PRESSURE AND CAPACITY DISTRIBUTIONS IN DEFORMABLE SPHERICAL SLIDE BEARING GAPS

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Abstract

This paper presents the pressure and capacity distributions in a thin layer of non-Newtonian, viscoelastic, lubricant inside the slide spherical bearing. Non-isothermal, unsteady and random flow conditions and thermal deformations of the bearing surfaces are taken into account. This problem finds application in ship power plants, electric locomotive designing, and precision engineering.

Keywords: *time depended hydrodynamic pressure, capacity distributions in spherical bearings*

1. General assumptions

This paper presents the general analysis of the pressure and capacity distributions for unsteady non isothermal flow of visco-elastic oil within the gap between two rotational spherical deformed bearing surfaces in random conditions (see Fig.1).

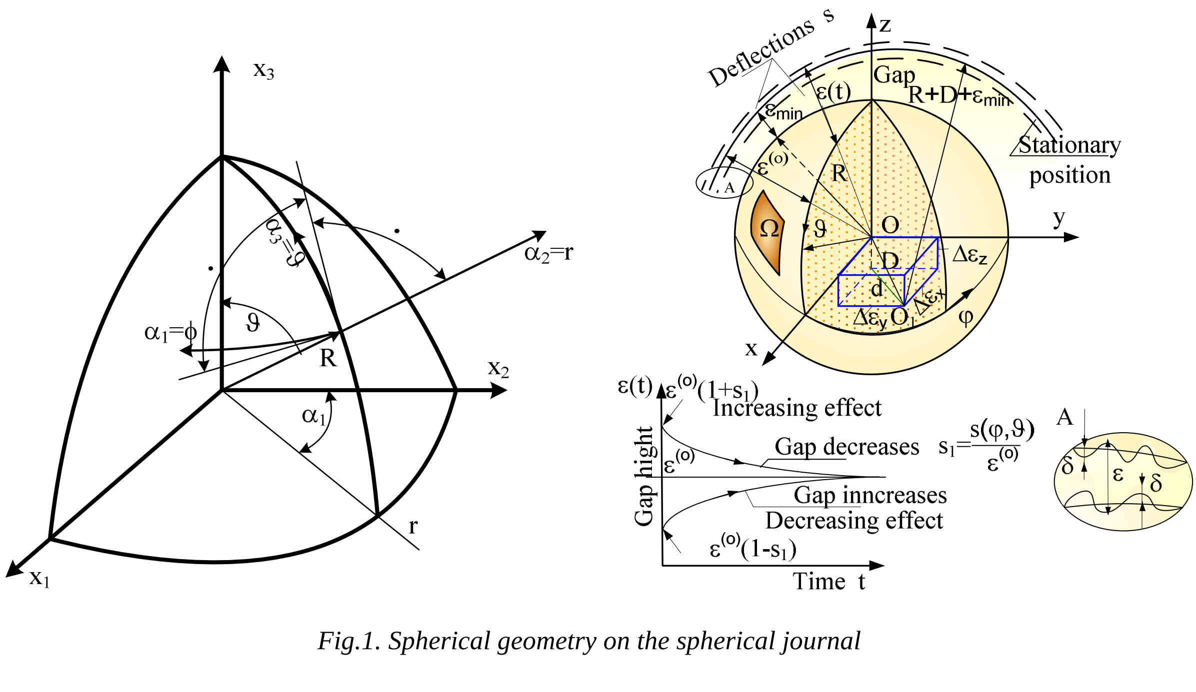


Fig.1. Spherical geometry on the spherical journal

We take into account following assumptions:

- unsteady asymmetrical oil flow in spherical bearing gap,
- spherical bearing gap changes in circumferential and meridional directions,
- hyper-elastic and random deformations of bearing surfaces.

The time- dependent, gap- height with impulsive perturbations has the following form [1]:

$$\varepsilon_{T1}=\varepsilon_{T1s}(\phi,\vartheta_1,t_1)+\varepsilon_{33}(t_1)=\varepsilon_{T1s}(\phi,\vartheta_1)\,[1+s_1\exp(-t_o t_1\omega_o)]+\varepsilon_{33}=\varepsilon_{T1s}(\phi,\vartheta_1,t_1)+p(t)/E_{33}=$$
$$=\varepsilon_{T1s}(\phi,\vartheta_1,t_1)+Ca\,p_{10}(t_1), \tag{1}$$

where

ε_{33} – dimensionless corrections caused by the hypo-elastic deformations in the gap-height directions,
$Ca=p_o/E_{33}$ – Cauchy number,
p_o – characteristic value of hydrodynamic pressure.

The time- independent part of value of gap- height for smooth bearing surfaces without deformations has the dimensional form: $\varepsilon_o\,\varepsilon_{T1s}(\phi,\vartheta_1)$.
If we take into account the unsteady effects in impulsive motion, then from the stochastic Reynolds equation presenting in spherical coordinates in papers [1], [2], [3], [4] it follows:

$$\frac{1}{\sin\vartheta_1}\frac{\partial}{\partial\phi}\left[\frac{E(\varepsilon_{T1}^3)}{\eta_1}\frac{\partial E(p_1)}{\partial\phi}\right]+\frac{\partial}{\partial\vartheta_1}\left[\frac{E(\varepsilon_{T1}^3)}{\eta_1}\frac{\partial E(p_1)}{\partial\vartheta_1}\sin\vartheta_1\right]=$$
$$=6\frac{\partial E(\varepsilon_{T1})}{\partial\phi}\sin\vartheta_1+12Str\frac{\partial E(\varepsilon_{T1})}{\partial t_1}\sin\vartheta_1, \tag{2}$$

In numerical calculations we use the real values of Young modulus and Poisson’s ratio of bearing soft material. We obtain from that the total Young modulus of bearing soft material in perpendicular direction to the sliding surface, has the value $E_{33}=0{,}1\ GPa$.

During the impulsive motion the dimensionless pressure p_1 in the lubrication region Ω: $\{0<\phi<\pi,\ \pi/8\le\vartheta_1\le\pi/2\}$ is determined by virtue of the modified Reynolds equations (2). The gap height (1), is taken into account.

2. Numerical calculations

The numerical calculations are performed in Matlab 7.2 by means of the finite differences method. In the calculations we assume: the radius of spherical journal R=0.025 *m*, angular velocity of the impulsive perturbations of bearing sleeve $\omega_o=15\ s^{-1}$, characteristic dimensional time $t_o=0.00001\ s$. To obtain real values of time we must multiply the dimensionless values t_1 by the characteristic time value $t_o=0.00001\ s$. For example $t_1=100000$ denotes one second after impulse.

We assume the following eccentricities of spherical journal: $\Delta\varepsilon_x=4.0\ \mu m$, $\Delta\varepsilon_y=0.5\ \mu m$, $\Delta\varepsilon_z=3\ \mu m$ and the characteristic gap-height value $\varepsilon_o=10\ \mu m$. Moreover we assume the following quantities: the dynamic viscosity of oil $\eta_o=0.030\ Pas$, pseudo-viscosity coefficient $\beta=0.0000002\ Pas^2$, density of the lubricant $\rho=890\ kg/m^3$, angular velocity of spherical journal $\omega=157\ s^{-1}$. The average minimum gap- height ε_{min} changes within the time interval $0.00001\ s\le t\le 10\ s$ and attains values from 6.1 μm to 10.1 μm; the average relative radial clearance amounts to $\psi\equiv\varepsilon_o/R=0.0004$.

Under the above assumptions we obtain the characteristic dimensional value of pressure $p_o=29.452431\ MPa$, Cauchy Number Ca=0.295, Strouhal Number Str=636.6 and additionally: $Re\cdot\psi\cdot Str=0.297$, $De\cdot Str=0.667$. In this case we have $0\le\beta/\eta_o t<1$. For the dimensionless time values: $t_1=1$, $t_1=10000$, $t_1=1\,000\,000$, i.e. for the dimensional time values: $t=0.00001\ s$; $t=0.1\ s$; $t=10.0\ s$, respectively, the dimensionless pressure distributions are presented in Fig. 2 (for s_1=+1/4), in Fig.3 (for s_1= –1/4) without hypo-elastic deformations of bearing soft material (the left column of Fig. 1, and Fig. 3 for Ca=0.00) and with hypo-elastic deformations of bearing soft material (the right column of Fig. 1, and Fig. 2 for Ca=0.295).

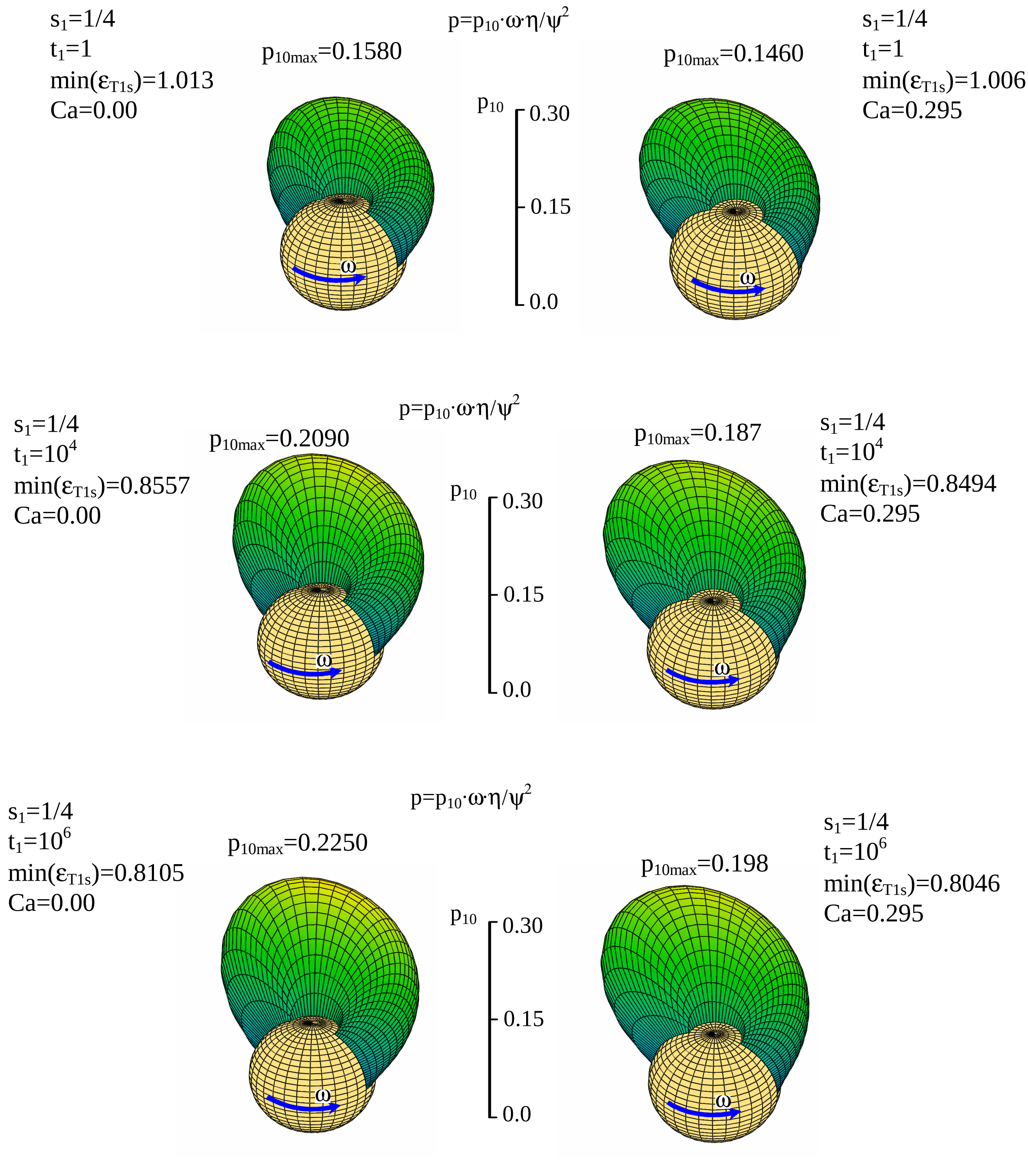


Fig. 2. Dimensionless hydrodynamic pressure distributions inside the gap of spherical bearing without bearing soft material deformations (the left column of figures for Ca=0.00) and with hypo-elastic soft material deformations (the right column of figures for Ca=0.295) in the region Ω: $0\le\varphi\le\pi$, $\pi R/8\le\vartheta\le\pi R/2$, for the dimensionless time values: t_1=1 (i.e. t=0.00001 s), t_1=1 0 000 (i.e. t=0.1 s), and t_1=1 000 000 (i.e. t=10 s) after the moment of the impulse causing first an increase and then a decrease of the gap height (s_1=+1/4).

The results were obtained for the following data:

$R=0.025$ m; $\eta_o=0.030$ Pas; $\rho=890$ kg/m^3; $p_o=29.452$ MPa, $\varepsilon_o=10$ μm; $\Delta\varepsilon_x=4$ μm; $\Delta\varepsilon_y=0.5$ μm; $\Delta\varepsilon_z=3$ μm; $\psi\equiv\varepsilon_o/R\approx0.0004$; $\omega=157$ s^{-1}; $\omega_o=15$ s^{-1}; $Str=636.6$; $Re\cdot\psi\cdot Str=0.297$; $De\cdot Str=0.667$

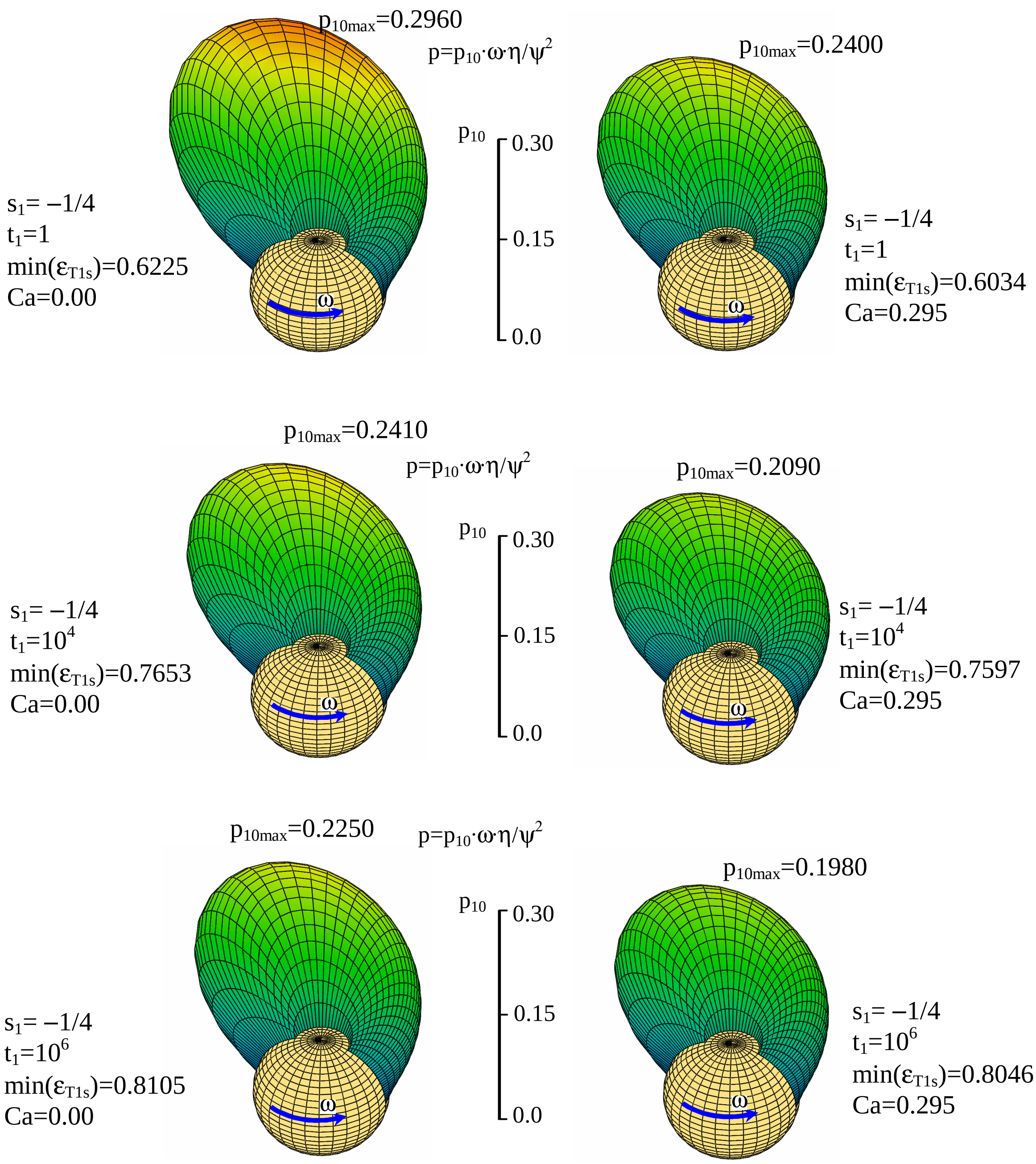


Fig. 3. Dimensionless hydrodynamic pressure distributions inside the gap of spherical bearing without bearing soft material deformations (the left column of figures for Ca=0.00) and with hypo-elastic soft material deformations (the right column of figures for Ca=0.295) in the region Ω*:* $0\le\varphi\le\pi$, $\pi R/8\le\vartheta\le\pi R/2$*, for the dimensionless time values:* $t_1=1$ *(i.e. t=0.00001 s),* t_1*=1 0 000 (i.e. t=0.1 s), and* t_1*=1 000 000 (i.e. t=10 s) after the moment of the impulse causing first an decrease and then a increase of the gap height, for* $s_1=-1/4$.

The results were obtained for the following data:

R=0.025 m; $\eta_o=0.030$ *Pas;* $\rho=890$ *kg/m*^{3}*;* p_o*=29.452 MPa,* $\varepsilon_o=10$ μ*m;* $\Delta\varepsilon_x=4$ μ*m;* $\Delta\varepsilon_y=0.5$ μ*m;* $\Delta\varepsilon_z=3$ μ*m;* $\psi\equiv\varepsilon_o/R\approx0.0004$*;* $\omega=157$ *s*$^{-1}$*;* $\omega_o=15$ *s*$^{-1}$*; Str=636.6; Re·*ψ*·Str=0.297; De·Str=0.667*

To obtain dimensional values of pressure we must multiply the dimensionless values indicated in Fig. 2, 3 by the factor $p_o=\omega\eta/\psi^2$.

The case of Ca=0.295 presents the influence of soft material deformations on the pressure values. The pressure values for Ca=0.295 are obtained in 6 steps. We assume the dimensionless pressure values $p_{10}\sim p_{10(1)}$ obtained from Eq. (2) for Ca=0.00 i.e. values of the pressure without influences of bearing soft material deformations, to be the first step of the pressure calculations. The dimensionless values $p_{10(1)}$ are multiplied by the non- zero Cauchy Number Ca and hence we obtain the dimensionless gap-height in the form: ε_{T1s}+Ca $p_{10(1)}$. For this dimensionless gap- height, from Eq. (2) we obtain the dimensionless pressure values p_{10}~$p_{10(2)}$ as the next step of the pressure calculations. The dimensionless values $p_{10(2)}$ are multiplied by the non-zero Cauchy Number Ca and hence we obtain the dimensionless gap-height in the form: ε_{T1s} +Ca $p_{10(2)}$. For this gap-height, from Eq. (2) we obtain the dimensionless pressure values p_{10}~$p_{10(3)}$ as the third step of the pressure calculations. The obtained series describing the dimensionless pressure values is convergent to the dimensionless boundary value of the pressure p_{10}. The dimensionless pressure value $p_{10(6)}$ obtained in sixth step of calculations can be found in the neighborhood of the dimensionless value of pressure p_{10}, where we have the inequality:

$$\frac{\left|p_{10(5)} - p_{10(6)}\right|}{\left|p_{10(6)}\right|} \leq 0.1\%$$

Fig. 4 presents the dimensional capacity distributions versus dimensional time in seconds in logarithmic scale (Fig. 4a) and in decimal gradation (Fig. 4b).

The numerical values of capacity are calculated for the following dimensionless times: t_1=1, t_1=10, t_1=50, t_1=100, t_1=500, t_1=1 000, t_1=2 000, t_1=4 000, t_1=6 000, t_1=8 000, t_1=10 000, t_1=20 000, t_1=40 000, t_1=60 000, t_1=80 000, t_1=100 000, t_1=1 000 000 i.e. for the dimensional times: t=0.00001 *s*; t=0.0001 *s*; t=0.0005 *s*; t=0.001 *s*; t=0.005 *s*; t=0.01 *s*; t=0.02 *s*; t=0.04 *s*; t=0.06 *s*; t=0.08 *s*; t=0.1 *s*; t=1.0 *s*; t=10.0 *s*, respectively. The first two curves depicted in the upper part of Fig. 3a and Fig. 3b, show the dimensional capacity values obtained without taking into account soft material deformations for Ca=0.00. The first two curves in the lower part of Fig. 3a and Fig. 3b, show the dimensional capacity values obtained for bearing soft material deformations with Ca=0.295. The upper curve of each two curves in Fig.3a and Fig. 3b denotes the dimensional capacities obtained for s_1=–1/4. The lower curve of each two curves in these figures denotes the dimensional capacities obtained for s_1=+1/4.

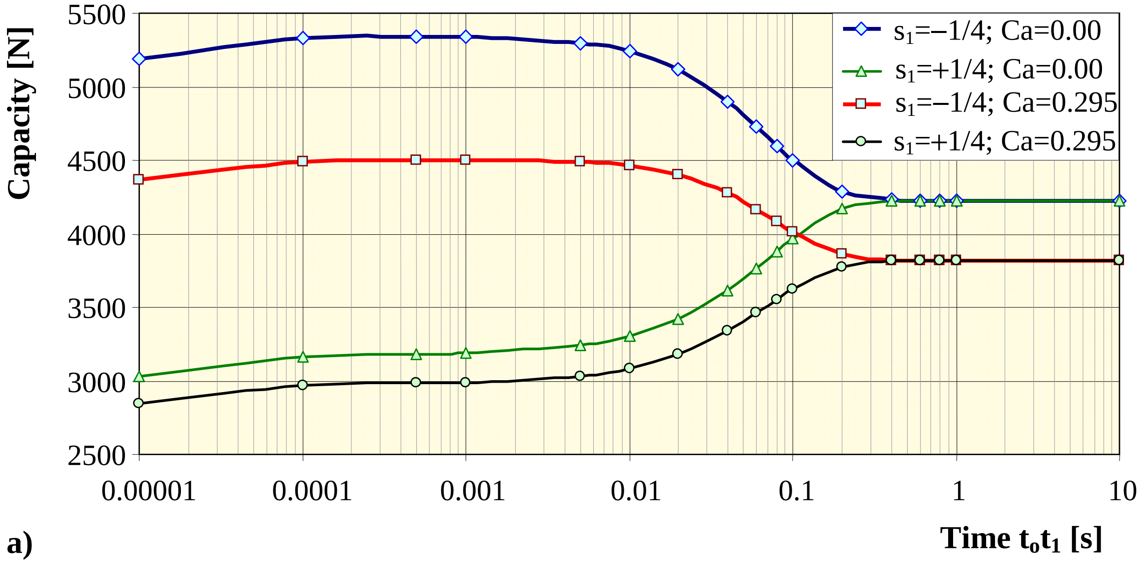
If the gap height increases (s_1>0) due to the impulse, then in the time after impulse the gap-height decreases and the pressure increases. In a sufficiently long time after impulse the gap- height and pressure attains the time- independent values of gap height and pressure, respectively.

If the gap height decreases (s_1<0) due to the impulse, then in the time after impulse the gap height increases and the pressure decreases. In a sufficiently long time after impulse the gap height and pressure attains the time- independent values of gap height and pressure, respectively.

If the time distant after the impulse moment is long enough, i.e. for $t_1 \rightarrow \infty$, then the pressure distributions for the increasing (s_1>0) and decreasing (s_1<0) effects of gap height changes caused by the bearing surface roughness, tend to identical pressure distributions. This limit of pressure distribution can be also obtained from the pseudo-classical Reynolds equation (2).

Pressures in joint gap attain identical distributions only when in both cases: s_1>0, s_1<0 the gap height gains the same but not durable damages in a sufficiently long time after impulse.

From the terms multiplied by the Cauchy number Ca in the formula (1) it follows that we can obtain durable and not identical deformations of joint gap height if the number Ca attains not the same values for s_1>0, s_1<0 in an enough long time after impulse moment.



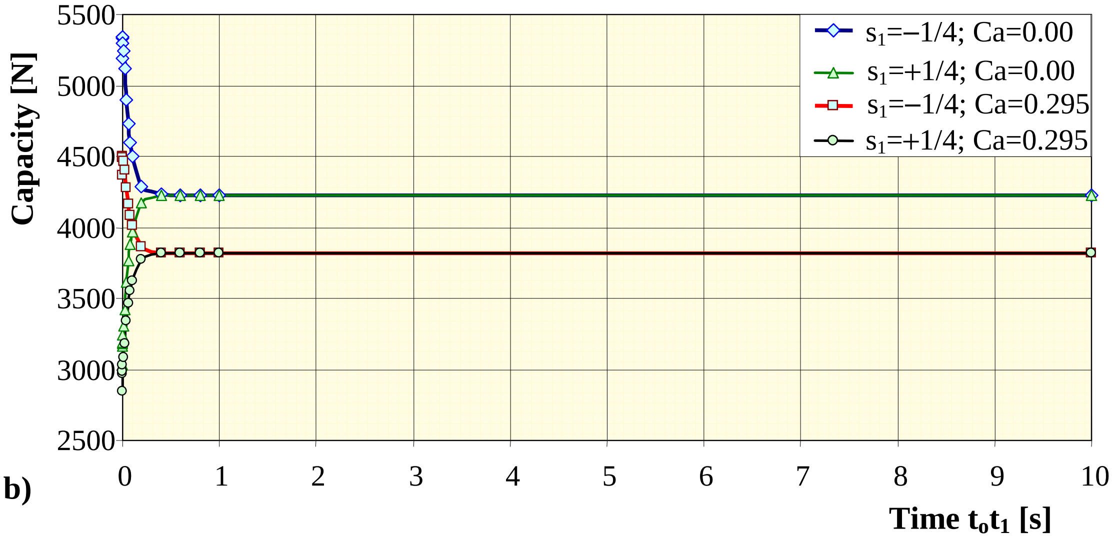


Fig. 4. Dimensional capacity distributions inside the gap of human spherical hip joint, in the region Ω: $0 \le \varphi \le \pi$, $\pi R/8 \le \vartheta \le \pi R/2$ versus dimensional time values from 0.00001 s to 10 s after impulse, expressed in logarithmic scale (Fig. 3a), and in decimal time scale (Fig. 3b).
The results were obtained for the following data: R=0.025 m; η_o=0.030 Pas; ρ=890 kg/m^3; ε_o=10 μm; $\Delta\varepsilon_x$=4 μm; $\Delta\varepsilon_y$=0.5 μm; $\Delta\varepsilon_z$=3 μm; $\psi \equiv \varepsilon_o/R \approx 0.0004$; ω=157 s^{-1}; ω_0=15 s^{-1}; Str=636.6; $Re \cdot \psi \cdot Str$=0.297; $De \cdot Str$=0.667

3. Conclusions

- ◇ If bearing soft material deformations are neglected and the stroke increases (decreases) the bearing gap height, then the gap decreases (increases), i.e. it returns to its initial shape in the time interval from 0.00001 *s* to the 10.0 *s*. For this case the pressure distributions are presented in the left column of Fig. 2 (Fig.3). In two above cases, a hundred seconds after injury the pressure attains the same values (see the lower figure in the left column of Fig. 1 and Fig. 3).

- ◇ If bearing soft material deformations are taken into account and the stroke increases (decreases) the bearing gap height, then the gap decreases (increases), i.e. it returns to its initial shape but with the same bearing soft material deformations in the time interval from 0.00001 *s* to the 10 *s*.

For this case the pressure distributions are presented in the right column of Fig. 2 (Fig. 3). In two above cases, a hundred seconds after injury the pressure attains the same values (see the lower figure in the right column of Fig. 2 (Fig. 3).

- ◇ The pressure distributions in the right column of figures of Fig. 2 and Fig. 3 are of much smaller values than those of the pressure distributions in the left column of figures of Fig. 2 and Fig. 3. From this fact it follows that the pressure distributions obtained without bearing soft material deformations are of greater values than those of the pressure distributions in spherical bearing with soft material deformations in every moment after injury within the time interval from 0.00001 *s* to 10 *s*.

- ◇ The spherical bearing capacity with bearing soft material deformations taken into account is much smaller than that obtained for not deformable bearing surface.

- ◇ The greatest changes in the capacity of spherical bearing are attained within the time interval from 0.01 *s* to 0.3 *s* after injury. In the time interval from 0.00001 *s* to the 0.01 *s* after injury, the capacities are not changed, irrespective of whether the gap height increases or decreases due to the stroke.

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