



## STOCHASTIC MODELS OF EMISSION OF TOXIC COMPOUNDS IN MARINE ENGINES EXHAUSTS

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### **Abstract**

*Development of marine transport and projected increase of emission of toxic compounds formed when combusting marine fuels enforces, apart from the emission research, the need of modelling those pollutants dispersion their immission in coastal regions of city agglomerations. The area of Gdansk Bay, just like sea ports or coastal regions, is vulnerable to the effect of pollutants present in exhaust gases coming from industrial plants, power plants, vehicles and vessels. The last source of pollution concerns the vessels both in port areas and in the roads.*

*In order to determine the share of vessels in environmental pollution and counteract the harmful effects of toxic compounds in marine engine exhaust gases, it is necessary to know the emission values of these compounds from particular vessels. This is possible with the knowledge of their movement parameters, concentration values of particular compounds for these parameters and the atmospheric conditions.*

*The first step towards formulating the emission models of toxic compounds in marine engines exhaust gases is determination of mathematical models of vessels movement, which can be later used when formulating dispersion and immission models of those pollutants.*

*In the paper there were presented conditions concerning modelling the emission of toxic compounds in vessels exhausts as well as the possibilities of using the queuing theory and Monte Carlo methods in order to formulate simulated model of vessels movement in a given area.*

### **1. Introduction**

Emission  $E$  and intensity (mainly mass) of pollutants dispersed in atmospheric air – immission  $I$ , in the urban areas are the subject of intensive research conducted in many research institutions in Poland and abroad. The relation between emission of toxic compounds in exhausts and their immission can be written down as follows [1]:

$$I(t) = \mathfrak{N}[E(t)], \quad (1)$$

where  $\mathfrak{N}$  means a mathematical operator (e.g. functional).

The intensity of emission  $E$  being a function of time  $m_t(t)$  from a particular source in relation to time  $t$ :

$$E(t) = \frac{dm_t(t)}{dt}, \quad (2)$$

where  $m_t$  – mass of a given toxic compound.

Road emission is defined [1] as emission derivative, being a function of the road  $m_s(s)$  from a source, which is the vessel, in relation to road  $s$  covered by her

$$b_s = \frac{dm_s(s)}{ds} \quad (3)$$

On the basis of equation (3) it can be written down that emission on road  $S$  will be equal to

$$m_s(S) = \int_0^S b_s(s) ds \quad (4)$$

and in time  $T$

$$m_t(T) = \int_0^T b_t(t) v(t) dt, \quad (5)$$

where  $v(t)$  – vessel speed.

Road emission [2] can be written down as the functional of value courses describing the combustion engine work state i.e. of torque  $M_o$ , rotational speed  $n$  and the vectors describing the thermal state of the engine  $\mathbf{R}(t)$ , conditions of the surroundings  $\mathbf{G}(t)$  (e.g. temperature of surrounding, pressure, air humidity) and the changing vessel resistances  $\mathbf{O}(t)$  (vessel resistance in shallow waters, vessel resistance during movement in a canal, air resistance and wave effect):

$$b_t = \wp [ M_o(t), n(t), \mathbf{R}(t), \mathbf{G}(t), \mathbf{O}(t) ] \quad (6)$$

where  $\wp$  - operator transforming torque, rotational speed and the vectors of the engine's thermal state, movement resistance and conditions of the surroundings into average road emission from a vessel.

The current research, concerning atmosphere pollution caused by emission of harmful compounds from traction engines, mainly concern vehicle engines [1,3,4,5,6] and aircraft engines [7]. They constitute a very large input into the development of modelling the immission of toxic compounds emitted from combustion engines, however, because of both different topographic, hydrometeorologic conditions and the specificity of vessel operation, they cannot be applied for immission estimation in coastal regions.

## 2. Factors determining exhausts emission

The subject of balancing the emission of the compounds in exhausts of vehicles and vessels engines, are the processes of global emission, averaged in sufficiently long period of time [2,8]. This time is determined first of all by the effectiveness of averaging variable conditions of the objects operation.

Main factors determining the global emission of substances present in the exhausts of marine engines can be classified as follows:

- vessel structure (with respect to their size and destination), engine size and kind, number of particular engine kinds on the vessel (main and auxiliary engines), and with respect to the vessel's technical condition, taking into consideration technical solutions, state of the hull and the wear of propulsion system elements as well as their number,
- vessel operational intensity,
- vessel movement model,

- conditions of the surroundings: atmospheric conditions (waving, strong winds, icing), navigational water areas (ports, straits, canals and other dangerous and difficult to navigate areas, open waters), sailing in ice,
- vessel economic properties with respect to operational fuel consumption,
- ecologic properties of engines applied on vessels,
- fuel properties (*inter alia*, with respect to fuel kind, composition and content of pollutants).

### 3. Determination of a vessel trajectory

In order to describe the trajectory of a vessel moving in a given shipping lane from point  $P_{i-1}$  to point  $P_i$  in time interval  $(t_0, t_k]$ , it is necessary to formulate a suitable mathematical model. There should be taken into account two options: a deterministic model and a stochastic one. Both models must consist of a track function describing the track of the individual vessels and a probability law on the total number of vessels en route during  $(t_0, t_k]$ , the position of those vessels at the initial time  $t_0$ , as well as their speed. It is necessary to assume that the times at which the ships depart each point  $P_{i-1}$  are Poisson distributed and the ship speeds and the routes the ships travel are statistically independent.

Description of a ship trajectory is possible if we formulate the function of the track the ship covers, function of the ship movement and function of the ship trajectory described by the first two functions.

The function of the track of a vessel describes a set of all possible roads from point  $P_{i-1}$  to point  $P_i$ , where the point  $P_{i-1}$  means the beginning of the journey, e.g. the departure port, and point  $P_i$  - the end of the journey, e.g. the port of destination. The bigger area is taken into account, the more pairs of points  $(P_{i-1}, P_i)$  must be considered. All possible routes between any two points are the function of distance  $x$ , measured from point  $P_{i-1}$  along the route that the vessel covers.

Vessels generally move along the shipping lanes, which means the areas of a given length and width [8,9]. Knowledge of the shipping lanes in a given area causes that for the majority of vessels, such as merchant vessels or passenger ferries travelling between ports, it can be assumed that the distance covered by them is along a known trajectory. However, there exist a group of ships, e.g. fishing vessels or recreational vessels or naval vessels, which can operate beyond shipping lanes. For these categories of vessels it can be necessary to formulate a more complicated model of ship trajectory.

Assume  $\Delta_N$  is an ordered set of points  $t_i, i \in \overline{1, N}$  dividing the interval  $[\alpha, \beta]$  on  $N$  subintervals, where  $\alpha = t_0 < \dots < t_N = \beta$ . To this division [9] corresponds a set  $N+1$  points  $P_0, P_1, \dots, P_N$  on arc. These points form a broken line, which approximates the route of a vessel. The length of the segment joining points  $P_{i-1}, P_i, i = 1, \dots, N$  can be expressed by a formula:

$$|P_{i-1}P_i| = \sqrt{[x(t_i) - x(t_{i-1})]^2 + [y(t_i) - y(t_{i-1})]^2} \quad (7)$$

Applying the Lagrange's theorem about average value, the investigated length of the segment joining points  $P_{i-1}, P_i, i = 1, \dots, N$  is:

$$|P_{i-1}P_i| = \sqrt{[x'(\theta_i)]^2 + [y'(\vartheta_i)]^2} \Delta t_i, \quad (8)$$

where  $\theta_i, \vartheta_i$  are the points from interval  $[t_{i-1}, t_i)$ , and  $\Delta t_i = t_i - t_{i-1}$ .

The knowledge of ship trajectories and statistical data obtained via AIS (*Automatic Identification System*) [11,12] for marine vessels passing through given test segments situated perpendicularly to shipping lanes approaching the ports of Gdynia and Gdansk, as well as the shipping lane splitting into those two shipping lanes (in the proximity of Hel port), it is possible to describe the marine vessels flow (fig.1) [9] and then to formulate models of emission of toxic compounds in these vessels exhausts.

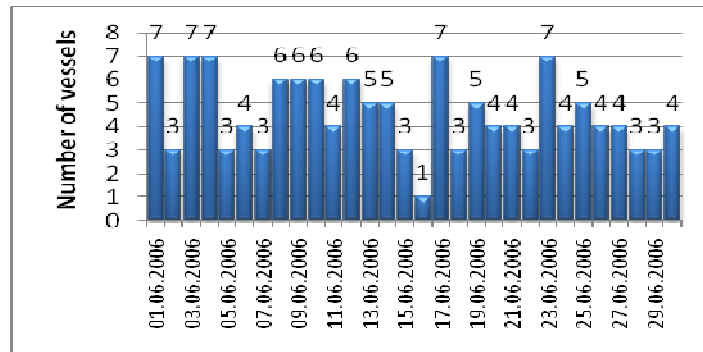


Fig. 1. A twenty-four hours distribution of the number of vessels outward bound passing through the 'gate' GD (01-30.06.2006) [9]

#### 4. Models applying the queuing theory

In modelling the movement of ships going along the shipping lane from point  $P_{i-1}$  to point  $P_i$ , it is possible to apply the queuing theory, which is based on the characteristics of the stream of incoming vessels ('incoming traffic') to the analysed area ('service canal') and the stream of vessels leaving the 'canal' [9]. For the vessels that entered the 'canal' (the shipping lane), service time means the time of the vessels journey in the analysed route.

If the moment of entering the first vessel to the system (moment of arrival at the queue) in the analysed time interval, e.g. a day, is the initial time  $t_0=0$ , then it seems to be extremely difficult or even impossible to predict the exact time of the next arrival (entering next vessels on the shipping lane), and also times of all following arrivals (the same day and the next days). The moments of all the following arrivals will not overlap. Therefore, both the moments of each incoming arrival, and the number of arrivals during a day, week, etc. are the random variables.

The process of incoming arrivals is stochastic process, and the incoming traffic can be described by a function  $X(t)$ , describing the number of arrivals requiring 'servicing' at the time interval  $(0, t)$ . The function  $X(t)$  is the random variable for each value of  $t$  [10]. In the figure 2 there is presented one of the realisations of the random function  $X(t)$  characterising the process of traffic incoming to the 'gate' GD in a given time period (48 hours) [9].

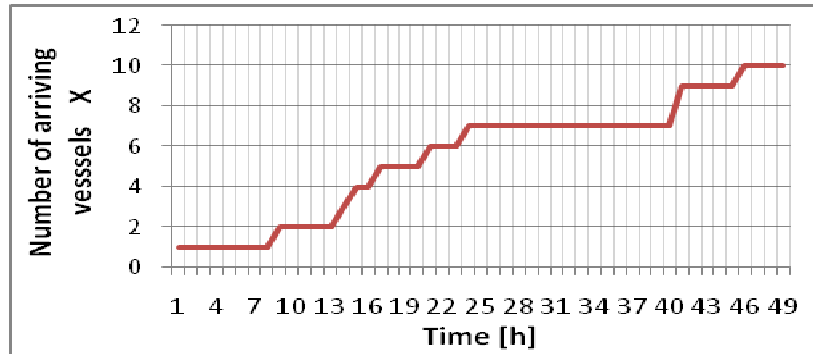


Fig. 2. Realisation of a random function  $X(t)$  of vessels arriving at the 'gate' GD [9]

In order to determine fully the incoming traffic, it is sufficient to know the probability of that in time  $(0, t_1)$  there will come  $k_1$  arrivals, in time  $(0, t_2)$  there will come  $k_2$  arrivals etc. If this probability is known for any group of positive integral numbers  $k_1, k_2, \dots, k_n$  and positive  $t_1, t_2, \dots, t_n$ , then the incoming traffic will be fully described.

A significant attribute of many real incoming traffic is their stationary character, which significantly simplifies the formulated model. The analysed incoming traffic of the vessels arriving to the shipping lane is also stationary. Moreover, this incoming traffic possesses also the character of the *traffic without sequence* or the *traffic of independent increments* (cumulative distribution function of a group  $X(t_1+a)-X(a)$ , where  $(i = 0, 1, 2, \dots, n)$  at  $t_i > 0$  and any  $a > 0$  does not depend on the value of  $X(t)$  at  $t < a$ ).

Apart from determining the probability of the number of arrivals to the shipping lane, it is also important to determine the time of their journey, which in the light of the queuing theory can be considered as *service time*. Of course, the time of their service is different for different vessels and depends on many factors: first of all, their speed, but also on random disturbances, such as conditions of the surrounding conditions, which means the atmospheric conditions (wind power and direction, state of the sea, sea currents, atmospheric pressure, air temperature), and the areas of the vessels operation (open sea, ports, straits, etc.), as well as the conditions referring to a particular vessel (hull resistances, technical state, nautical possibilities etc.).

Taking into account the above, in the analysed case for the vessels, the service time is the random variable, so it can be described by the cumulative distribution function:  $F(t) = P\{\gamma < t\}$  for  $t \geq 0$ , where  $\gamma$  is the service time. The function  $F(t)$  describes the probability of that the service time  $\gamma$  will be shorter than previously determined  $t$ . (Function  $F(t)$  should be a positive function monotonically increasing and should not be greater than one).

## 5. Models applying Monte Carlo methods

In practice there are situations where incoming traffic does not possess the characteristic of prime traffic or is not stationary and non-homogeneous, there can be any distribution of service time, or organisation in service time is complex and consists of many stages. Problems of this kind are very difficult or impossible to solve with the analytic methods.

Simulation models of Monte Carlo methods permit to solve these problems. Simulations of the vessels 'service' [9], which means the traffic of vessels arriving at the analysed shipping lane, the time spent on the lane, can be performed with the assumption (previously preceded with the identification research) that the incoming traffic is a homogeneous Poisson process of parameter  $\lambda$ , and time spent on the lane is of normal distribution. In simulation performed with so-called *method of uniform step of a stationary*

system 'without waiting', the incoming traffic is simulated in an assumed period of  $M$  days, e.g. with the step of 24 hours. Probability of occurrence of  $l$ -arrivals in each time interval of 24 hours can be calculated from the following formula:

$$P(l) = \frac{\lambda^l}{l!} \exp(-\lambda) \quad \text{for } l = 0, 1, 2, 3, \dots, D. \quad (9)$$

For the calculated values of probabilities there can be assigned e.g. 10 000 pseudorandom numbers of steady distribution, which results from the accuracy of computation. If  $\lambda = 6$  vessels/24 h, the probability of that an arrival will not occur during that time ( $l = 0$ ) is equal to 0.0025 and to that probability there are assigned the 25 consecutive numbers, as it is presented in the fig. 3 ( $\lambda_{p,l} = 0.0025$ ).

For simulation of the process of the arrivals 'servicing', there can be analysed the distribution of speeds of vessels moving along the analysed shipping lane or distribution of time spent on that shipping lane. It is also possible to simulate the number of vessels leaving the shipping lane in the analysed period of time.

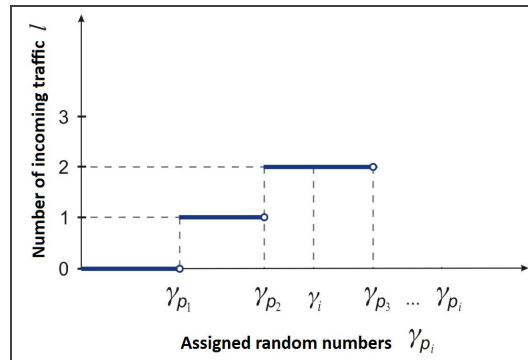


Fig. 3. An example of the simulation of the number of incoming traffic to the system [9]

The capacity of the 'service canal', expressed by the number of vessels leaving the shipping lane at  $i$ -unit of time, can be calculated from the formula:

$$W_i = \alpha_i \cdot \sigma + a, \quad (10)$$

where:

- $\alpha_i$  – sampled random number of normal distribution  $N(0,1)$ ,
- $\sigma$  – standard deviation of the number of vessels leaving the shipping lane,
- $a$  – average value of the number of vessels leaving the shipping lane (being serviced by the 'canal') at a time unit.

If the average number of vessels 'serviced' in a unit of time is  $\mu$ , then  $1/\mu$  means the average time of 'servicing'. And if the length of the 'canal' is known, then it is possible to determine the average speed of vessels operating in the analysed shipping lane.

In the *method of consecutive occurrences*, time of each consecutive arrival to the 'canal' and its time of 'servicing' in that 'canal' are simulated. Consecutive values of time  $\tau_i$  intervals between arrivals can be computed by generating the random numbers of steady distribution. The following relation is used:

$$\int_0^{\tau} \lambda \exp(-\lambda t) dt = \xi, \quad (11)$$

from which it results that:

$$\tau = -\frac{1}{\lambda} \ln(1 - \xi). \quad (12)$$

As the distribution of value  $1 - \xi$  is the same as the value  $\xi$ , it can be written down that:

$$\tau = -\frac{1}{\lambda} \ln \xi. \quad (13)$$

The formula (13) is directly helpful when computing values of  $\tau_i$ , at different values of sampled random numbers  $\xi_i$ .

Time of ‘service’ of each arrival is simulated by sampling separately the consecutive random numbers  $\alpha_j$  of normal distribution  $N(0,1)$  and computing simulated time spent by  $j^{\text{th}}$  arrival ( $j^{\text{th}}$  vessel) in the area (on the shipping lane) with the formula:

$$t_j = \sigma \cdot \alpha_j + t_o^{\text{sr}}, \quad (14)$$

where:

$\sigma$  – standard deviation of the time spent by a vessel on the shipping lane,

$t_o^{\text{sr}}$  – average value of time spent by vessels in the area,

$j = 1, 2, 3, \dots, m$  – a consecutive random number of ‘servicing’ the  $j^{\text{th}}$  arrival,

Course of computation starts at the moment of occurrence the first arrival (arriving a vessel to the entry to the shipping lane). Time spent on the shipping lane is calculated from the formula (14).

## 6. Modelling emission of toxic compounds in marine vessels exhausts

To determine the toxic compounds emission in exhausts on the basis of data obtained via AIS, there were formulated some statistical models describing the momentary power  $P_e^*$  for the momentary speed  $v^*$ , regarding the vessel size, its time spent in the area of research, and emission of toxic compounds. These values permit to estimate the emission intensity  $E_{\text{NO}_x}$  at that point at the level of (0,6 – 576) kg/h [13].

However, it should be noticed that the time a vessel spends in a region of a ‘gate’ is only a few minutes, and the absolute time a vessel requires for covering the distance Hel – Gdynia, depending on the vessels speed, accounts for 20-144 minutes.

Thus, determining the value of emission intensity  $E_{\text{NO}_x}$  in kilograms per nautical mile [kg/Mm] (fig.4) or in grams per nautical mile during one hour [g/(Mm·h)] (fig. 5), seems to be more expedient in this case (fig.3).

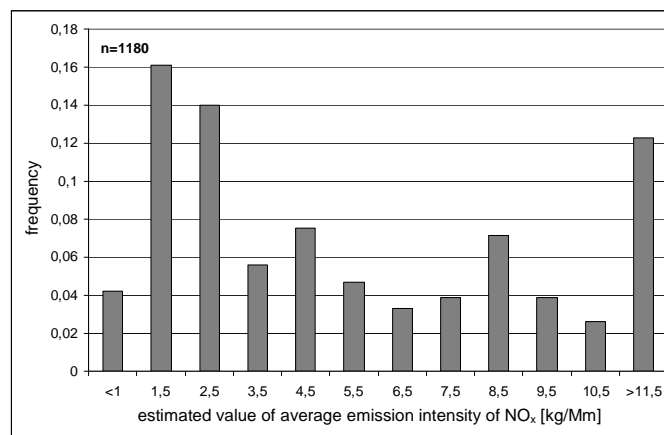


Fig.4. Distribution of estimated value of average emission intensity  $E_{\text{NO}_x}$  [kg/Mm] for vessels operating in the approach area of Hel – Gdynia

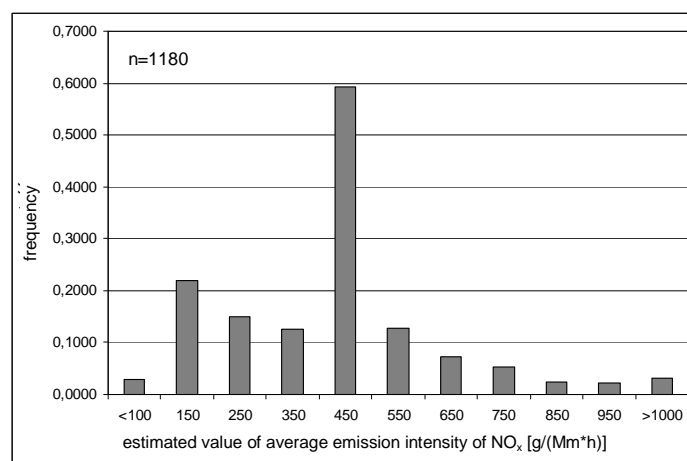


Fig.5. Distribution of estimated value of average emission intensity  $E_{NO_x}$  [g/(Mm·h)] for vessels operating in the approach area of Hel – Gdynia

## 7. Conclusion

Modelling emission, and then dispersion and imission of toxic compounds in marine engines exhausts constitutes a very important and very complex issue. Currently conducted research devoted to the pollutants dispersion, refer mainly to motorization, which because of, among other things, the size of marine engines, disqualifies them in modelling emission of toxic compounds from marine engines, as the model structure depends not only on its destination, but also on the quantity and quality of input data.

Additionally, apart from the problems appearing when modelling the toxic compounds emission from road vehicles, in case of marine vessels, among the parameters disturbing the appropriate determination of particular compounds emission (due to the lack of data or its changeability) are also technical state of engine, and especially of fuel apparatus, as well as atmospheric conditions (mainly wind power and direction).

## References

- [1] Chłopek Z.: *Modelowanie procesów emisji spalin w warunkach eksploatacji trakcyjnych silników spalinowych*. ZN Politechniki Warszawskiej, Mechanika, z.173, Warszawa 1999.
- [2] Kniaziewicz T.: *Problemy modelowania imisji szkodliwych składników spalin z silników okrętowych w rejonach miejskich aglomeracji nadmorskich*. Zeszyty Naukowe Politechniki Częstochowskiej 162, MECHANIKA 26, Częstochowa 2006.
- [3] Brzozowska L., Brzozowski K.: *Komputerowe modelowanie emisji i rozprzestrzeniania się zanieczyszczeń samochodowych*. Wydawnictwo Naukowe „Śląsk”, Katowice 2003.
- [4] Brzozowska L., Brzozowski K., Wojciech S.: *Computational Modeling of Car Pollutant Dispersion*. Wydawnictwo Naukowe „Śląsk”, Katowice 2001.
- [5] Drag Ł.: *Modelowanie emisji i rozprzestrzeniania się zanieczyszczeń ze środków transportu drogowego*. Archiwum Motoryzacji nr 1/2007.
- [6] Joumard R.: *Methods of estimation of atmospheric emission from transport*. European scientific state of the art, Action COST319 final report, LTE 9901 report, 1999.
- [7] Kotlarz W.: *Turbinowe zespoły napędowe źródłem skażeń powietrza na lotniskach wojskowych*. Wyższa Szkoła Oficerska Sił Powietrznych, Katedra Płatowca i Silnika, Dęblin 2003.
- [8] Kniaziewicz T., Piaseczny L.: *Model of NO<sub>x</sub> emission by sea-going vessels navigating in the gulf of Gdańsk region*. Silniki Spalinowe PTNSS – 2007-SC3. Poznań. 2007.



- [9] Pawlak M., Piaseczny L.: *Evaluation of suitability of existing harmful compounds dispersion models for assessment of land pollution with marine engines exhaust gases*. Silniki Spalinowe/Combustion Engines, No. 2007-SC3, p. 62 – 71
- [10] Pawlak M., Piaseczny L.: *Modele symulacyjne ruchu jednostek morskich*. Konstrukcja, badania, eksploatacja, technologia pojazdów samochodowych i silników spalinowych. Teka Komisji Motoryzacji. Oddziału PAN w Krakowie, Zeszyt Nr 33-34, s.325 – 334, Kraków 2008.
- [11] Felski A., Piaseczny L.: *Monitoring of the movement of the objects on the Gdansk bay in order to recognize the characteristics of their main propulsion systems*. Silniki Spalinowe/Combustion Engines, No. 2007-SC1, p. 377 – 382.
- [12] Felski A.: *Implementation of AIS in air pollution investigations*. Materials of European Navigation Conference Global Navigation Satellite Systems. Geneva 2007.
- [13] Kniaziewicz T., Piaseczny L.: *Model symulacyjny emisji NOx w spalinach jednostek pływających*. Konstrukcja, badania, eksploatacja, technologia pojazdów samochodowych i silników spalinowych. Teka Komisji Motoryzacji. Oddziału PAN w Krakowie, Zeszyt Nr 33-34, s.185 – 198, Kraków 2008.