THE EXPLICITNESS OF VECTOR BALANCING THE UNSUSTAINABLE PRODUCTION AND CONSUMPTION MODE

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Abstract. In this article, the analysis of the explicitness of vector balancing the unsustainable production and consumption model was conducted. In this analysis, contractors' options and concessions price had been included.

Key words:

JEL Codes: C02, C30

The previous articles [1], [2] have proposed an iterative method to balance the unsustainable production and consumption model.

With the data:

a) supply-demand vector $\mathbf{p} = (p_1, \ \ldots, \ p_n)$ such that $p_1 + p_2, \ldots + p_n \ \langle \ 0$

b) maximum concession vector $\mathbf{u} = (u_1, \ \ldots, \ u_n)$ such that $p_1 + u_1 + p_2 + u_2 + \ldots + p_n + u_n \geq \ 0$, $u_i \geq \ 0$, $i \ = \ 1, \ \ldots, n$

c) contractors' concession vector $\mathbf{a} \ = \ (a_1, \ \ldots, a)$ such that $a_1 + \ldots + a_n = 1$, $a_i \geq \ 0$, $i \ = \ 1, \ \ldots, \ n$ and assuming $\mathbf{p}^o \ = \ \mathbf{p}$, $\mathbf{a}^o \ = \ \mathbf{a}$ we can define for $s \ \geq \ 1$ the quantities as follow:

$$r^s = -\left(p_1^{s-1} + \ldots + p_n^{s-1}\right)$$

$$N^s = \left\{ \ i \leq n \ : \sum_{j \ \leq \ s} \ a_i^{j-1} r^j \leq u_i \right\}$$

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$$\mathbf{a}^{s} = \left(a_1^s, \ \ldots, a_n^s\right) \quad \text{where}$$

$$a_i^s = \begin{cases} 0 & for \ \ i \notin N^s \\ a_i^{s-1}\left(B^{s-1}\right)^{-1} & for \ i \in N^s \end{cases}$$

$$B^{s-1} = \sum_{j \in N^{s-1}} a_j^{s-1}$$

$$\overline{N}^s = N^{s-1} \ \setminus \ N^s \ \text{where} \ N^o = \{1, \ \ldots, n\}$$

$$\mathbf{p}^{s} = \left(p_1^s, \ldots, p_n^s\right) \text{where}$$

$$p_i^s = \begin{cases} p_i^{s-1} + r^s \cdot a_i^{s-1} & for \quad i \in N^s \\ p_i + u_i & for \quad i \in \overline{N}^s \\ p_i^{s-1} & for \quad i \in \bigcup\limits_{j \langle s} \overline{N}^j \end{cases}$$

The iterative process, described by above formulas, ends for s_o such that $r^{s_o} = 0$. The upper index in the following symbols: p^s, r^s, $N^s, \overline{N}^s, a^s$ mean the s-th iteration.

Below, the economic interpretation of these formulas will be conducted. The r^s number equals the difference remains to balance the model after the $s-1$ iterations. Vector $\mathbf{a}^s$ sets the contractors' concessions price under the s-th iteration criteria. The I-th vector's coordinate $\mathbf{r}^s \mathbf{a}^{s-1}$ determines the quantity under which, the verification of contractors' supply and demand in s-th iteration has been proposed. During the verification it should be careful not to exceed the maximum concession of the *i*-th contractor. For this reason, sets N^s and $\overline{N}^s$ are entered. The N^s set is a set of contractors, for which, the total proposed concession after the *s*-th iteration, does not exceed their maximum concession. The $\overline{N}^s$ set is a complementation of N^s set, it means, it is a set of contractors, for which, the total concession has exceeded their limits in *s*-th iteration. The reduced supply and demand in the *s*-th iteration sets p_i^s . When defining the $\mathbf{p}^s$ vector, the principle of distribution the contractors into the 3 sets has been applied:

- a set of contractors, who as a result of supply's and demand's verification, in the previous iterations, have set supply and demand on limit of their ability. The contractor's supply or demand in *s*-th iteration has remain the same $\left(p_i^{s-1} = p_i^s\right)$
- a set of contractors, who as a result of supply's and demand's verification in *s*-th iteration, have exceed their limits to make concession. The contractor's supply and demand has been set at their limits to make concession $\left(p_i^s = p_i + u_i\right)$.
- A set of contractors, who as a result of proposed changes in supply or demand in the *s*-th iteration, still have some reserve concession. These contractors' supply or demand, calculated in the previous iteration, has been changed by the $r^s a_i^{s-1}$ it means $p_i^s = p_i^{s-1} + r^s a_i^{s-1}$.

Summarize: having given supply, demand, limits of contractors' concession and their concession price – the process of balancing the unsustainable production and consumption model is being carried out by the scheme below.

Step 1: as a first approximation of the vector $\mathbf{p}$, which balance the model, the known supply and demand of all contractor have been assumed.

Step 2: for every contractor, the proposed change of supply and demand has been calculated, using the p_i^s term

Step 3: a set of contractor divides to 2 subsets, for which:

- The proposed change of supply and demand does not exceed the limit of their ability (the N^1 set),
- The proposed change of supply and demand exceeds the limit of their ability (the $\overline{N}^1$ set).

Step 4: as a second approximation of the vector, which balance the model, it is assumed:

- For the contractors from the N^1 set, the previous approximation changed by the quantity calculated in the step 2.
- For the contractors from the N^1 set, the first approximation, changed by the limit of their ability. For these contractors it is the final answer and it has not been changed till the end of the iteration process.

Step 5: eliminating the $\overline{N}^1$ set from the iteration process till its end and assuming the first approximation of vector $\mathbf{p}$ which balances the model – vector calculated in step 4, the calculation are being continued from the step 2.

The algorithm is terminated when the vector's scalar product, calculated in the step 4, together with unit vector is equal to zero. Then as a final answer is the vector calculated in step 4. This vector balances the model.

The problem of clear representation vector $\mathbf{p}^s$ is presented by the following statement.

If $\mathbf{p}$, $\mathbf{u}$ are supply and demand vectors, maximum concession and $\mathbf{a}$, $\mathbf{b}$ are the vectors of contractors' scale concession, $\mathbf{p}^s$, $\overline{\mathbf{p}}^s$ are s-repeated vector's $\mathbf{p}$ iterations, obtained by the vectors $\mathbf{u}$&$\mathbf{a}$ and $\mathbf{u}$&$\mathbf{b}$ according to each $i \in N_a^s \cap N_b^s = M^s$ there is $p_i^s = \overline{p}_i^s \Leftrightarrow a_i^{s-1} = b_i^{s-1}$ where

$$N_a^s = \left\{ i \le n : \sum_{j \le s} r^j a_i^{j-1} \le u_i \right\}$$

$$N_b^s = \left\{ i \le n : \sum_{j \le s} \overline{r}^j b_i^{j-1} \le u_i \right\}$$

The proof of the theory is conducted by induction due to *s*.

For each $i \in M^s$ there are equals as follows:

$p_i^{s-1} + r^s a_i^{s-1} = p_i^s$ and $\overline{p}_i^{s-1} + \overline{r}^s b_i^{s-1} = \overline{p}_i^s$.

- Let the $s=1$. Then $p_i^1 = p_i + r^1 a_i^o$, $\overline{p}_i^1 = \overline{r}^1 + b_i^o$ but $r^1 = -(p_1 + \ldots + p_n) = \overline{r}^1$ it means $p_i^1 = \overline{p}_i^1$ then and only then, when $a_i^o = b_i^o$, what ends the proof of the theory for $s=1$.
- It is assumed that the statement's thesis is true $s \le m$.
- Let the $s = m+1$. Then $p_i^{m+1} = p_i^m + r^{m+1}\ a_i^m$, $\overline{p}^{m+1} = \overline{p}^m + \overline{r}^{m+1} b_i^m$.

From the induction hypothesis $p_i^m = \overline{p}_i^m$ then:

$$r^{m+1} = -\left(p_i^m + \ldots + p_n^m\right) = -\left(\overline{p}_i^m, + \ldots + \overline{p}_n^m\right) = \overline{r}^{m+1}.$$

Therefore $p_i^{m+1} = \overline{p}_i^{m+1}$ then and only then, when $a_i^m = b_i^m$ what ends the proof of the theory. In the above statement was showed that supply and demand vector in every iteration, the vector which balances the model, for the contractors which have not exceeded the limit of their concession, is determined to an accuracy of their weight concession to the criteria. For the remaining contractors, it is possible to speak about certain uniqueness of the solution. In their case, no matter the weight of their concession to the criteria, it means no matter to, eventually, crossing the concession limit, the vector's coordinates, which balances the model, are verified to size $p_i + u_i$.

References

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JEDNOZNACZNOŚĆ WEKTORA RÓWNOWAŻĄCEGO NIEZBILANSOWANY MODEL PRODUKCYJNO - KONSUMPCYJNY

Abstrakt: W niniejszej pracy dokonano analizy jednoznaczności wektora bilansującego niezrównoważony model produkcyjno-konsumpcyjny. W analizie uwzględniono możliwości i ceny ustępstw kontrahentów.

明晰載體的平衡，不可持續的生產和消費模式

摘要:在這篇文章中，進行矢量平衡不可持續的生產和消費模式明晰的分析。在這種分析中，承包商的選擇和優惠的價格已包括在內。