

A METHOD TO DETERMINE FUNCTIONAL AVAILABILITY OF TECHNICAL OBJECTS

Józef Żurek ¹, Jarosław Ziółkowski ²

¹ *Instytut Techniczny Wojsk Lotniczych
(Air Force Institute of Technology)
skr. poczt. 96, ul. Księcia Bolesława 6, 01-494 Warszawa, Poland
tel.: +48 22 6852173, fax: +48 22 6852163,
e-mail: jozef.zurek@itwl.pl*

² *Wojskowa Akademia Techniczna
(Military University of Technology)*

Abstract

The intended aim of the paper is to present a problem of reaching engineering maturity by a technical object being introduced into service and designed for emergency activities. These are objects and systems associated with those of basic performance. Failures to the latter ones are hazardous to safety. Models of availability analysis, in particular of servicing by teams of specialists have been based on the theory of Markov/semi-Markov processes. The method of analysis has been introduced using a ship-helicopter as an example.

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1. Introduction

The question of reaching engineering maturity of objects newly introduced into service usually consists in coming up to the nominal level of availability, starting however from the underrated one.

The underrated level of availability results from both higher failure rate and longer servicing and repair times throughout initial stage of operational phase. From this standpoint, particular attention should be paid to emergency/rescue vehicles. Any ship-helicopter exemplifies such objects.

When consideration is given to such technical objects, prior to formulating a model of availability estimation, some simplifying assumptions should be made:

- all transitions from the state $i \in E$ to the state $j \in E$ show discrete nature (are described with a discrete random variable),
- mean time of the object's staying in the state 'i' before transition thereof to the state 'j' is described with a random number or function,
- both operational states $S = \{1, 2 \dots r\}$ and duration thereof t_{ij} are mutually independent, i.e. they cannot occur simultaneously,
- all possible transitions can be described with an operation/maintenance graph that does not change its form throughout the time of examining the problem, i.e. T_0 .

If the operational phase of these objects satisfies all the above-mentioned assumptions, a model of a Markov/semi-Markov process of some finite number of states can prove a proper analytical model.

2. A model to calculate functional availability of technical objects

2.1. Determination of boundary probabilities for the Markov chain

On the grounds of analyses of the operational phase of any technical object that performs indivisible tasks (sea voyages, flights) within emergency and rescue systems it has been found that such an object can remain in one of the following states:

S_1 – being on duty (waiting for a task),

S_2 – operating, and

S_3 – emergency object under refurbishment (becoming replaced with a capable one).

Furthermore, the following assumptions have been made to formulate a mathematical model of the operational phase from the point of view of availability analysis:

- any technical object can remain at any time instance in only one of possible states,
- in the course of performing tasks, the objects fail at random time instances,
- time needed for the object's refurbishment (replacement with a capable one) is strictly determined,
- duration of the process T_0 has been pre-set.

A diagraph representing the operational process (Fig. 1) is a representation thereof. There are interrelations between all the states (they 'communicate' with each other), thus generating a reducible chain [3, 4].

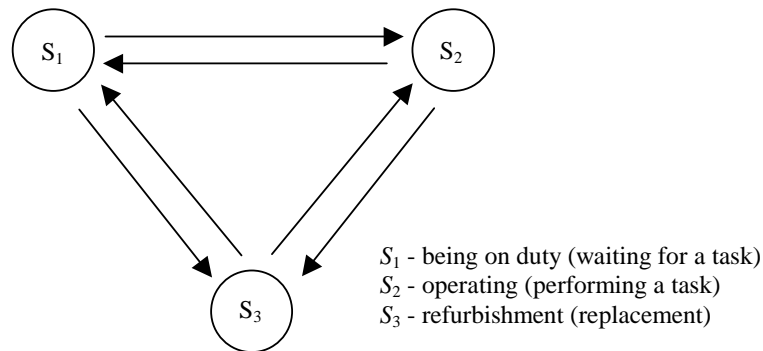


Fig. 1. A graph illustrating the technical object's operation

Presented in Fig. 1 is a 3 x 3 square matrix of transitions of the process; all transitions from the state 'i' to the state 'j' are possible for this matrix:

$$M_1 = [p_{ij}]_{3 \times 3} = \begin{bmatrix} 0 & p_{12} & p_{13} \\ p_{21} & 0 & p_{23} \\ p_{31} & p_{32} & 0 \end{bmatrix}. \quad (1)$$

Ergodic probabilities p_j can be calculated from the boundary of the matrix of transition in n steps $M_n = M_1^n$ by means of solving either a system of linear equations or an equivalent matrix equation [3]:

$$\hat{\wedge}_j p_j = \lim_{n \rightarrow \infty} p_{ij}(n) = \sum_i p_i p_{ij} \Leftrightarrow M_1^T [p_j] = [p_j], \text{ when } \sum_j p_j = 1, \quad (2)$$

where:

M_1^T - transpose of the transition matrix M_1 ,
 p_j - boundary probability,
 p_{ij} - probability of transition from state 'i' to state 'j'.

$$M_1^T(p_{ij}) = \begin{bmatrix} 0 & p_{21} & p_{31} \\ p_{12} & 0 & p_{32} \\ p_{13} & p_{23} & 0 \end{bmatrix}. \quad (3)$$

According to both the relationship $M_1^T \cdot [p_j] = [p_j]$ and the normalisation condition $\sum_j p_j = 1$, in the matrix notation the boundary (ergodic) probabilities p_j are calculated according to (4) and the system-normalisation condition (5):

$$M_1^T(p_{ij}) = \begin{bmatrix} 0 & p_{21} & p_{31} \\ p_{12} & 0 & p_{32} \\ p_{13} & p_{23} & 0 \end{bmatrix} \cdot \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix} = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix}, \quad (4)$$

$$\sum_{j=1}^3 p_j = 1, \quad (5)$$

or as the forms of linear equations (6):

$$\begin{cases} p_{21} \cdot p_2 + p_{31} \cdot p_3 = p_1 \\ p_{12} \cdot p_1 + p_{32} \cdot p_3 = p_2 \\ p_{13} \cdot p_1 + p_{23} \cdot p_2 = p_3 \\ p_1 + p_2 + p_3 = 1 \end{cases}. \quad (6)$$

After having solved the system of equations (6), the following formulae to find boundary probabilities are arrived at:

$$p_1 = \left(\frac{1 - p_{12} \cdot p_{21} - p_{23} \cdot (p_{12} \cdot p_{31} + p_{32})}{(p_{12} \cdot p_{31} + p_{32}) \cdot p_{13}} \right) \cdot p_2, \quad (7)$$

$$p_2 = \frac{1}{1 + \frac{1 - p_{12} \cdot p_{21}}{p_{12} \cdot p_{31} + p_{32}} + \frac{1 - p_{12} \cdot p_{21} - p_{23} \cdot (p_{12} \cdot p_{31} + p_{32})}{(p_{12} \cdot p_{31} + p_{32}) \cdot p_{13}}}, \quad (8)$$

$$p_3 = \left(\frac{1 - p_{12} \cdot p_{21}}{p_{12} \cdot p_{31} + p_{32}} \right) \cdot p_2. \quad (9)$$

Table 1 gives estimates of probabilities p_{ij} that the object remains in a particular state found for a real operational process:

Tab.1. Probabilities p_{ij} that the object remains in particular operational states

$S_{i/j}$	S_1	S_2	S_3
S_1	0	0,91	0,09
S_2	0,95	0	0,05
S_3	0,5	0,5	0

After having substituted the assumed values of p_{ij} from Tab. 1 into eqs (7) – (9), the boundary probabilities for the Markov chain are arrived at (Fig. 2).

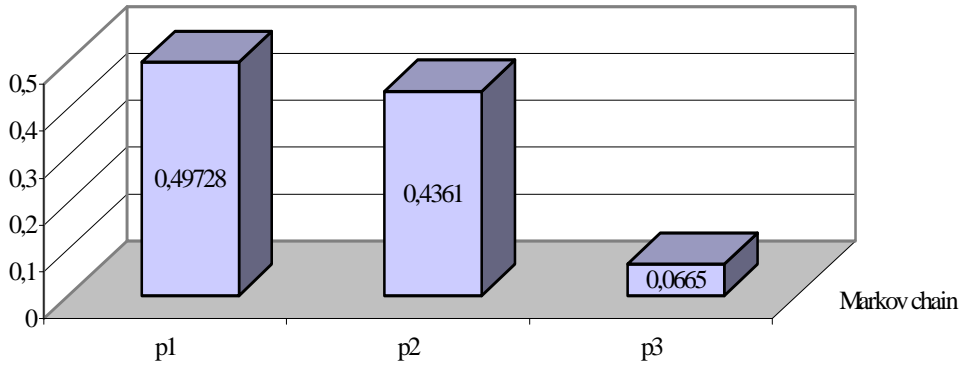


Fig. 2. Boundary probabilities for the Markov chain

2.2. Finding boundary probabilities for the Markov/semi-Markov process

Technical objects under operation are featured throughout their whole life cycles with different times of performing servicing and/or repairs. Fig. 3 shows characteristics of changes in servicing time and of coming up to the nominal time.

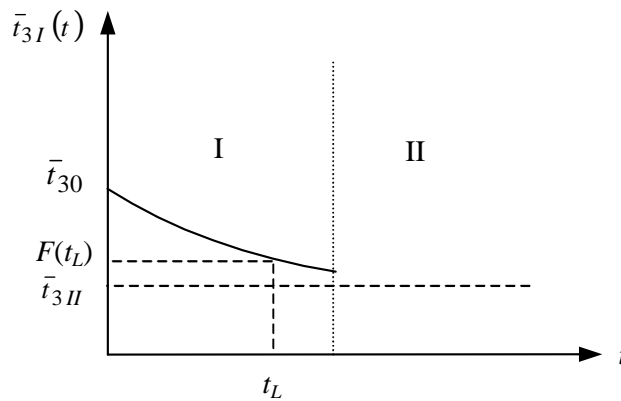


Fig. 3. Characteristics of changes in servicing time and of coming up to the nominal time

Interval I – the time of learning, featured with relatively long and gradually decreasing servicing and repair times. This is an effect of coming up to the engineering maturity in production and operational phase, including the teaching/learning of the staff expected to provide new objects introduced into service with maintenance (getting rid of errors in maintenance procedures and techniques, acquiring new habits).

Interval II – a ‘plateau’ period, i.e. time of proper operation/maintenance, in the course of which the failure rate keeps constant or nearly constant. It usually results from random failures and errors made by servicing personnel.

For Interval I (Fig. 3), a model of the semi-Markov process might prove a proper mathematical apparatus to describe operational phase. The semi-Markov process is a Markov process, throughout which mean times of the emergency object's staying in the state 'i' prior to transition to the state 'j' are random functions of time.

In phase I, mean time of transition from the state of refurbishment $\bar{t}_{3I}(t)$ to other states for $t = 0$ reaches maximum and decreases down to reach the mean time of refurbishment calculated in Phase II (Fig. 3). The time dependence can be described with the exponential function as:

$$\bar{t}_{3I}(t) = (\bar{t}_{30} - \bar{t}_{3II}) \cdot \exp^{-t/\tau} + \bar{t}_{3II}, \quad (10)$$

where:

- $\bar{t}_{3I}(t)$ - time of transition from the state of refurbishment to other states within Interval I,
- \bar{t}_{30} - assumed maximum time of performing the refurbishment,
- \bar{t}_{3II} - the expected value of time of performing the refurbishment in phase II (Fig. 3).

Operational data show that the maximum time of 'learning' (t_L) how to provide new emergency objects with servicing amounts to 7 days. Since usually $t_L = t_{90\%}$, the learning constant can be found (Fig. 3):

$$\frac{F(0) - F(t_L)}{F(0) - F(\infty)} = 0,9 \quad \text{therefore,} \quad \frac{\bar{t}_{30} - \bar{t}_{3II(90\%)}}{\bar{t}_{30} - \bar{t}_{3II}} = 0,9, \quad (11)$$

$$\ell^{-\frac{t_L}{\tau}} = 0,1, \quad (12)$$

$$\ell^{\frac{t_L}{\tau}} = \ln 10 \cong 2,303, \quad (13)$$

$$\tau = \frac{t_L}{2,303} = \frac{7}{2,303} = 3,04 \approx 3 \text{ days.} \quad (14)$$

Hence, the form of time dependence for the learning period is as follows:

$$\bar{t}_{3I}(t) = 20 \cdot \ell^{-\frac{t}{3}} + 10, \quad (15)$$

where:

- \bar{t}_{3I} - random function of time of staying in the state of refurbishment at stage I,
- $t = \{0, 1, \dots, 7\}$ - random variable of physical time of 'learning' measured with days.

Mean times of staying in the state of refurbishment are what is arrived at after having substituted possible values of t in eq (15) for stage I. Then, transition rates are found according to (18) and (19) and substituted in the system of equations (21); values of probabilities p_j (Tab. 2) are calculated for variable rates of transition λ_{3j} from the state of refurbishment to other states.

Tab.2. Expected values of time of staying in the state of refurbishment for stage I

t [days]	$\bar{t}_{3I}(t)$	$\lambda_{3I}(t)$	$p_{1I}(t)$	$p_{2I}(t)$	$p_{3I}(t)$
0	30	44	0,735216	0,248457	0,016327
1	24,3804	54,1427	0,737471	0,249219	0,013309
2	20,3397	64,8975	0,739102	0,24997	0,011128
3	17,3605	76,0368	0,740309	0,250178	0,009513
4	15,2764	86,4077	0,741155	0,250464	0,008381
5	13,8053	95,6161	0,741754	0,250666	0,00758
6	12,7089	103,88	0,742138	0,250797	0,00698
7	11,9477	110,4824	0,742511	0,250922	0,006567
∞	10	132	0,7427	0,25081	0,00552

at:

$$\bar{t}_{3I}(t) = \bar{t}_{31}(t) + \bar{t}_{32}(t), \quad (16)$$

where:

$\bar{t}_{3I}(t)$ - function of time of refurbishment at stage I,

$\bar{t}_{31}(t)$ - function of time of refurbishment prior to transition to the state of being on duty,

$\bar{t}_{32}(t)$ - function of time of refurbishment prior to transition to the state of operation.

Figs 4 – 6 illustrate probabilities $p_j(t)$ throughout the period of learning.

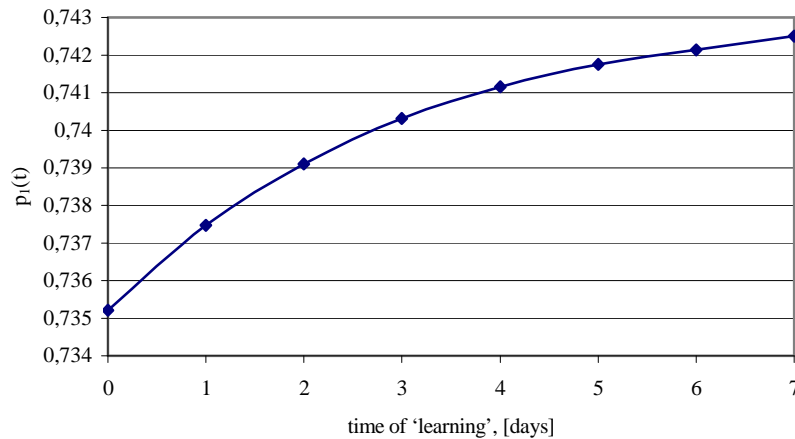


Fig. 4. Values of probabilities $p_1(t)$ for the semi-Markov process, the state of being on duty

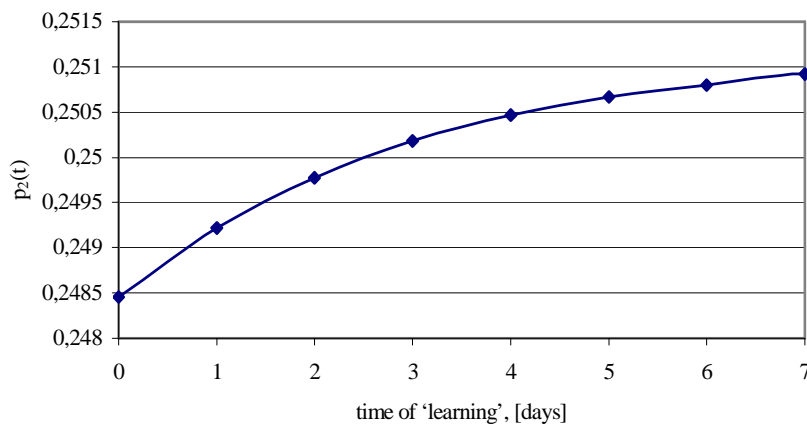


Fig. 5. Values of probabilities $p_2(t)$ for the semi-Markov process, the state of operation

Figs 4 and 5 prove that for both the state of being on duty and the state of performing the task, respectively, values of probabilities p_1 and p_2 gradually increase in the real time, whereas for the state of refurbishment decrease is observed (Fig. 6).

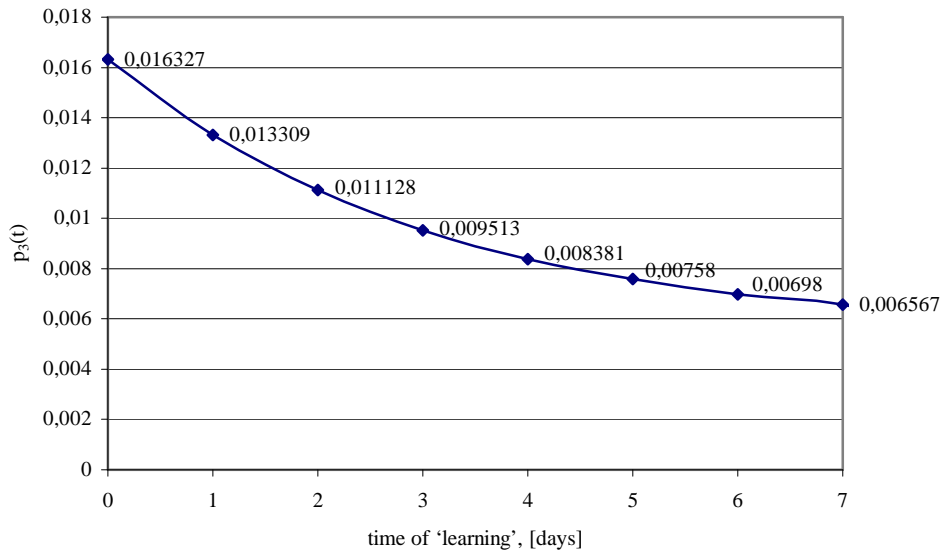


Fig. 6. Values of probabilities $p_3(t)$ for the state of refurbishment

As time passes by, i.e. time for progress in learning how to service new devices/systems, the number of errors made by the staff who operate/provide maintenance of objects recently introduced into service decreases, which means lower and lower probability that a failure occurs due to incorrect operational use and maintenance.

$$K_I = \frac{p_1(t) + p_2(t)}{\sum_{j=1}^3 p_j(t)} = 0,983 \div 0,993.$$

For stage II (Fig. 3) featured with constant failure rates it has been assumed that the Markov process $X(t)$ of finite phase space $\Omega = \{S_1, S_2, S_3\}$ is a model of operating/servicing technical objects.

If:

- $X(t) = 1$, at time instance t the object is in the state of being on duty (waiting for a task),
- $X(t) = 2$, at time instance t the object is in the state of performing a task,
- $X(t) = 3$, at time instance t the object is in the state of refurbishment.

The stochastic process $X(t)$, i.e. the Markov process of some finite set of states S can be completely determined by means of the following:

- initial distribution of the process $X(t) = [1,0,0]$,
- matrix M of probabilities of changes of the states for the Markov chain,
- matrix of intensities of transitions Λ of the process.

The Λ matrix has been built on the basis of the digraph shown in Fig. 1:

$$\Lambda = \begin{bmatrix} -\lambda_{11} & \lambda_{12} & \lambda_{13} \\ \lambda_{21} & -\lambda_{22} & \lambda_{23} \\ \lambda_{31} & \lambda_{32} & -\lambda_{33} \end{bmatrix}. \quad (17)$$

Intensities λ_{ij} and λ_{ii} of the states $S_1 - S_3$ for the Markov process under consideration, calculated by test, have been given in Table 3.

Tab. 3. Matrix of intensities of transitions while operating a technical object throughout stage II

$\lambda_{ij}/\lambda_{ii}$	λ_1	λ_2	λ_3
λ_1	-34,879	32,154	132
λ_2	100,32	-98,0392	66
λ_3	132	132	-132,15

with λ_{ij} calculated according to the relationship:

$$\lambda_{ij} = \frac{1}{\bar{t}_{ij}}, \quad (18)$$

where:

\bar{t}_{ij} - mean time of staying in the state i before transition to the state j .

Diagonal rates have been found as:

$$\lambda_{ii} = 1/\bar{t}_i = \frac{1}{\sum_{j \in J_{ij}} \bar{t}_{ij}} = \frac{1}{\sum_{j \in J_{ij}} n_{ij} \cdot \bar{t}_{ij}} = \frac{1}{\sum_{j \in J_{ij}} \omega_{ij} \cdot \frac{1}{\lambda_{ij}}}, \quad (19)$$

where:

ω_{ij} - frequency of transitions from the state i to the state j ,

λ_{ij} - rate of transitions from the state i to the state j .

According to the theory of Markov processes, for ergodic processes the matrix equations are satisfied [3]:

$$\Lambda^T \cdot [p_j] = 0, \quad (20)$$

where:

$\Lambda = [\lambda_{ij}]$ - matrix of rates for diagonal λ_{ii} and non-diagonal elements λ_{ij} .

According to eq (20) for the matrix notation with the normalisation condition, the following is arrived at:

$$\begin{bmatrix} -\lambda_{11} & \lambda_{21} & \lambda_{31} \\ \lambda_{12} & -\lambda_{22} & \lambda_{32} \\ \lambda_{13} & \lambda_{23} & -\lambda_{33} \end{bmatrix} \cdot \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad (21)$$

$$\sum_{i=1}^3 p_i = 1,$$

i.e.

$$\begin{cases} -\lambda_{11} \cdot p_1 + \lambda_{21} \cdot p_2 + \lambda_{31} \cdot p_3 = 0 \\ \lambda_{12} \cdot p_1 - \lambda_{22} \cdot p_2 + \lambda_{32} \cdot p_3 = 0 \\ \lambda_{13} \cdot p_1 + \lambda_{23} \cdot p_2 - \lambda_{33} \cdot p_3 = 0 \\ \sum_{i=1}^3 p_i = 1 \end{cases} \quad (22)$$

After having solved the above-shown system of equations the following formulae for boundary probabilities of the Markov process are arrived at:

$$p_1 = \left[\frac{\lambda_{21}(\lambda_{11} \cdot \lambda_{31} + \lambda_{32} \cdot \lambda_{11})}{\lambda_{11}(\lambda_{11} \cdot \lambda_{22} + \lambda_{12} \cdot \lambda_{21})} + \frac{\lambda_{31}}{\lambda_{11}} \right] \cdot p_3, \quad (23)$$

$$p_2 = \left[\frac{\lambda_{11} \cdot (\lambda_{31} + \lambda_{32})}{\lambda_{11} \cdot \lambda_{22} - \lambda_{12} \cdot \lambda_{21}} \right] \cdot p_3, \quad (24)$$

$$p_3 = \frac{1}{\frac{\lambda_{11} + \lambda_{31}}{\lambda_{11}} + \frac{\lambda_{21} \cdot \lambda_{11} \cdot (\lambda_{31} + \lambda_{32})}{\lambda_{11}(\lambda_{11} \cdot \lambda_{22} - \lambda_{12} \cdot \lambda_{21})} + \frac{\lambda_{11} \cdot (\lambda_{31} + \lambda_{32})}{\lambda_{11} \cdot \lambda_{22} - \lambda_{12} \cdot \lambda_{21}}}. \quad (25)$$

With values taken from Tab. 3 and substituted into eqs (23) – (25), boundary probabilities (Fig. 7) are obtained for the Markov process (stage II).

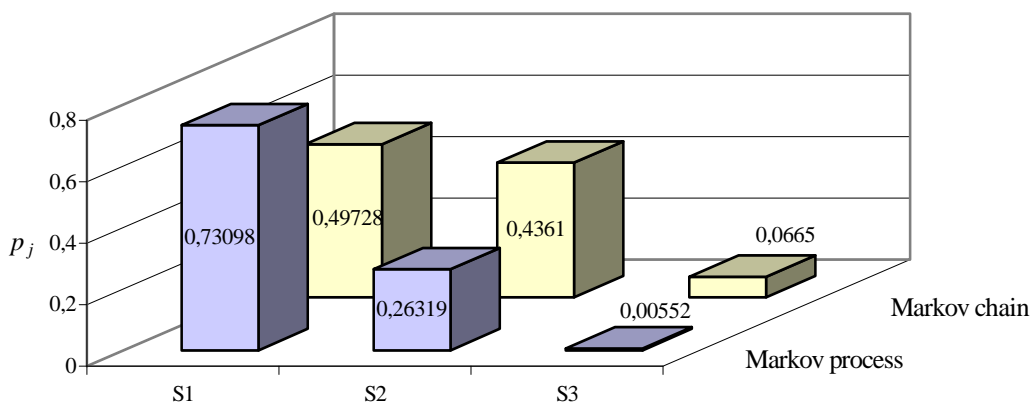


Fig.7. Boundary probabilities p_j at stage II for an emergency object

The calculated rate of functional availability for stage II is:

$$K_{II} = \frac{p_1(t) + p_2(t)}{\sum_{j=1}^3 p_j(t)} = 0,993.$$

3. Conclusion

The paper has been intended to introduce a method of determining functional availability of technical objects introduced into service, in particular, within rescue systems. The required characteristics of such objects are availability and readiness to perform tasks that arise at random time instances during the duty. Hence, the objects usually perform tasks, or keep waiting in the fit-for-use state.

A three-state Markov/semi-Markov model has been used for analysis; the object's lifetime has been divided into three stages. The first one: for objects of a new type recently introduced into service, the rate of failures due to servicing errors gradually decreases. The second one is the time of constant failure rate and constant functional availability. For the first stage (Fig. 3) the semi-Markov process has been assumed, for which mean times of refurbishment have been described with the exponential function. The second stage has been described with the Markov process. The effect is that, on the basis of data, rates of functional availability have been calculated, according to the following relationship:

$$K = \frac{p_1(t) + p_2(t)}{\sum_{j=1}^3 p_j(t)}.$$

The rates for particular stages are as follows:

- 1) stage I, for which some increase in the rate of availability $K_I = 0.983 \div 0.993$ is observed; this results from the improvement in quality of performing services,
- 2) stage II of constant availability $K_{II} = 0.993$, in the course of which constant errors are made

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