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GAS COUPLING OF LOADS IN COMPLEX GLASS PANELS

Introduction

A complex glass panel (a insulating glass unit) is manufactured from two sheets of glass that are hermetically connected with a spacer. Hermetically sealed gas space between two sheets of glass has large influence on physical and mechanical parameters of the complex glass panel. Hermetic seal is gained by two-stage sealing (Fig. 1). A glass unit which lost its hermetic seal is considered not to be usable in buildings anymore [1-3].

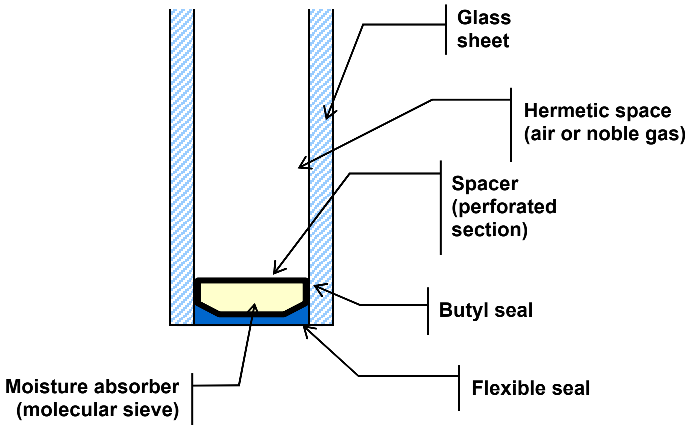

Fig. 1. Diagram of the complex glass panel construction with two-stage sealing

Complex glass panels are the result of searching solutions for limiting heat loss in a building. Hermetically sealed gas space limits convection exchange of heat. However the sealed gas space is an integral part of a unit considering its mechanical properties.

In operational conditions the physical gas state is defined by pressure p, temperature T and volume v. This state describes the gas law with accuracy sufficient for practical purposes:

$$\frac{p_0 \cdot v_0}{T_0} = \frac{p_k \cdot v_k}{T_k} = \text{const} \tag{1}$$

where:

p_0, T_0, v_0 - initial parameters of gas in the gas space - gas parameters gained during production process, or other, assumed as a reference level for stress in glass shees,

p_k, T_k, v_k - operational parameters of gas in the gas space - gas parameters in the gas space with assumed load for glass sheets.

Changing gas parameters cause loads - connected with the change of internal pressure - influencing both sheets in a glass unit. Dependence (1) is the basis of further consideration involving characteristics of these loads.

It ought to be remembered that there are agents absorbing water vapour in an insulating glass unit construction. Considering this fact, there is a possibility that reliable p_0 is lower of water vapour partial pressure absorbed during operating than pressure at the moment of glass sheets hermetic sealing. However further analyses performed by the Author have shown that mistakes in assuming initial pressure do not have a meaningful influence on the calculations result of the resultant load of glass sheets.

1. Displacement of sheets in a glass unit

Feldmeier in his elaboration [4] tries to define static values in glass units loaded by climatic factors. He proposes treatment of loads caused by changes in the external temperature and pressure as surface-loads evenly distributed on the glass surface. When the temperature of external air rises or when the atmospheric pressure falls down, overpressure in the gas space arises. Glass sheets of a unit bulge outwards. In the opposite case, the pressure below atmospheric arises in the gas space which causes concavities in glass sheets (Fig. 2).

Some physical phenomena, which influence displacement and loading glass sheets, were not taken into consideration in the process of describing an analytical model. They are as follows:

- displacement of glass sheets caused by diversification of their surface temperatures,
- diversification of the atmospheric pressure on both sides of a partition; this diversification might be caused by the difference in temperatures or the wind.

A mathematical model presented below is an alternative for the solution provided by Feldmeier with the consideration of influence which was omitted by him.

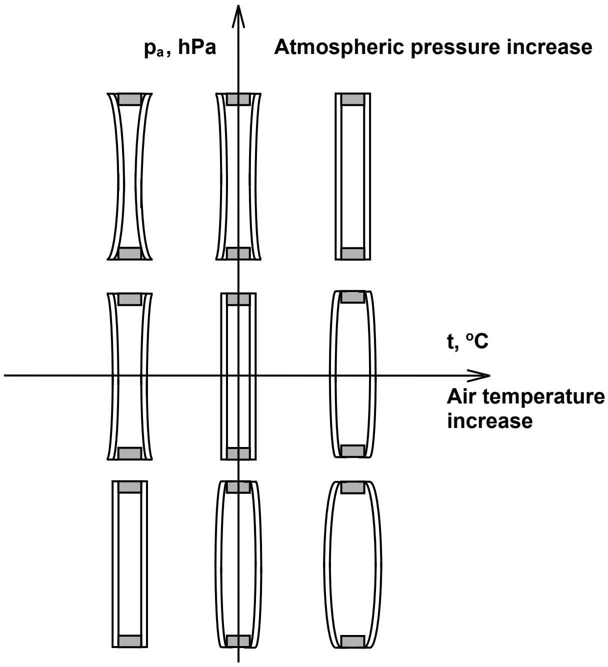

Fig. 2. Displacement of glass sheets in a glass unit caused by climatic factors [4]

2. Volume of a single glass sheet displacement

Let us look at the displacement of a single glass sheet.

A glass sheet displaces under an influence of any surface load q (Fig. 3a). Volume of a shape limited by a deformed and non-deformed glass sheet surface is called volume of a single glass sheet displacement Δv_1. This value might be defined by integration of a glass sheet function deflection w(x,y):

$$\Delta v_1 = \pm \int_P w(x, y)dxdy \tag{2}$$

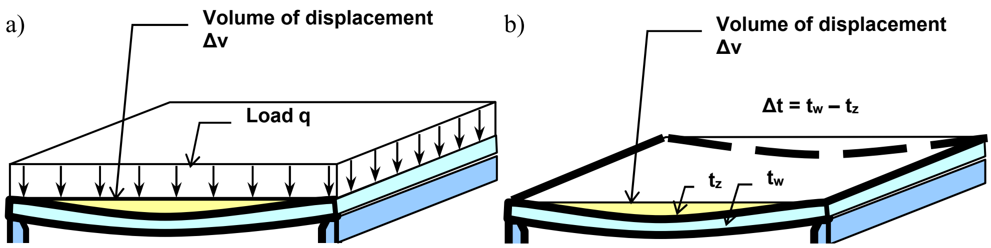

Fig. 3. Volume of a single glass sheet displacement with: a) surface load, b) temperature load

The general theory of plates (2) says that with a thin plate both, deflection and field volume of a glass sheet displacement are linear functions of a load q. Assuming factor of proportionality α_v formula (2) might be written as follows:

$$\Delta v_1 = \pm\alpha_v \cdot q \tag{3}$$

A sheet of glass loaded by linear (in glass thickness) temperature changes Δt is displaced as well, that is if the outer surface of a glass sheet has the temperature different from the outer (Fig. 3b). In this case deflection and field volume of a glass sheet displacement are linear functions of the temperature change in glass thickness Δt. Assuming the factor of proportionality α_v:

$$\Delta v_1 = \pm\alpha_t \cdot q \tag{4}$$

In case of a complex glass unit (two sheets with hermetic space, Fig. 4) the volume of glass unit displacement $\Delta v = \Delta v_1 + \Delta v_2$ is the sum of volumes of both glass sheets displacement. In other words it is the change of the space between glass sheets volume caused by the displacement of glass sheets forming a unit, under a load.

3. Values characterizing sheets cooperation in a glass unit

To calculate the resultant operational load of glass sheets it is necessary to define the change of gas pressure Δp in the gas space of a glass unit, that is the difference between the pressure in operational conditions and the initial pressure ($\Delta p = p_k - p_0$). Value Δp is called the gas coupling of loads. This coupling is tightly connected with changes in volume and gas pressure in the gas space of a loaded glass sheet, caused by a field volume of glass sheets limiting the gas space displacement [3].

Besides emergency situations, the most common cases of loading in using windows with glass units are:

– Change of the external atmospheric pressure.
– Change of gas pressure in the gas space caused by its gas temperature change.
  In both above cases climatic loads arise which would not arise in case of a non-hermetic gas space; however the gas coupling diminishes glass sheets load in relation to a unit with perfectly rigid glass sheets.
– Loads caused by the pressure of wind speed (pressure and suction); in this case there is a favourable gas coupling; the part of load influencing directly an external glass sheet is transmitted to an internal glass sheet.
– Loads caused by linear (in glass thickness) temperature changes; gas pressure in the gas space does not allow for glass sheets displacement in a unit without any stress; the gas coupling causes unfavourable surface loads.

Above mentioned cases consider loads which might be treated as evenly distributed on the whole surface of a particular glass sheet and they can occur at the same time.

Operational loads of glass sheets might be estimated into two ways:

– All the influence should be considered together - characteristic values should be estimated from one general formula.
– The principle of superposition should be applied, meaning each external load should be considered separately and resulting stress in glass sheets or resultant loads should be summed up.

Analyses performed by the Author have shown that both ways lead to almost identical results (the difference amounts to less than 1%).

Following values, besides Δp, will be used in the analysis of a glass unit:

p_0, p_k [kPa], T_0, T_k [K], v_0, v_k [m^3] - look at the description for formula (1),

p_a [kPa] - atmospheric pressure p_a, in surroundings of glass sheets,

Δp_a [kPa] - atmospheric pressure change, in relation to the initial pressure p_0,

p [kPa] - operational resultant load of the single glass sheet surface - the sum of loads of both glass sheets surfaces,

α_v [m^5/kN] - volume of displacement under a single surface load - volume of a single glass sheet displacement under the load of 1 kN/m^2,

α_t [m^3/K] - volume of displacement under a single load by linear (in glass thickness) temperature changes - volume of a single glass sheet displacement with the temperature difference on its both surfaces amounting to 1 K.

Some of the described values might have indexes (1, 2). These indexes exist in case of necessity to indicate a particular glass sheet.

4. Gas coupling of loads - general case

We are considering a general case of climatic load during a glass unit usage. External surface loads q_1 and q_2 influence glass sheets of a glass unit. In the neighbourhood of a glass sheet 1 atmospheric pressure is p_{a1} and causes its loading $\Delta p_{a1} = p_{a1} - p_0$. Analogous load influences a glass sheet 2 (Fig. 4). Above mentioned external loads were considered to be positive if they were turned towards the middle of a unit. Volume of displacement under a load of glass sheets causes a change of gas pressure in the gas space Δp. Δp is also influenced by the gas temperature changes in the gas space $\Delta T = T_k - T_0$.

Volume of a glass sheet 1 displacement Δv_1 is described by the dependence

$$\Delta v_1 = -\alpha_{v1} \cdot p_1 \tag{5}$$

where p_1 means the resultant load of a particular glass sheet. The minus sign means that positive external loads correspond to diminishing volume of the gas space. Coefficient α_{v1} is dependent on a glass sheet rigidity and the way of its connection with the frame. Volume of a glass sheet 2 displacement might be described by analogous dependencies.

The formula for the change of glass sheets loads caused by the gas coupling Δp might be derived during one stage by solving the system of equations.

The first equilibrium equation of the system is the gas law:

$$(p_0+\Delta p)\cdot(v_0+\Delta v)\cdot T_0 = p_0\cdot v_0\cdot(T_0+\Delta T) \tag{6}$$

The second is the balance of field volume of the gas space displacement:

$$\Delta v = \Delta v_1+\Delta v_2 = -\alpha_{v1}\cdot(q_1+p_{a1}-p_0-\Delta p)-\alpha_{v2}\cdot(q_2+p_{a2}-p_0-\Delta p) \tag{7}$$

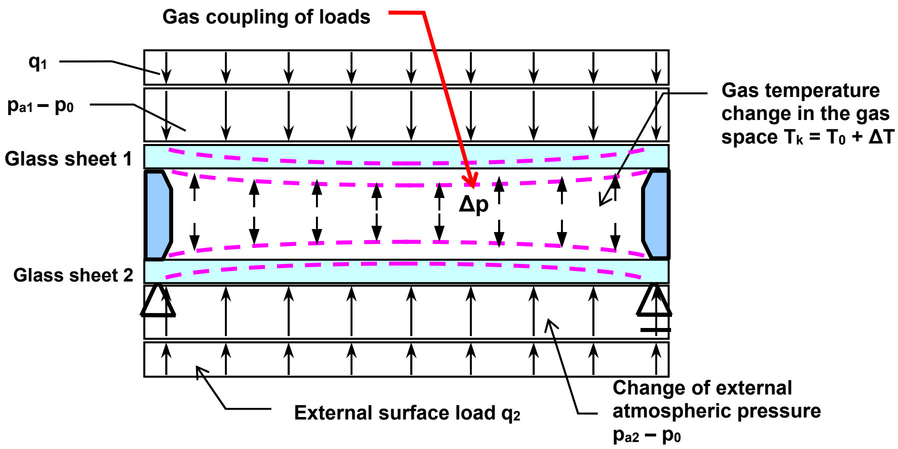

Fig. 4. Diagram of glass sheets load in a hermetic unit

The solution of the above system is a general formula:

$$\Delta p = -p_0 + \frac{p'_{a1}\cdot\alpha_{v1}+p'_{a2}\cdot\alpha_{v2}-v_0}{2\cdot(\alpha_{v1}+\alpha_{v2})} + \sqrt{\left(\frac{p'_{a1}\cdot\alpha_{v1}+p'_{a2}\cdot\alpha_{v2}-v_0}{2\cdot(\alpha_{v1}+\alpha_{v2})}\right)^2+\frac{p_0\cdot v_0\cdot T_k}{(\alpha_{v1}+\alpha_{v2})\cdot T_0}} \tag{8}$$

In formula (8) following designations have been applied:

$p'_{a1} = p_{a1}+q_1$ $\qquad$ $p'_{a2} = p_{a2}+q_2$

Ultimately resultants of particular glass sheets loads amount to:

– For glass sheet 1 $\qquad p_1 = q_1+p_{a1}-p_0-\Delta p$ (9a)

– For glass sheet 2 $\qquad p_2 = q_2+p_{a2}-p_0-\Delta p$ (9b)

5. Gas coupling of loads by changing the atmospheric pressure and gas temperature in the gas space

In case of: lack of external surface loads $q_1 = q_2 = 0$, equal air pressure on both sides of a unit $p_{a1} = p_{a2} = p_a$, equal rigidities and support conditions of both glass sheets $\alpha_{v1} = \alpha_{v2} = \alpha_v$, the formula (10) for the change Δp of glass sheets load has been derived:

$$\Delta p = -p_0 + \frac{p_a}{2} - \frac{v_0}{4 \cdot \alpha_v} + \sqrt{\left(\frac{p_a}{2} - \frac{v_0}{4 \cdot \alpha_v}\right)^2 + \frac{p_0 \cdot v_0 \cdot T_k}{2 \cdot \alpha_v \cdot T_0}} \tag{10}$$

When only the temperature changes ($p_a = p_0$) it has been derived:

$$\Delta p = -\frac{p_0}{2} - \frac{v_0}{4 \cdot \alpha_v} + \sqrt{\left(\frac{p_0}{2} - \frac{v_0}{4 \cdot \alpha_v}\right)^2 + \frac{p_0 \cdot v_0 \cdot T_k}{2 \cdot \alpha_v \cdot T_0}} \tag{11}$$

When only external pressure changes ($T_k = T_0$):

$$\Delta p = -p_0 + \frac{p_a}{2} - \frac{v_0}{4 \cdot \alpha_v} + \sqrt{\left(\frac{p_a}{2} - \frac{v_0}{4 \cdot \alpha_v}\right)^2 + \frac{p_0 \cdot v_0}{2 \cdot \alpha_v}} \tag{12}$$

6. Interaction of a single load of one of the glass sheets

Particular pressure $p_a = p_0$ and temperature $T_k = T_0$ of gas in the gas space and a single external load q of one of the glass sheets (Fig. 5) have been assumed. In case of a surface load of glass sheet 1, the gas coupling causes secondary loading of a glass sheet 2 and equal unloading of a glass sheet 1. The result of the gas coupling Δp is the interaction between glass sheets that is transmission of loads between glass sheets of a glass unit. Considering above-mentioned assumptions, the gas coupling amounts to:

$$\Delta p = -\frac{p_0}{2} + \frac{q \cdot \alpha_{v1} - v_0}{2 \cdot (\alpha_{v1} + \alpha_{v2})} + \sqrt{\left(\frac{p_0}{2} + \frac{q \cdot \alpha_{v1} - v_0}{2 \cdot (\alpha_{v1} + \alpha_{v2})}\right)^2 + \frac{p_0 \cdot v_0}{(\alpha_{v1} + \alpha_{v2})}} \tag{13}$$

If rigidities and support conditions of both glass sheets are equal:

$$\Delta p = -\frac{p_0}{2} + \frac{q}{4} - \frac{v_0}{4 \cdot \alpha_v} + \sqrt{\left(\frac{p_0}{2} + \frac{q}{4} - \frac{v_0}{4 \cdot \alpha_v}\right)^2 + \frac{p_0 \cdot v_0}{2 \cdot \alpha_v}} \tag{14}$$

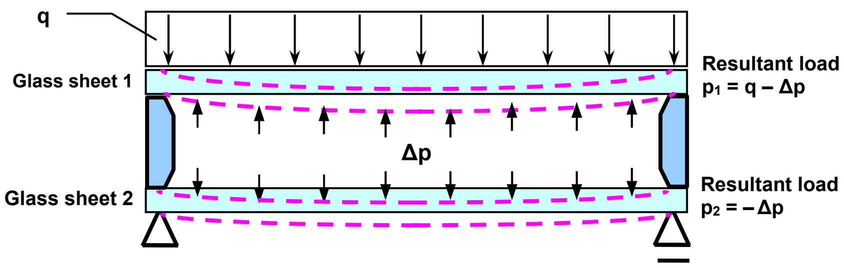

Fig. 5. Diagram of transmitting surface load between glass sheets of a glass unit

7. The influence of the gas coupling caused by linear (in glass thickness) temperature changes on glass sheets load

The influence might be easily analysed with omitting external pressure changes and gas temperature in the gas space. It has been assumed that surfaces of one of the glass sheets in a glass unit have different temperature and that temperature changes in a linear manner in glass thickness. Such a distribution of temperature causes that a single glass sheet bends with its convexity towards higher temperature.

In case of losing its hermetic seal and a loose connection between a glass sheet and a spacer, a loaded glass sheet displaces (Fig. 6a). This displacement does not cause stress in a glass sheet (besides edge stress, described in the plate theory [5]). The volume of this displacement equals $\Delta v = -\alpha_t \cdot \Delta t$. The 'minus' sign means that the difference in temperature Δt, assumed on the diagram (Fig. 6a) as positive, corresponds to reducing the gas space volume.

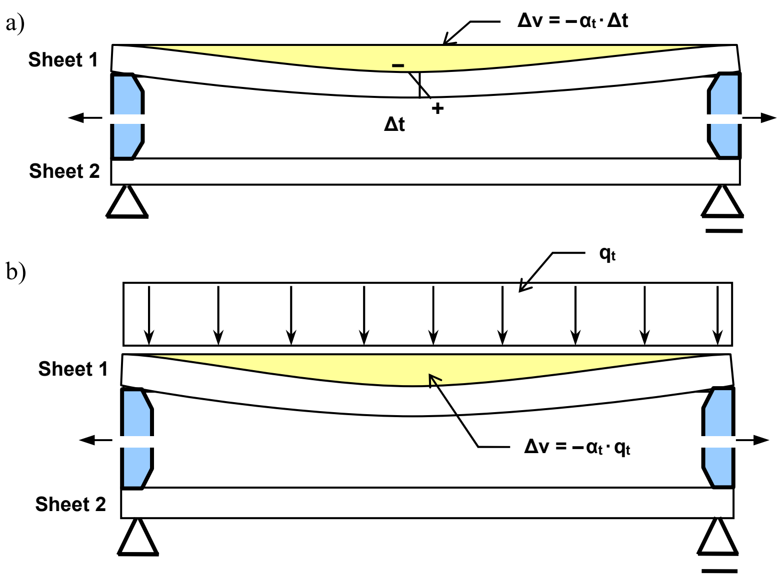

Fig. 6. Volume of displacement caused by linear (in glass thickness) temperature changes with a gas space without a hermetic seal: a) actual load, b) substitute surface load

In order to define the gas coupling Δp of the discussed temperature load, a substitute surface load has been introduced q_t (Fig. 6b). It has been also assumed that the volume of displacement under the influence of load q_t and the volume of displacement of a glass sheet with a temperature load are equal:

$$\Delta v = -\alpha_t \cdot \Delta t = -\alpha_v \cdot q_t \tag{15}$$

Formula (15) results in:

$$q_t = \frac{\alpha_t}{\alpha_v} \cdot \Delta t \tag{16}$$

Taking into consideration the influence of described load on the gas coupling Δp with a hermetic gas space, means adding a substitute load (q_{t1}, q_{t2}) and a surface load q_1 or q_2 applied in the formula (8). Of course q_t is not a constituent of the resultant load of a glass sheet, it only influences the change of the gas coupling value.

In case of using the above-described method of superposition, temperature influence might be considered differently. In case of a hermetic gas space, after assuming the surface temperature changes of glass sheets as in Figure 7 and neglecting other influence, the balance of the field volume of the gas space displacement might be defined with a formula:

$$\Delta v = -\alpha_t \cdot \Delta t_1 - \alpha_t \cdot \Delta t_2 + 2\Delta p \cdot \alpha_v \tag{17}$$

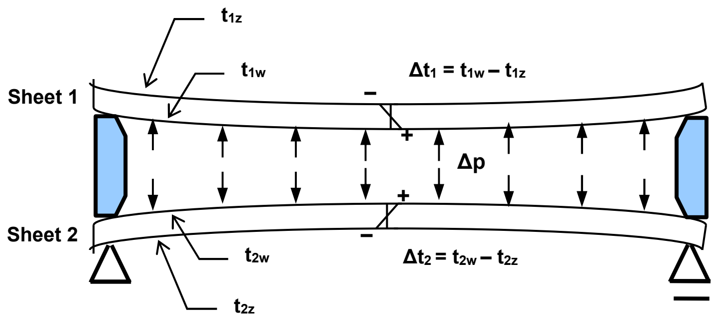

Fig. 7. Gas couplings of loads caused by temperature changes in both glass sheets

After having considered the gas law, the formula (18) for the gas coupling of the discussed load has been derived:

$$\Delta p = -\frac{p_0}{2} + \frac{\alpha_t \cdot \Sigma\Delta t - v_0}{4 \cdot \alpha_v} + \sqrt{\left(\frac{p_0}{2} + \frac{\alpha_t \cdot \Sigma\Delta t - v_0}{4 \cdot \alpha_v}\right)^2 + \frac{p_0 \cdot v_0}{2 \cdot \alpha_v}} \tag{18}$$

where $\Sigma\Delta t = \Delta t_1 + \Delta t_2$.

In case of different proportionality coefficients for both glass sheets, reliable α_v and α_t might be defined from formulas:

$$\alpha_v = (\alpha_{v1} + \alpha_{v2})/2, \quad \alpha_t \cdot \Sigma\Delta t = \alpha_{t1} \cdot \Delta t_1 + \alpha_{t2} \cdot \Delta t_2 \tag{19}$$

Calculated gas coupling Δp is the resultant load of both glass sheets in this case.

8. Values of coefficients α_v and α_t

Values of these coefficients might be indicated by means of the classic plate theory with the assumption of particular flexibility of glass sheets in connecting with the frame. In the easiest case (but sufficient for practical purposes) of free connection, coefficients α_v and α_t might be calculated from formulas:

$$\alpha_v = \alpha_v' \frac{a^6}{D}, \quad \alpha_t = \alpha_t' \frac{a^4 \cdot \beta \cdot (1+\nu)}{g} \tag{20}$$

where:
a - glass sheet width (shorter dimension),
β - glass linear expansion coefficient $\beta \approx 9 \cdot 10^{-6}$ 1/K,
ν - Poisson's ratio of glass $\nu = 0.2 \div 0.22$,
g - glass thickness, m,
D - plate rigidity of a glass sheet expressed by the formula:

$$D = \frac{E \cdot g^3}{12 \cdot (1 - \nu^2)} \quad [\mathrm{kN \cdot m}]$$

where:
E - Young's modulus of glass, $E \approx 70$ GPa,
α_v', α_t' - dimensionless coefficients according to Table 1.

TABLE 1

Coefficients α_v', α_t' depending on the relation of a glass sheet length to a glass sheet width s

s	1.0	1.1	1.2	1.3	1.4	1.5
α_v'	0.001703	0.002246	0.002848	0.003499	0.004189	0.004912
α_t'	0.03513	0.04233	0.04982	0.05753	0.06540	0.07339
s	1.6	1.7	1.8	1.9	2.0	3.0
α_v'	0.005659	0.006427	0.00721	0.008004	0.008808	0.017055
α_t'	0.08147	0.08962	0.09782	0.10605	0.11432	0.19745

References

[1] Respondek Z., Parametry techniczne nowoczesnych przegród szklanych. Cz. 1. Rodzaje szkła budowlanego. Konstrukcja szyb zespolonych, Świat Szkła 2007, 10.

[2] Respondek Z., Rajczyk M., Sposob opriedielienia ekspluatacyonnogo davlenia v stieklopakietah. Doklady 60 nauchnoy konferenci professorov, prepodavateley, nauchnyh rabotnikov, inzhenerov i aspirantov universiteta, Sankt Peterburg 2003.

[3] Respondek Z., Sprzężone gazowo płyty szklane w budownictwie. Sposoby badań i obliczeń, Seria Monografie nr 151, Wydawnictwo Politechniki Częstochowskiej, Częstochowa 2008.

[4] Feldmeier F., Obciążenia temperaturowe zespolonych szyb izolacyjnych, Świat Szkła 1997, 6 (part 1), Świat Szkła 1998, 1 (part 2).

[5] Timoszenko S., Woinowsky-Krieger S., Teoria płyt i powłok, Arkady, Warszawa 1962.

Abstract

The paper describes the mathematic model for calculating glass sheets loads taking into consideration the cooperation of the gas layer sealed in the hermetic gas space of an complex glass panel. In loaded glass unit, the gas coupling of loads takes place. By means of numbers it is expressed by the gas pressure change in the gas space of a loaded glass sheet in relation to the initial pressure assumed as a reference level. It is connected with volume and gas pressure changes in the gas space caused by the volume of glass sheets displacement limiting the gas space. The result of gas coupling is the interaction between glass sheets that is transmitting loads between glass sheets of a glass unit.

Sprzężenie gazowe obciążeń w szybach zespolonych

Streszczenie

W artykule przedstawiono model matematyczny do obliczania obciążenia szyb z uwzględnieniem współpracy warstwy gazowej zamkniętej w szczelnej komorze szyby zespolonej. W obciążonej szybie zespolonej zachodzi sprzężenie gazowe obciążeń. Liczbowo wyraża się ono zmianą ciśnienia gazu Δp w komorze szyby obciążonej w odniesieniu do ciśnienia początkowego, przyjętego za poziom od-
niesienia. Jest ono związane ze zmianami objętości i ciśnienia gazu w komorze szyby, spowodowanymi objętością pola przemieszczenia szyb ograniczających komorę. Efektem sprzężenia gazowego jest interakcja międzyszybowa, tzn. przekazywanie obciążeń między szybami zestawu.