

## A MULTI-PRODUCT AND MULTI-PERIOD FACILITY LOCATION MODEL FOR REVERSE LOGISTICS

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**Abstract :** Reverse logistics has become an important entity in the world economy. Businesses increasingly have to cope with product returns, mandated environmental regulations and increasing costs associated with product disposal. This study presents a cost-minimization model for a multi-time-step, multi-type product waste reverse logistics system. The facility location is a central issue of the logistics networks. In this article we are interested in optimizing of the sites facility location for a reverse logistics network for product end of life. Specifically, we present a Mixed Linear Program model for the strategic problem of collection sites facility location, cannibalization and recycling. This model allows determining to open or to close the sites previously in the reverse logistics network. All of these decisions are to minimize the costs of end of life product returns at various time periods considered in the planning. To solve the mathematical program, we have used the evaluation process and separation implemented in CPLEX commercial solver

**Keywords:** Reverse logistics; Facility location; End of life product, location problem, Mixed Linear Program model

### Introduction

According to the American Reverse Logistics Executive Council, Reverse Logistics is defined as [13]:“*The process of planning, implementing, and controlling the efficient, cost effective flow of raw materials, in-process inventory, finished goods and related information from the point of consumption to the point of origin for the purpose of recapturing value or proper disposal.*”

Because of changes in legislation, both for environmental protection and for economic and service reasons, an increasing number of companies now take into account reverse flows, going backwards from customers to recovery centers, within their logistics systems [7]. However, little research has been devoted to the planning and optimization of reverse logistics systems for network design. A reverse logistics system comprises a many activities. These activities include collection, cleaning, disassembly, test and sorting, storage, transport, and recovery operations. The latter can also be represented as one or a combination of several main recovery options, like reuse, repair, refurbishing, remanufacturing, cannibalization and recycling [3]. The management of the reverse flows is an extension of the traditional supply chains with used product or material either returning to reprocessing organizations or being discarded. Reverse supply chain management (RSCM) is defined as the effective and efficient management of the

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series of activities required to retrieve a product from a customer and either dispose of it or recover value [14].

The importance of studying reverse supply chains (RSCs) has increased for several reasons:

- the amount of product returns can be very high, with some industries experiencing returns at over 50% of sales [17]
- end-of-life take-back laws have proliferated over the past decade both in the European Union and in the United States, requiring businesses to effectively manage the entire life of the product [5]
- landfill capacity has become limited and expensive. Alternatives such as repackaging, remanufacturing and recycling have become more prevalent and viable [16].

As in a traditional logistics network, three types of decisions are involved in the decision problem related to the design of the reverse logistics network: the first concerns the sites facility location. The second concerns the flow of matter and information between these entities. Finally, the third is investment in labor and equipment in each of these facilities. In literature, these three categories of decisions refer to the problems of facility location, allocation and capacity [15]. For the facility location decision, it usually means selecting the most appropriate among previously identified potential sites in order to optimize one or more objective functions. In our research work, we focus specifically on modeling the problem of sites facility location in a reverse logistics network for product end of life.

Fleischmann [6] address the problem of localization in two approaches. The first is to add reverse logistics to a network of existing logistics while the second is to make a new logistics network. It is in second position that we're interested. Specifically, we seek to determine:

- the sites to open or close from a set of previously defined sites
- the flow of material from these sites.

The decisions mentioned above are taken to minimize the cost of products recovery at the end of life.

We wish to solve the problem of site facility location site in a reverse logistics network for various periods of time considered in strategic planning. The inclusion of such an element means to solve a multi-period problem: decisions taken at a given time are related to those taken in previous periods and take account of future needs. Consideration of several periods can also introduce environmental costs generated by the landfill of hazardous materials.

The rest of the paper is structured as follows: following this introduction, a literature review and related research work are illustrated in Section 2; this is followed by Section 3 which provides a discussion of our research problem, the conceptual model and the formulation model using Mixed Linear Program; In section 4 we present an illustrative example; Section 5 concludes the paper and provides suggestions for future research.

### Literature review

The facility location problem for the reverse logistics network has been the subject of several studies.

One of the first models to localize the center of recovery is that proposed by Caruso [4]. They describe a management system for solid waste in Lombardy region (Italy) including collection, transportation, recycling and disposal. They use a multi-objective model. Kroon L, Vrijens G. [10] propose a model to minimize the total treatment cost of reusable containers using a special case of a localization classic model. It is a mixed linear program model. Barros [2] present a model for the spent recycling in the Netherlands in order to minimize the total cost of recovery network.

Marin & Pelegrin [12] describe a Mixed Linear Program model for secondary products (uses) recovery. They assume that the quantity of secondary products is proportional to the amount of primary (new). In addition, they assume that returns are not necessarily returned to the outlet that delivered the product of departure, they can be returned at any open site.

Jayaraman [8] present a linear programming model for the design of a network of recovery of the end of life products. This model is composed of several collection sites, a treatment site and several clients whose returns are known. The problem is to determine the number of collection sites and to initiate treatment in order to minimize the total cost of distribution around. They assume that returns can be routed directly to the treatment center without the intermediary of a collection center, the capacity is limited.

Ahluwalia and Nema [1] propose a multi-objective linear model for the recovery of obsolete computers in New Delhi. It adds the transport of hazardous materials risks between collection sites of treatment and risks in the different sites that constitute the network.

Hai-Jie [9] propose a linear programming model to give decision support to determine the collection sites, recycling and landfill open for the recovery of waste materials while limiting the capacity of sites. It aims to minimize the operational costs of the logistics network for product recovery at the end of life.

Lu [11] propose a linear programming model to locate sites in a reverse logistics network. These intermediate centers where used products are disassembled, cleaned and sorted to be transported to the remanufacturing sites (remanufacturing centers) in which the parts from the used products are used to manufacture new products. They assume that the capacity of sites is unlimited and that the poor quality may be disposed in the centers as intermediaries in the remanufacturing sites.

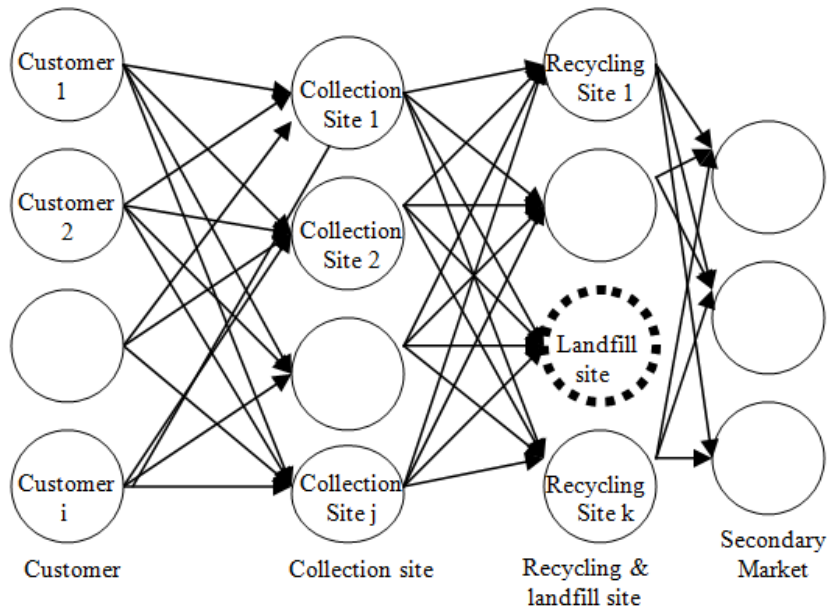
When we review the literature on the facility location of sites in a reverse logistics network, we find that the models developed so far are based on a classical model of locating warehouses which are added one or two elements for reverse logistics as capacity, number of facilities open, non-negativity of decision variables, on the one hand. On the other hand, we note that the majority of models are single-period

which cannot measure the impact of long-term decisions. Note also that most models proposed in the literature were developed for a reverse logistics network structure (no authorization flow between sites). Some models are easily adaptable from one network to another. Finally, they do not reflect environmental costs (emission of toxic gases) generated by hazardous materials.

**Problem statement**

Most location models developed so far are models of single-product or single period. Indeed, some models are not adaptable for a network to another. Moreover, they do not take account of dynamism of reverse logistics program. To overcome these drawbacks, we propose a generic model multi-product and multi-period site locations for the reverse logistics of products at the end of life in order to minimize logistics costs. The proposed model can be used to solve the facility location problem of sites for varied structure networks.

The proposed model refers to the structure of the reverse logistics network shows in Fig 1. In this network, the company gets the products at the end of life returned by its customers through its collection sites. After their yards, they are transported to treatment sites. Some will be disposed of in landfill (eg hazardous materials). Further, they will be recycled (metal and plastic). Once processed, finished products from the recycling are used for the manufacture of new products which will then be offered to consumers and consumed.



**Figure 1. Reverse logistics network Structure for product end of life**

The proposed model aims to determine the sites to open or to close each period and the flow of goods between the different sites that make up the reverse logistics network (site collection, site recycling and landfill).

In this model, we assume that:

- the location of potential sites for the collection and treatment is known at period
- the costs of opening the site and transportation costs are known in advance
- the capacity of each site is limited to the period
- the cost of investment and divestiture of a portion of capacity at a site from one period to another are fixed
- the various costs considered in the different nodes are: opening site cost and cost of unit transportation of products at the end of life
- several products at the end of life to be recovered by the compan
- no storage in the collection site.

### Indices

$p$	End of life Product index, $p = \{1, \dots, P\}$
$i$	Customer, $i = \{1, \dots, I\}$
$j$	Potential collection site, $j = \{1, \dots, J\}$
$t$	Time period, $t = \{1, \dots, T\}$ , ( $T$ : planning horizon)
$k$	Potential Recycling site, $k = \{1, \dots, K\}$ .
$k'$	Potential landfill site, $k' = \{1, \dots, K'\}$ .

### Parameters

$F_{jt}$	Fixed cost of collection site $j$ opening in period $t$
$F_{kt}$	Fixed cost of recycling site $k$ opening in period $t$
$F_{k't}$	Fixed cost of landfill site $k'$ opening in period $t$
$C_{pijt}$	Cost of end of life product $p$ transporting from customer $i$ to the collect site $j$ at time $t$
$C_{pjkt}$	Cost of end of life product $p$ transporting from collection site $j$ to the recycling site $k$ at time $t$
$C_{pj k' t}$	Cost of end of life product $p$ transporting from collection site $j$ to the landfill site $k'$ at time $t$
$B_{jt}$	Collection site $j$ capacity at time $t$
$D_{kt}$	Recycling site $k$ capacity at time $t$
$E_{k't}$	Landfill site $k'$ capacity at time $t$
$Y_{\max}$	Maximum number of collection sites to open
$Z_{\max}$	Maximum number of recycling sites to open
$W_{\max}$	Maximum number of landfill sites to open

$Y_{\min}$	<i>Minimum number of collection sites to open</i>
$Z_{\min}$	<i>Minimum number of recycling sites to open</i>
$W_{\min}$	<i>Minimum number of landfill sites to open</i>
$G_t$	<i>The sum of the customer returns at time t</i>
$M$	<i>A constant size A</i>

Decision variables

$Y_{jt}$	<i>Binary variable equal to 1 if site j is open at time t</i>
$Z_{kt}$	<i>Binary variable equal to 1 if site k is open at time t</i>
$W_{k't}$	<i>Binary variable equal to 1 if site k' is open at time t</i>
$X_{pijt}$	<i>End of life products quantity stored at customer i and transported to the collection site j in period t.</i>
$X_{pjkt}$	<i>End of life products quantity to recycled and transported from the collection site j to recycling site k at period t.</i>
$X_{pjk't}$	<i>End of life products quantity to eliminate and transported from the collection site j to landfill site k' at period t.</i>

Using above indices and parameters; the mathematical formulation standard for this problem can be stated as follows.

$$\begin{aligned} \text{Min } \Lambda = & \sum_j \sum_t F_{jt} Y_{jt} + \sum_k \sum_t F_{kt} Z_{kt} + \sum_p \sum_t \sum_j \sum_t C_{pijt} X_{pijt} \\ & + \sum_p \sum_j \sum_k \sum_t C_{pjkt} X_{pjkt} + \sum_p \sum_j \sum_{k'} \sum_t C_{pjk't} X_{pjk't} \end{aligned} \quad (1)$$

Subject to

$$\sum_j X_{pijt} \geq G_t \quad \forall p, \forall i, \forall t \quad (2)$$

$$\sum_j X_{pijt} = \sum_k X_{pjkt} + \sum_{k'} X_{pjk't} \quad \forall p, \forall i, \forall t \quad (3)$$

$$\sum_p \sum_i \sum_j X_{pijt} \leq B_{jt} Y_{jt} \quad \forall k, \forall t \quad (4)$$

$$\sum_p \sum_i \sum_k X_{pjkt} \leq D_{kt} Z_{kt} \quad \forall j, \forall t \quad (5)$$

$$\sum_p \sum_i \sum_{k'} X_{pjk't} \leq E_{k't} W_{k't} \quad \forall j, \forall t \quad (6)$$

$$\sum_p \sum_i \sum_j X_{pijt} \leq M Y_{jt} \quad \forall k, \forall t \quad (7)$$

$$\sum_p \sum_i \sum_k X_{pjkt} \leq M Z_{kt} \quad \forall j, \forall t \quad (8)$$

$$\sum_p \sum_i \sum_{k'} X_{pjkt} \leq M W_{k't} \quad \forall j, \forall t \quad (9)$$

$$Y_{\min} \leq \sum_j Y_{jt} \leq Y_{\max} \quad \forall t \quad (10)$$

$$Z_{\min} \leq \sum_k Z_{kt} \leq Z_{\max} \quad \forall t \quad (11)$$

$$W_{\min} \leq \sum_{k'} W_{k't} \leq W_{\max} \quad \forall t \quad (12)$$

$$Y_{jt}, Z_{kt}, W_{k't} \in \{0,1\} \quad \forall j, \forall k, \forall k', \forall t \quad (13)$$

$$X_{pijt}, X_{pjkt}, X_{pjkt} \geq 0 \quad \forall p, \forall i, \forall j, \forall k, \forall k', \forall t \quad (14)$$

The main objective of this mathematical model is the determination of the collection and treatment sites (site of recycling and landfill) location in each period and the flow between these sites. This model aims to minimize the costs of end of life products recovery. The mathematical model specifies is the variety of end-of-life product and multiple periods.

The constraint (2) describes that all the end of life products are collected by the company. Float balance between the different sites is assured by constraint (3).

The respect of the available capacity is provided by the constraint (4,5,6). Constraints (7), (8) and (9) ensure that if a site is closed, the flow of incoming and outgoing products are zero, M is a size constant. The respect of opening site constraint is provided by the constraint (10,11,12). Constraint set (13) check for binary variables and the last constraint s (14) check for the non-negativity of decision variables.

### An illustrative example and insight into the model

We apply our model on a dataset taken from the literature on the reverse logistics of electronic products at end of life in India, Indeed, electronic products end of life will reach 217 440 tones in 2010 in India [1]. Using a numerical example, we will illustrate how the model works in the proposed framework a gain some insights into the proposed model. A small set of data is prepared reflecting the real business situation. Indeed, the company recovers the end of life product of its customers to be sorted and disassembled in the collection sites. Hazardous products will be disposed of in landfills. Materials will be transported to recycling sites.

The reverse logistics network for the application is composed of: 3 customer, 9 collection site, 7 recycling site, 2 landfill site, 4 periods' time and 6 end of life products. We obtained the results using a Windows XP, Pentium 4, 2.4 GHz and 160 GB of memory (Figure 2). Fig 3 shows potential site of collection, recycling and landfill to open in every period.



Figure 2. Implementation of the proposed model with the case of application using the CPLEX

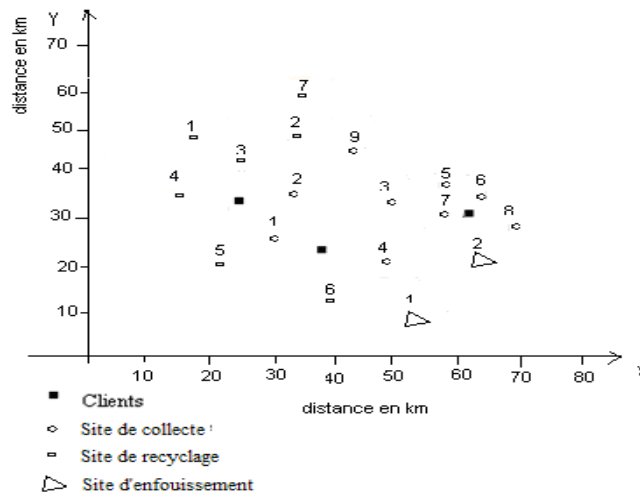


Figure 3. Customers and sites to open location



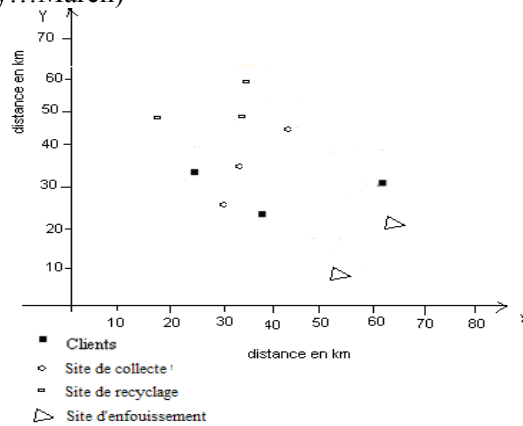
The results of the case of application considered are shown in the table below.

**Table 1. Numerical results of the application**

Constraint number	Total variable	Binary variable number	Execution time (s)	Optimal cost
885	2665	72	33.5	693150

The optimal cost is found 693,150 including three collection sites, three recycling sites and two landfills are open. CPLEX found the optimal solution after 33.5 minutes second. The sites selected for each period (quarter) are represented in figures 4, 5, 6 and 7.

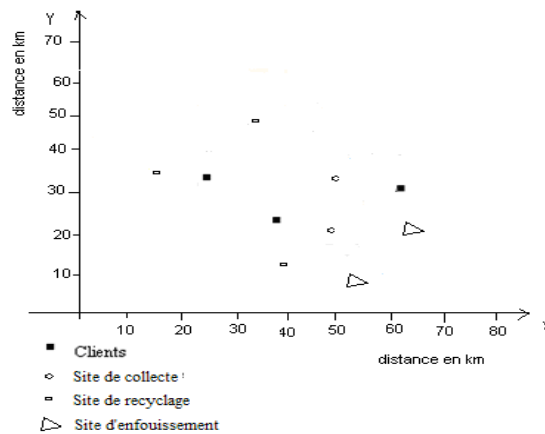
• Period 1( January...March)



**Figure 4. The different sites to open in the period 1**

• Period 2(April...june)

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**Figure 5. The different sites to open in the period 2**

• Period 3(July...September)

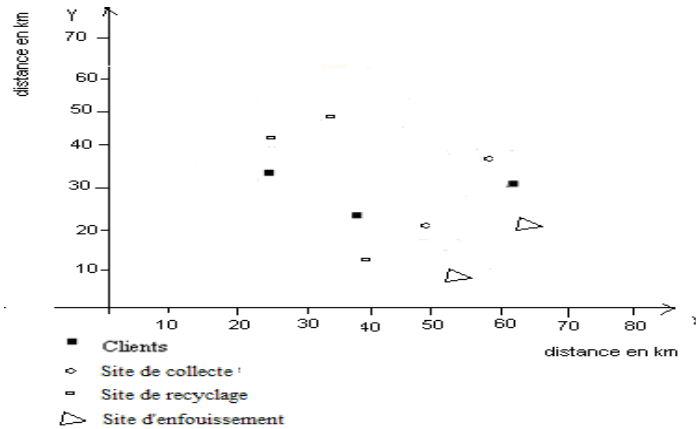


Figure 6. The different sites to open in the period 3

• Period 4 (October...December)

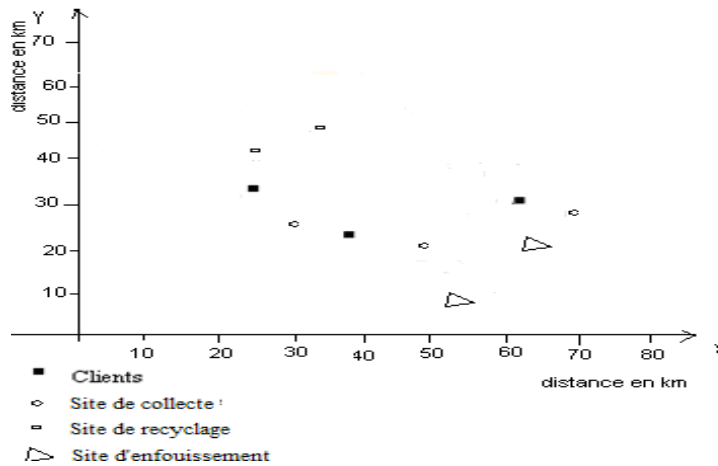


Figure 7. The different sites to open in the period 4

In the period 4, three collection sites will be open because the quantity of returns less than in the first period. Thus, the proposed model allows taking into account the dynamism of a reverse logistics system. Moreover, we note that when we increase the number of periods considered in the strategic planning, logistics costs are declining. This can be explained by increasing the quantity of products at the end of life recovered.

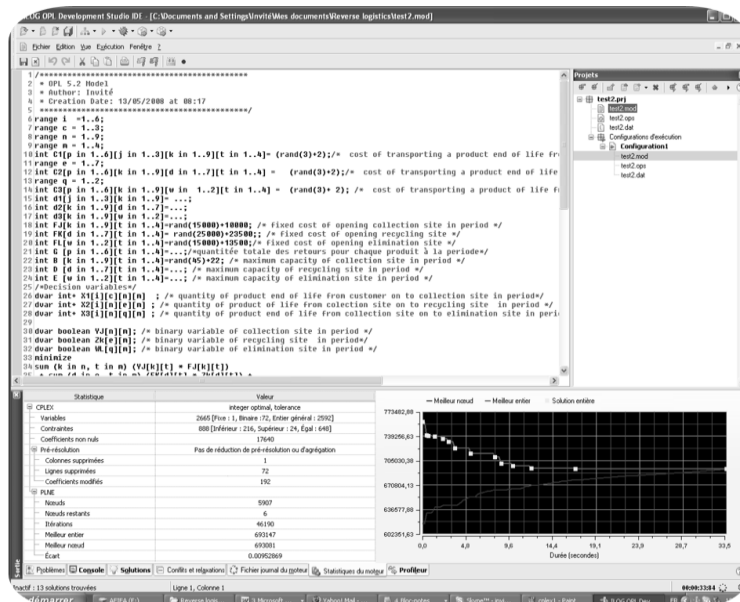


Figure 8. Execution time for the resolution of illustrate example

However, we note that the costs increase significantly in establishing a strategic planning on 9 periods (Fig.9).

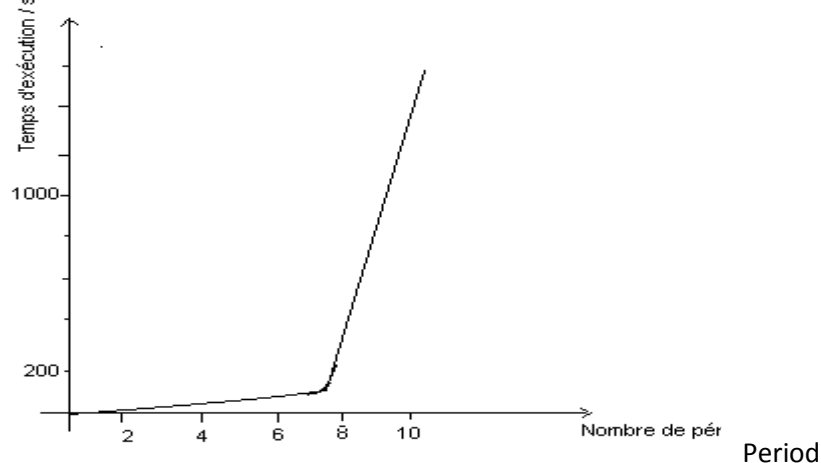


Figure 9. Execution time depending on the number of periods in the strategic planning

### Summary

This paper has presented a cost-minimization model for minimizing the total operating costs of a multi-period, multi-type product reverse logistics system. By identifying the critical activities and related basic requirements involved

in the process of end of life reverse logistics operations, a discrete-time linear objective function coupled with thirteen groups of constraints are formulated. Compared to early literature on addressing end life recovery and facility location, the model found in this study has two distinctive features.

First, by coordinating the critical activities of reverse logistics management, the proposed method addresses the classical network of end life product treatment problem with a generic model. Second, In this work, we established a multi-product and multi-period location of sites for the reverse logistics of the end of life products. The proposed model can be applied to varied structure reverse logistics network. It can determine the state of sites, their openness, closure, available capacity and material flow between the various entities of the logistics network. All decisions shall be taken to minimize logistics costs.

However, we considered that the returns quantity is determinist and that investment in the capacity of a site is fixed. In literature the problem of facility localization is an NP-hard problem. To solve the mathematical program, we use the evaluation and separation process located in a commercial solver Cplex. The modeling problem of the facility location in reverse logistics network sites is an open area of research.

The multi-product and multi-period Model, aims to minimize the costs in considered time in strategic planning, ignores the negative effects generated by the opening of landfill sites. We can envisage a multi-objective model minimizes both the end of life products recovery cost and environmental costs. To obtain a solution of the multi-product and multi-period model second in a reasonable time, we proposed to use other technique like the genetic algorithm.

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#### MODEL LOKALIZACJI WIELOASORTYMENTOWEJ I WIELOOKRESOWEJ W LOGISTYCE ODWROTNEJ

**Streszczenie:** Logistyka odwrotna stała się bardzo ważnym elementem świata ekonomii. Przedsiębiorstwa coraz częściej muszą radzić sobie ze zwrotami produktów, obowiązującymi przepisami dotyczącymi środowiska oraz rosnącymi kosztami usuwania wyrobu. W artykule zaprezentowano model minimalizacji kosztów dla systemu wielu kroków czasowych usuwania wielu typów produktów w logistce odwrotnej. Problem lokalizacji jest główną kwestią sieci logistycznych. W niniejszej pracy skupiono się przede wszystkim na optymalizacji miejsc lokalizacji w sieciach logistyki odwrotnej w przypadku końcowego życia produktu. Zaprezentowano model Liniowego Programowania Mieszanego w strategicznym problemie lokalizacji w miejscu zbierania odpadów, kanibalizacji i recyklingu. Ten model umożliwia określenie wcześniejszego otwarcia lub zamknięcia miejsc zbierania odpadów w sieci logistyki odwrotnej. Celem tych decyzji jest minimalizacja kosztów końcowego życia zwrotu wyrobu w różnych okresach czasu na etapie planowania. Aby rozwiązać ten problem matematyczny zastosowano proces oceny i separacji zaimplementowany w komercyjnym solverze CPLEX.

#### 逆向物流的多产品多周期设备选址模型

**摘要:** 本项研究为多时间步骤, 多类型产品浪费的逆向物流系统提出了一个消耗最小化的模型。在本文中, 我们对于逆向物流系统中的终端产品设备基地选址颇感兴趣。特别是, 我们提出混合线性程序模型以解决收集站选址, 拆解, 回收循环等问题。为了解决数学问题, 我们使用评价机制和CPLEX商业解决方案中的求解分离。