

OPTIMIZATION OF PRODUCTION PROBLEMS USING MATHEMATICAL PROGRAMMING

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Abstract: In production activity, starting manufacturing of a new product involves taking a particular risk. Therefore, the need arises for investigating the rational basis for starting such projects. This process should begin from the verification of the opportunities of reaching the expected effects of the new production. One of the methods of solving the complex problems is mathematical programming, whose utility was demonstrated with an example of a manufacturing enterprise.

Keywords: mathematical model, linear programming, production optimization

Introduction

In management of production processes, the essential importance, similarly to any problem-generating area, is from decision-making process. There are a variety of factors which contribute to the development and implementation of optimization decision models. The essence of the decision is to make right choices due to the required solution or solutions. Depending on the number and the quality of the factors which influence on possible decision variants, optimization of production is distinguished by one- or multi-criteria methods of decision-making.[1] Multi-criteria decision problem concerns the performance of the following conditions:[2]

- objectives are defined through the determination of the common set of acceptable solutions
- each objective has a priority of achievement, which affects the degree of achievement of other objectives
- a particular problem has a limited number of objectives planned to be achieved
- choice of a decision variant is determined by a group of criteria defined for each objective.

A relatively frequent case, which requires making a multi-criteria decision, is adjustment of the production line, required to start a new production or function[3].

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Mathematical Programming

The decision on selection of an optimal solution consists in the determination of such an acceptable decision for whom the objective function takes an extreme value (maximal or minimal, depending on the type of the production problem).[4] Searching for the most favourable solution involves the four stages [5] (Fig. 1).

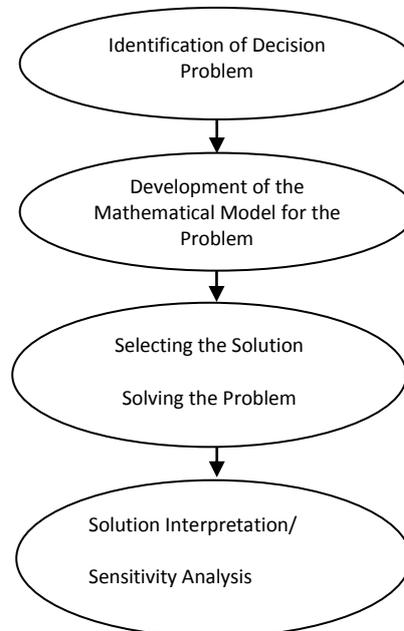


Figure 1. Stages of formulation and definition of optimal solutions

Source: author's own elaboration based on: Sawicki P., *Zarządzanie systemami transportu drogowego. Programowanie liniowe*, etacar.put.poznan.pl/piotr.sawicki/Dydaktyka/Zstd/.../Progr_linowe.pdf

Identification of the decision problem concerns the characterization of the current state, i.e. currently performed tasks in production process and its trouble spots, where specific difficulties with selection of the satisfactory solution arise. After recognition of the problem, another step is to reflect it in the form of a mathematical model, which will take into consideration the limitations of the model situation and decision variables and criteria of assessment being the basis for the set of acceptable solutions for the problem[6]. Reducing the decision problem to the form of a mathematical model allows for searching for an optimal solutions through development of the problem for mathematical programming (in the case of the linear model this will mean the linear programming), with the general formula being as follows.[6]

find the acceptable decision d^{\wedge} belonging to the set D of acceptable decision for which

$$f(d^{\wedge}) = \text{extreme } \{f(d) : d \in D\}$$

where f is a criterion function, whereas extreme means the minimal or maximal value of the objective function, which, in specific form, is given by[7]:

maximization/minimization of the objective function

$$\text{Max/Min } F(d^{\wedge}) = c_1 d_1 + c_2 d_2 + \dots + c_n d_n$$

and constraints

$$a_{11}d_1 + a_{12}d_2 + \dots + a_{1n}d_n \leq b_1$$

$$a_{21}d_1 + a_{22}d_2 + \dots + a_{2n}d_n \leq b_2$$

$$a_{m1}d_1 + a_{m2}d_2 + \dots + a_{mn}d_n \leq b_m$$

and boundary conditions

$$d_1 \geq 0, d_2 \geq 0, \dots, d_n \geq 0$$

where:

c_j – increment of j activity in assessment of the set D ($j = 1, 2, \dots, n$)

b_i – amount of i resource available for the activity ($i = 1, 2, \dots, m$)

a_{ij} – amount of i resource used by the j activity.

Solving a linear problem (using e.g. algebraic method) allows for carrying out the sensitivity analysis in an easy way, obtaining an optimal decision variant for solution of the problem[5]. The use of mathematical programming limits the risk of failure of the planned production project and leads to enhanced effectiveness of management.[5]

Solving a Multi-Criteria Problem: Case Study

An enterprise which manufactures goods made of special ceramics took a decision on production of ceramic bearings at a particular time of their activity. The product has practical applications as machine parts as it exhibits high resistance to variable atmospheric and chemical conditions. A ceramic bearing is

composed of the two parts: ceramic balls (C_1) and ceramic race (C_2), which are cast using the means of production i.e. silicon nitride (P_1) and glass mass (P_2) (Fig.1).

In order to minimize the costs of manufacturing, the enterprise limited the availability of the components of ceramic mass to 56 units of P_1 and 54 units of P_2 . In order to manufacture the ceramic balls for one bearing it is necessary to have 8 units of silicon nitride and 9 units of glass mass and 7 and 6 units, respectively, in the case of the race. Manufacturing of one bearing necessitates 5 units of C_1 and 4 units of C_2 . The cost of unit production is not high, however, as a result of high contribution of defects, the finished products are added a high profit margin. Before the process of manufacturing was started, it was estimated that the production will reach break-even point at the level of sales of 10 pieces. In consideration of the planned production, the enterprise set and assessed the achievement of operating objectives by means of multi-criteria optimization. These objectives assumed:

1. Reduction in investment expenditures maximally to the level of 45 cash units for production of one piece of product P, with the unit costs of casting being 1.5 cash units for ceramic balls and 3.5 units for ceramic race.
2. Reaching maximal profit on sales, not lower than 115 cash units per product, with the value of individual components of C_1 and C_2 at the level of 9 cash units and 7 cash units.
3. Maintaining the number of employees in production division at the level of 47 people working shifts, with 3 people at each shift participating in manufacturing of ceramic ball and 2 people making casts of ceramic race.

These objectives were listed according to the degree of importance; however, it is known that not all of them can be equally achieved. Hence, the entity ordered the values of penalty coefficient to individual objectives within the probability of their non-achievement. For the objective 1, the value of this coefficient reached the value of 2 for positive deviation (\hat{y}^+), with the value of 7 for the objective 2 for negative deviation (\hat{y}^-). For the objective 3, this coefficient reached 3 (\hat{y}^+) i 1 (\hat{y}^-).

In order to assess the objectives, the first step was to build the mathematical model in consideration of the constraints and values of variables in planned production.

Definition of decision variables in the form of: x_1 – level of production of the product C_1 and x_2 – level of production of the product C_2 and, in consideration of the constraint of availability of means of production, the following system of inequalities was obtained:

$$8x_1 + 7x_2 \leq 56$$

$$9x_1 + 6x_2 \leq 54$$

$$5x_1 + 4x_2 \geq 10$$

where: $x_1, x_2 \geq 0$

Then, the equations which define individual objectives and the corresponding penalty coefficients were collected in the table (Table 1).

Objective Id		Deviations $y^+ (+)$	Deviations $y^- (-)$	Priority
1	$1.5x_1 + 3.5x_2 \leq 45$	2	-	1
2	$9x_1 + 7x_2 \geq 115$	-	7	2
3	$3x_1 + 2x_2 = 47$	3	1	3

Table 1. Linear equations for objectives and penalty coefficients [8]

The obtained mathematical model started to be solved according to the order of the degree of importance of achievement of the objectives.

The objective with priority 1: limitation of investment expenditures to the maximal level of 45 cash units, with the unit costs of the cast being 1.5 cash units for C_1 and 3.5 for C_2 .

$$1.5x_1 + 3.5x_2 \leq 45 \qquad y^+_{1(+)} = 2$$

In order to minimize negative deviations for this objective, the linear programming problem A1 was build.

Objective function: $2 y^+_{1(+)} \rightarrow \text{MIN}$

Constraints from primary problem:

$$8x_1 + 7x_2 \leq 56$$

$$9x_1 + 6x_2 \leq 54$$

$$5x_1 + 4x_2 \geq 10$$

Constraints which are a result of determination of individual objectives:

$$1.5x_1 + 3.5x_2 - y_3(+)+ y_3(-) = 45$$

$$9x_1 + 7x_2 - y_1(+)+ y_1(-) = 115$$

$$3x_1 + 2x_2 - y_2(+)+ y_2(-) = 47$$

the assumed boundary conditions:

$$y_1(+), y_1(-), y_2(+), y_2(-), y_2(+), y_2(-), x_1, x_2 \geq 0$$

When solving the problem using simplex method, a number of alternative solutions for the problem were obtained for $y_1(+)=0$:

$$x_1 = 2 \quad x_2 = 0$$

for which the following values of deviations were obtained:

$$\begin{array}{lll} y_1(+) = 0 & y_2(+) = 0 & y_3(+) = 0 \\ y_1(-) = 97 & y_2(-) = 41 & y_3(-) = 42 \end{array}$$

$$2 y_1 (+) \rightarrow \text{MIN} \quad \text{Objective Function} = 2y_3 = 2 \cdot 0 = 0$$

Level of positive deviation for the problem A1 amounts to 0.

Analogously, for the goals 2 and 3, linear programming problems were built and solved, with the same primary constraints resulting from the set objectives and the constraints determined through the previous solutions.

The results obtained for the objective 2, which was reaching maximal profit on sales not lower than 115 cash units per piece, with the value of individual components C_1 and C_2 which should be, respectively, at the level of ca. 9 cash units and 7 cash units, were as follows:

$$x_1 = 2.8 \quad x_2 = 4.8$$

for which the following deviations were obtained:

$$\begin{array}{lll} y_1(+) = 0 & y_2(+) = 0 & y_3(+) = 0 \\ y_1(-) = 56.2 & y_2(-) = 24 & y_3(-) = 29 \end{array}$$

$$7y_1(-) \rightarrow \text{MIN} \quad \text{Objective Function} = 7 y_1 = 7 \cdot 56.2 = 393.4$$

Negative deviation for the problem A2 amounted to 56.2.

The results obtained for the objective 3 were similar to the objective 2, thus the objective function is given by:

$$3y_2(+) + y_2(-) \rightarrow \text{MIN} \quad \text{FC} = 3y_2(+) + y_2(-) = 3 \cdot 0 + 1 \cdot 24 = 24$$

With the negative deviation for $A2 = A3 = 24$ and the negative deviation for $A3 = 0$.

In order to minimize the value of negative deviations for achievement of the objectives, an optimal solution will be $x_1 = 2.8$ $x_2 = 4.8$.

Assessment of the Likelihood of Achievement of the Set Objectives

The initially adopted assumption that the employment should be maintained at the level of 47 people was not confirmed since the negative deviation amounted to 24 people, which meant a reduction in employment by half.

Also in the case of the objective connected with reaching the profit at the level of at least 115 cash units, the objective was not achieved since the planned profits amounted to 58.8 cash units, i.e. substantially lower than 115.

In the case of the assumption of non-exceeding investment expenditures at the level of 45 cash units, the objective will be achieved as the positive deviation reduces the level of the required expenditure by almost one third, leaving almost 70% of the unused capital.

Finally, the enterprise made the decision on temporary postponing the plans connected with production of ceramic bearings.

Summary

Choice of trade-off solutions concerns a variety of domains of activities in enterprises, including the area of production management. However, the set objectives necessitate the definition of a number of criteria of selection of an optimal decision variant, which might be contradictory to each other (not all the criteria can reach the demanded value within the set of acceptable solutions).

Similarly to the above presented analysis, the difficulty in finding a trade-off solution can arise, because the set objectives concerning the decision on starting a production of bearings are not consistent with each other. Even in the case of a problem situation of a deterministic character, obtaining an optimal solution is not always possible.

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OPTIMALIZACJA ZAGADNIENIA PRODUKCYJNEGO Z WYKORZYSTANIEM PROGRAMOWANIA MATEMATYCZNEGO

Abstrakt: W działalności produkcyjnej rozpoczęcie produkcji nowego wyboru wiąże się z podjęciem pewnego ryzyka, stąd też istnieje potrzeba uprzedniego zbadania racjonalnych podstaw przystąpienia do takiego przedsięwzięcia. Proces ten powinien rozpocząć się od zbadania możliwości osiągnięcia spodziewanych efektów nowej produkcji. Jednym ze sposobów rozwiązywania złożonych problemów jest programowanie matematyczne, którego użyteczność przedstawiono na przykładzie przedsiębiorstwa produkcyjnego.

生產問題的優化使用數學編程

在生产活动中，开始新产品的生产涉及到特定的风险。因此，需要对启动这个项目的合理性进行分析。这个过程应该从确认达到新产品的预期效果的机率开始。解决这个复杂问题的方法之一是数学规划。这将通过一个制造企业例子来证明数学规划的效用。