proved that the critical buckling force would be exceeded if the frame was fully loaded. Since the frame began to incline when it was loaded only in 80%, other reasons of buckling must also be considered. Although we can't eliminate designer's mistake, it is more probable, that the buckling resistance of the frame was reduced by inappropriate operation of hydraulic stackers. The photographs show that the construction was so tightly loaded with palettes, that the overloading was the most probable cause of the catastrophe. The bending moment originated during the loading process could also reduce the buckling resistance of the construction.

Keywords: *frame stability, buckling, Euler method, Rayleigh's method*

1. Introduction

At present, development of computer technology and professional CAD/CAM codes the ability of applying appropriate design and simulation software is a constitutive part of engineering education. However, the practical use of this software requires a detailed training. A cursory study is not satisfactory and may lead to catastrophic consequences in operation of designed constructions. The analyzed case of collapsed multi-level frame for palettes storing shows that a stability analysis is an indispensable part of a design process. The majority of commercial civil engineering software includes modules of a stability analysis. Nevertheless, it is instructive to compare and evaluate most known and applied methods of stability and buckling analysis that may serve as a handy check of computerized design process. The other problem is an appropriate and strictly observed system of loading and unloading multi-level storing frames. It is essential for a safe life cycle operation of this type of construction to preserve the loading limits. When the maximum utilization of storing space of warehouses turns a main objective and substitutes a safety criterion, damage may be very extensive.

The inspiration for the present work was collapsing and the total destruction of a multi-level storing frame that was 16,44 m long, 9,60 m deep and 8,00 m high. The frame provided three levels for storing palettes. The construction consisted of u-channel bearing columns with tops connected by horizontal square section tubes. As it is shown in fig.1, the palettes were stored side by side without leaving any distance between them. Palettes stored with maximum concentration might cause an overloading of the construction. The other reason might be an inappropriate system of loading the construction with palettes. The loading and unloading of multi-level high storing frame was performed by telescopic hydraulic stackers. Operators controlled the process with help of camera and, in case of inattention; the forks of stacker might hit a bearing column and cause the loss of stability or deformation and local buckling of u-channel column.

In the time of the catastrophe the frame was loaded in 80% with palettes of 7700 N unitary weight. The process of frame destruction, since the moment of light noticeable inclination of the first external span until the complete destruction, lasted about two hours. In order to preserve the content of the palettes stays between the frame and the building's girders were installed. The static forces in the curved frame were so great, that the bearing columns began to crack. The enclosed picture (fig. 2.) shows the extension of destruction caused by buckling of bearing columns.

Fig. 1. Palettes stored in the multi-level frame

Fig. 2. Destroyed multi-level storing frame after collapsing

2. Definition of construction stability and its analysis

We consider a dynamic system described by set of regular differential equations:

$$\dot{\mathbf{y}} = \mathbf{f}(\mathbf{y}, t) \, . \tag{1}$$

If the vectorial function **f** does not depend in explicit way from time t, then the system is stationary. Otherwise the system is nonstationary. Accepting the partial solution $\tilde{y}(t)$ of the above-mentioned equation as equation of undisturbed motion, and the remaining solutions y(t) as disturbed motion equations, we observe the evolution of disturbances $y_i(t_0) - \tilde{y}_i(t_0)$, i = 1,…, m, for the initial moment t = t_0. For so defined solutions Liapunov (1892) introduced the following definition of stability:

Undisturbed motion $\tilde{y}(t)$ of system (1) we call stable in relation to variables $y_1, y_2, \ldots, y_m$ if for every ε > 0 exists δ > 0 such, that for every solution y(t) of the system (1) satisfying condition $\left\| y(t_0) - \tilde{y}(t_0) \right\| < \delta$, inequity $\left\| y(t) - \tilde{y}(t) \right\| < \varepsilon$ is valid for every $t \geq t_0$ [1].

According to the above-mentioned definition, small variations from the initial conditions remain finite in time for stable motion. Equations of disturbed motion we express by means of deviations $x_i(t) = y_i(t) - \tilde{y}_i(t)$. Substituting this equation to (1) and expanding the right side of the resulting equation into the Tylor series we receive the equation of disturbed motion in the vectorial form:

$$\dot{\mathbf{x}} = \mathbf{A}(t)\mathbf{x} + \eta(\mathbf{x}, t) \, , \tag{2}$$

where the coefficients $a_{ij}(t) = \partial f_i / \partial y_j$ are estimated for y=$\tilde{y}(t)$, and η_i are higher order derivatives from the Tylor expansion. Taking into account exclusively linear equation $\dot{\mathbf{x}} = \mathbf{A}(t)\mathbf{x}$ we carry out a linearization of equation of disturbed motion.

The linear analysis of stability called singular values method permits a determination of theoretical buckling resistance (the point of bifurcation) of the ideally linear elastic construction. However, imperfections and nonlinearities present in the majority of real constructions inflict, that buckling occurs before they achieve their theoretical buckling resistance. Therefore, in everyday engineering practice a nonlinear buckling analysis should be applied, which is available in professional engineering software. The finite element method (MES) is predominantly used in civil engineering software. The application of MES in stability analysis is clearly presented in [2].

However, the aim of this article is to present and compare methods that evaluate the correctness of computer procedures. The analyzed case of collapsed frame serves as an example.

3. Euler's method of buckling analysis

Stability of steel structures is the essential safety criterion during their design and life cycle operation. The research on the structure stability dates from 1744, when Euler published his work on bar's stability. The classical method of buckling analysis still bears his name. It is a simple method that assumes one of four buckling types. As it was noticed in the first paragraph the frame collapsed by inclination of the construction in the plane of figure 3. Since the tops of bearing columns were joined by horizontal square section tubes and the base of each column was fixed in the ground, we assume that in the analyzed case the first derivatives of deflection line equals zero at external points of each column. It is evident from fig. 3, that the buckling length of the column is the same as in the second Euler's buckling type i.e. the buckling coefficient μ=1. As forces in the analyzed column are applied between nodes, we predict that the buckling coefficient of the column should be even less than one (μ<1). Fig. 3 presents the sketch of the analyzed frame. In the case of the internal column four palettes of the same level rest their corner on the same column (see fig. 1.). Therefore we admit that the single column is loaded with the weight of one palette on each level.

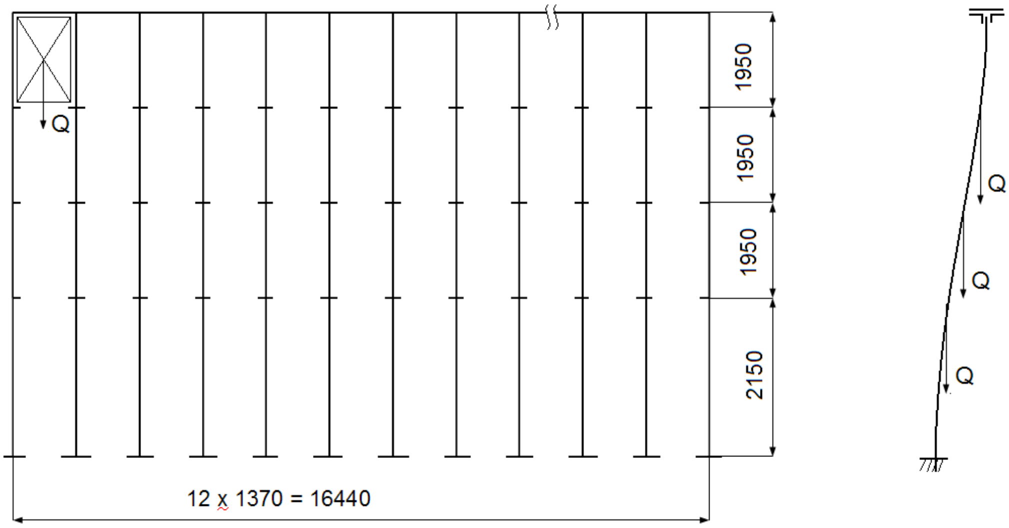

Fig. 3. The scheme of the multi-level storing frame and its buckling model

The geometrical data of u-shaped bearing columns are as follows: cross section area F = 3,48 cm^2, moment of inertia of the cross section I_x = 41,04 cm^4, radius of inertia i = 3,43 cm.

Stress of pure compression caused in the column by the loading 3Q amounts to:

$$\sigma_c = \frac{3Q}{F} = \frac{3 \cdot 0{,}0077}{0{,}000348} = 66{,}37 \quad MPa \, .$$

The stress caused by buckling is greater than originated by pure compression. The measure of the column strain is the critical force it can support without buckling. The value of this critical force is given by the subsequent formula:

$$P_{kr} = \frac{\pi^2 \cdot E \cdot I_x}{L_w^2} \, . \tag{3}$$

In this formula L_w is the buckling length of a column and it can be written as $L_w = L \cdot \mu$, where L is a real length of the column. For the adopted buckling type the value of buckling coefficient μ we calculate by means of the following formula:

$$\mu = \frac{1}{2 - \left(\frac{Q}{Q_t} \right)^{0,25}} = \frac{1}{2 - \left(\frac{7700}{23100} \right)^{0,25}} = 0{,}805 \, , \tag{4}$$

where Q is the weight of a single palette and Q_t is the total load of a column. Consequently the buckling length L_w = 0,805·8,0 = 6,44 m. The slenderness of a column is given by the formula:

$$s = \frac{L_w}{i} = \frac{644}{3,43} = 187,7\ . \tag{5}$$

The boundary slenderness we compute assuming the proportionality limit σ_{prop} = 235 MPa and elastic modulus E = $205 \cdot 10^3$ MPa:

$$s_{gr} = \pi \cdot \sqrt{\frac{E}{\sigma_{prop}}} = \pi \cdot \sqrt{\frac{205 \cdot 10^3}{235}} = 92,74\ . \tag{6}$$

Since the column slenderness is greater than boundary slenderness, we may use the Euler formula (3) to calculate the critical buckling force, which results P_{kr} = 19980 N. For the internal fully loaded column we have:

$$Q_t = 3Q = 23100 \text{ N} > P_{kr} = 19980 \text{ N}\ . \tag{7}$$

The stability analysis carried out by Euler's method reveals that the critical force P_{kr} for internal columns of the structure is exceeded. Treating the case more precisely, we notice that 38 external columns are loaded only with forces P = Q/2 on each level and 4 corner columns only with forces P = Q/4 on each level. This slightly increases the stiffness of the frame. The critical force calculated for the whole structure approximately equals the total load of the frame.

On the other hand the palettes, probably of varying weight, were stored side by side increasing the load of the single column. Applying the formula (4) was also an approximation. Concluding we may say, that the frame was in the state of boundary stability and any incorrectness in loading process might cause the buckling. Because of adopted approximations we also calculated the critical buckling force applying Rayleigh's method.

4. The Rayleigh's method of stability analysis

Our purpose is to check the stiffness of the frame against buckling. The analysed structure has in fact infinite number of degrees of freedom. To find the buckling criterion we may reduce the problem to a single column treated as a single degree of freedom (SDOF) system. This method is called Rayleigh's method and it assumes, that during free vibrations a column adopts a single shape function $\psi(x)$, changing only the amplitude A(t). So the function $y(x) = \psi(x) \cdot A(t)$ determines the position of all points of bearing column. Applying the boundary conditions of the deflection line of a column we assume the shape function in the following form:

$$\Psi(x) = 0,5\left(1 - \cos\frac{\pi x}{L}\right). \tag{8}$$

Applying the principle of virtual work we formulate the equation of motion of generalized SDOF system:

$$\delta W_E = \delta W_I\ , \tag{9}$$

where δW_E is the virtual external work done by external loadings on their corresponding displacements. It includes the work of excitation force on palette's mass (m_Q), the work of palette's inertia force on its displacement $\delta y(x_i)$ and the work of palette's weight on its displacement δe_i. δW is the virtual internal work done by internal bending moments M(x, t) on their corresponding changes in curvature δy"(x). The analysed model is presented in fig. 4.

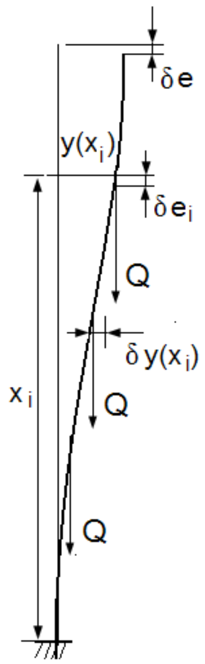

Fig. 4. The model of bearing column as a generalized SDOF system

In our model we neglect the weight of the frame elements supposing that they are much smaller than the weight of the palettes and we assume the constant stiffness of the columns EI. Now we may write the virtual external and internal work in the subsequent form:

$$\delta W_E = -\sum_i m_Q \ddot{y}(x_i,t)\cdot\delta y(x_i) + \sum_i m_Q \ddot{y}_{EX}(t)\cdot\delta y(x_i) + \sum_i Q_i\cdot\delta e_i \tag{10}$$

$$\delta W_I = \int_0^L M(x,t)\cdot\delta y''(x)dx\,. \tag{11}$$

In the generalized SDOF system we may write the following relations:

$$\begin{aligned} y(x,t) &= \Psi(x)A(t) & \dot{y}''(x,t) &= \Psi''(x)\dot{A}(t) \\ y'(x,t) &= \Psi'(x)A(t) & \delta y(x,t) &= \Psi(x)\delta A \\ y''(x,t) &= \Psi'(x)A(t) & \delta y'(x,t) &= \Psi'(x)\delta A \\ \ddot{y}(x,t) &= \Psi(x)\ddot{A}(t) & \delta y''(x,t) &= \Psi''(x)\delta A \end{aligned} \tag{12}$$

that are the consequence of adopting the shape function ψ(x).

According to the Euler-Bernouli hypothesis plane sections remain plane after deformation. We assume this hypothesis as well as the linear relation between damping stresses and strain velocity. Made assumptions lead to the following relation [3]:

$$M(x,t) = EI\left[y''(x,t) + c\cdot\dot{y}''(x,t)\right], \tag{13}$$

where c is a damping constant.

The displacements of buckling forces δe_i we calculate from the following relation [3]:

$$\delta e_i = \int_0^{x_i} y'(x,t)\delta y'(x)dx \,. \tag{14}$$

Finally the equation of motion of the column has the form:

$$m_G \ddot{A}(t) + c_G \dot{A}(t) + k_G A(t) = P_G(t) \,, \tag{15}$$

where:

$m_G = \sum_i m_Q [\Psi(x_i)]^2$ - generalized mass

$c_G = c \cdot EI \int_0^L [\Psi''(x)]^2 dx$ - generalized damping

$k_G = EI \int_0^L [\Psi''(x)]^2 dx - Q\left\{ \int_0^{x_1} [\Psi'(x)]^2 dx + \int_0^{x_2} [\Psi'(x)]^2 dx + \int_0^{x_3} [\Psi'(x)]^2 dx \right\}$ - generalized stiffness

$P_G(t) = -\ddot{y}_{EX}(t) \sum_i m_Q \cdot \Psi(x_i)$ - generalized effective load.

The column will lose its stability when the generalized stiffness equals zero, so the critical weight of one palette we calculate from the following relation:

$$k_G = 0 \Rightarrow Q_{cr} = \frac{EI \int_0^L [\Psi''(x)]^2 dx}{\int_0^{x_1} [\Psi'(x)]^2 dx + \int_0^{x_2} [\Psi'(x)]^2 dx + \int_0^{x_3} [\Psi'(x)]^2 dx} \,. \tag{16}$$

Taking into account that x_1 = L·6050/8000 = 0,75625·L, x_2 = 0,5125L, x_3 = 0,26875L we calculate the integrals of equation (16). Since we assumed the shape function in the form of (8), the results are as follows:

$$\begin{aligned}
&\int_0^L [\Psi''(x)]^2 dx = \int_0^L \frac{\pi^4}{4L^4}\left(\cos\frac{\pi x}{L}\right)^2 dx = \frac{\pi^4}{8L^3} \\
&\int_0^{x_1} [\Psi'(x)]^2 dx = \int_0^{0,75625\cdot L} \frac{\pi^2}{4L^2}\left(\sin\frac{\pi x}{L}\right)^2 dx = 0,45765\frac{\pi^2}{4L} \\
&\int_0^{x_2} [\Psi'(x)]^2 dx = \int_0^{0,5125\cdot L} \frac{\pi^2}{4L^2}\left(\sin\frac{\pi x}{L}\right)^2 dx = 0,26249\frac{\pi^2}{4L} \\
&\int_0^{x_3} [\Psi'(x)]^2 dx = \int_0^{0,26875\cdot L} \frac{\pi^2}{4L^2}\left(\sin\frac{\pi x}{L}\right)^2 dx = 0,2134\frac{\pi^2}{4L}
\end{aligned} \tag{17}$$

Substituting these results into (16) we receive:

$$Q_{cr} = \frac{EI \cdot \pi^4 \cdot 4L}{8L^3\pi^2(0{,}45765 + 0{,}26249 + 0{,}2134)} = 6942 \text{ N.} \qquad (18)$$

Since the average palette's weight is Q = 7700 N, the critical loading of the bearing column is exceeded. The total critical load of the column is 3Q = 20826 N. If we compare this result with (7), we note that the difference is about 3%. Unfortunately, both methods are approximate. In the case of Rayleigh's method we had to assume a shape function. If the admitted shape function were the true one, the calculated critical buckling force would be the exact one too. Every shape function other than the true one yields greater critical buckling force.

The shape function we admitted would be the true one if the loading was applied on the top of the column. Therefore, we may assume a sufficient correctness of our calculations.

5. Conclusions

The analysed multi-level storing frame was designed with aid of a professional computer program by experienced engineer. Therefore, the most probable cause of buckling of the construction was overloading (fig. 1.).

The buckling of the frame began then the construction was loaded in 80%. This suggests that during the loading of the frame, the telescopic hydraulic stackers could hit any bearing column and cause the local buckling.

Moving the palettes on the higher levels by hydraulic stackers also could produce a bending moment that contributes to the buckling.

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