THE EVALUATION OF CONFLICTS' DEGREE IN GROUP DECISION MAKING

Jiří Mazurek*

Abstract: In group decision making conflicts arise from the fact that individuals or groups of individuals have often different opinions about problems' solutions. These conflicts might have various degrees ranging from almost complete agreement to an absolute opposition of two equally strong sides. The aim of this article is to extend the evaluation of conflicts' degree introduced by Z. Pawlak and others originally in the rough set theory context. The scope of this generalization embraces an arbitrary number of groups or individuals involved in a conflict, an arbitrary number of their attitudes as well as different degrees of agreement or disagreement on an issue (fuzzy conflicts). The evaluation of a conflict degree is divided into two levels, as both conflict situation as a whole and each individual in a conflict can be evaluated. The proposed measures of conflicts' degree can be used in initial stages of decision making processes to the quantitative evaluation of conflict intensity, so they can provide useful information for a conflict potential resolution; and also, they might help to describe conflicts' dynamics. Numerical examples of the evaluation of conflics' degree are provided as well.

Keywords: conflict, conflict degree, conflict analysis, group decision making, fuzzy conflict.

JEL codes: C69.

Introduction

Conflict analysis and conflict resolution can be found in many areas of decision making such as business, management, law, politics, military or environmental protection, see e.g. [1] or [10]. Conflicts arise from the fact that when a solution of a problem have to be found by a group of subjects, different opinions about possible solution emerge naturally. There are many tools for conflict resolution, e.g. graph theory, topology, the rough set theory or game theory. However, there is no universal theory of conflicts yet according to Pawlak and Skowron [7].

The conflict analysis in the rough set theory context was proposed by a Polish computer scientist Z. Pawlak and his colleagues in [7] and [9]. For the rough set theory see [5] or [6]. To avoid misunderstandings, in this article *conflict analysis* predominantly refers to the theoretical framework developed in [7] and [9]. In this context, conflict analysis deals with individuals, which are referred as

* **PhD. Jiří Mazurek**, Sileasian University in Opava, School of Business Administration in Karviná, Department of Mathematical Methods in Economics
✉ corresponding author: mazurek@opf.slu.cz

agents, and their votings on some issue(s) with three feasible attitudes: *favorable, against* or *abstention (neutral)*.

However, real-world conflicts are more complex, for they are not restricted to voting procedures, they usually enable more than three options, and an agreement or a disagreement between individuals or groups can have different intensity. Therefore, the aim of the article is to propose the following extensions to the conflict analysis introduced in [7] and [9]:

- To conflicts with a range of a voting function extended to a discrete (and finite) set of integer codes or linguistic variables (to more than three feasible attitudes of individuals or groups involved in a conflict).
- To conflicts with a range of a voting function extended to a real and bounded set (that is individuals or groups can express their attitudes in a continuum of values).
- To fuzzy conflicts, which allow expressing degree of agreement or disagreement among individuals or groups of individuals.
- For all three cases an overall degree of a conflict is defined as well as a degree of a conflict for each individual.

The evaluation of a conflict degree might be important in all areas of human action, where different interests and judgments interfere, as it can facilitate its potential resolution. The evaluation of individuals can be used to the identification of the most conflicting individuals, which in turn can be used to more intensive negotiation with such persons, or even to their elimination from decision making in some cases.

The paper is organized as follows: in Section 1 a brief overview of the conflict analysis is provided, in Section 3 aforementioned extensions are proposed with numerical examples and Conclusions close the article.

A brief overview of the conflict analysis

In this section notation and basic definitions of the conflict analysis based on [7] and [9] is provided as well as the evaluation of a conflict degree for a voting function with three-valued range.

Definition 1. Let *Ag* be a finite, non-empty set called *universe*, let elements *ag* of *Ag* be *agents*, and let $v: Ag \rightarrow \{-1, 0, 1\}$ be a *voting function*. The voting function assigns each agent $ag \in Ag$ his opinion about some issue, and $\{-1,0,1\}$ means that an agent is *against*, *neutral* or *favorable* to an issue respectively.

Definition 2. The pair $CS = (Ag, V)$, where V is a set of voting functions, is called a *conflict situation*.

Definition 3. For each pair of agents (ag, ag') and a voting function v an auxiliary function $\Phi_v(ag, ag')$ is given as Pawlak and Skowron (2007):

$\Phi_v(ag, ag') = 1$ if $v(ag)v(ag') = 1$ or $ag = ag'$

$\Phi_v(ag, ag') = 0$ if $v(ag)v(ag') = 0$ and $ag \neq ag'$

$\Phi_v(ag, ag') = -1$ if $v(ag)v(ag') = -1$

If $\Phi_v(ag, ag') = 1$, agents ag and ag' agree on a issue; if $\Phi_v(ag, ag') = 0$, then one of agent is neutral on an issue; and if $\Phi_v(ag, ag') = -1$, then agents disagree on an issue. The auxiliary function Φ_v enables to introduce three binary relations on a set Ag: agreement, neutrality and disagreement. Based on these relations, a conflict graph is associated with every conflict situation situation (see Figure 1), where solid lines between coalitions denote a conflict while dashed lines express alliances.

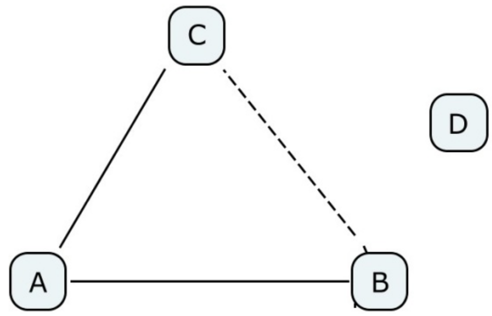

Figure 1. An example of a conflict graph

Definition 4. With a conflict situation CS a conflict degree Con (CS) is associated:

$$Con(CS) = \frac{\sum_{\{(ag,ag'):\Phi_v(ag,ag')=-1\}} |\Phi_v(ag, ag')|}{2\left\lceil \frac{n}{2} \right\rceil \times \left(n - \left\lceil \frac{n}{2} \right\rceil\right)}, \tag{1}$$

where n = card Ag. The conflict degree given by (1) is introduced only for two groups of conflict, and its maximal value Con (CS) = 1 is achieved when both groups have the same number of participants (in case when n is even), or the number of participants differ by 1 (in case when n is odd).

Example 1. Suppose there is a conflict situation represented by Table 1. Using formula (1), the conflict degree associated with the given conflict we get:

$$Con(CS) = \frac{\sum_{\{(ag,ag'):\Phi_v(ag,ag')=-1\}} \left|\Phi_v(ag,ag')\right|}{2\left\lceil \frac{n}{2} \right\rceil \times \left(n - \left\lceil \frac{n}{2} \right\rceil \right)} = \frac{32}{2\cdot(5\cdot 5)} = 0.64$$

.

Agents	***v***
x_1	1
x_2	1
x_3	1
x_4	1
x_5	0
x_6	0
x_7	–1
x_8	–1
x_9	–1
x_{10}	–1

Table 1. A conflict situation with a three-valued range of a voting function

For a conflict resolution, negotiations in the rough set framework and use of Petri nets were proposed in [7] and [9]. Each agent $ag \in Ag$ is allowed to perform actions from a set of possible actions denoted as *Action* (*ag*). Each action includes input and output condition, which represent conditions for a possible performation of an action, and its result respectively. To find a plan of negotiations, the Petri nets [8] can be used. In these nets places correspond to inputs and outputs of actions, and transitions to actions. Another theoretical approach for conflict resolution provides the game theory; see e.g. [3] or [4]. However, in many situations there are conflicts of more than two individuals or groups. Moreover, individuals or groups might hold more than three opinions on an issue, especially a complex one. Also, conflicts among agents can have different intensity. In the Section 3, some generalizations to conflict analysis based on issues mentioned above are proposed.

Extensions

Extension to conflicts with a range of a voting function extended to a discrete set of integer codes or linguistic variables

Suppose that a range of a voting function is not restricted to three value set $\{-1,0,1\}$, but it is extended to any discrete (but finite) set I of integer codes or linguistic variables. Then the voting function $v: Ag \rightarrow I$ assigns each agent a linguistic term or an integer coding his attitude (opinion).

In such setting, agents can have an arbitrary (but finite) number of possible attitudes and can form an arbitrary number of groups. The maximum degree of a conflict is achieved, when each agent has a different attitude. In such a case, there is no pair of agents in agreement, as each agent disagrees with all the others. On the other hand, when all agents are in accord, then a conflict degree is zero.

Because agents' attitudes are only labels, a conflict degree (or a 'distance') between two agents cannot be estimated numerically; however, it is possible to evaluate a conflict degree of a given conflict situation from the number of pairs of agents, who agree and disagree respectively on an issue.

Definition 5. Let Ag be a set of n agents ag, $ag \in Ag$, and let v be a voting function $v: Ag \rightarrow I$, where I is a value set of v. The coefficient of conflict $C(S)$ of conflict situation S is given as:

$$C(S) = \frac{N_D}{N_A + N_D} = N_D \binom{n}{2}^{-1}, \tag{2}$$

where N_A is the number of all pairs of agents (i, j) from Ag, who are in agreement ($v(i) = v(j)$), and N_D is the number of all pairs of agents (i, j) from Ag, who are in a disagreement ($\mathrm{v(i)} \neq \mathrm{v(j)}$). It is clear from Definition 5 that $C(S) \in \langle 0,1 \rangle$. Also, as only agreement or disagreement among agents is feasible according to Definition 5, a concept of neutrality was eliminated from the theory.

Pair-wise conflicts among agents allow for a conflict evaluation of individual agents. Each agent i can be assigned an individual degree of conflict I_D equal to the number of agents in conflict with:

$$I_D(i) = \left|(i, j); v(i) \neq v(j)\right| \tag{3}$$

This approach can identify the most conflicting and the most conforming agents.

Example 2. Consider the conflict situation shown in Table 2. In this particular setting, 10 agents (managers of an investment company denoted as x_1 to x_{10}) expressed their view about a possible investment into a set of bonds $I = \{A, B, C, D, E\}$. Table 2 summarizes opinions of agents. From (2) we get the coefficient of conflict: $C(S) = \frac{N_D}{N_A + N_D} = \frac{35}{45} = 0.78$, which is rather high. The most conflicting individual is x_4 with $I_D(x_4) = 9$, while the least conflicting agents are x_2, x_3, x_5 and x_8.

Agents	***v***
x_1	A
x_2	B
x_3	B
x_4	D
x_5	C
x_6	B
x_7	A
x_8	B
x_9	C
x_{10}	A

Table 2. A conflict situation with a three valued voting function.

Extension to fuzzy conflicts

In the previous sections it was assumed that only two (or three) situations might occur: agents agreed or disagreed (or were neutral) on an issue. However, agreement or disagreement between two agents can be generalized to express different intensity on a proper scale, e.g. on the interval [0,1], where the value 0 represents absolute agreement and 1 absolute disagreement. Such a scale enables to define a *fuzzy conflict*.

In the fuzzy set theory (see e.g. [2] or [11]), an element x of a universal set X belongs into a given set A with some degree expressed by the membership function:

$$\mu_x(A) \to [0,1]$$

If $\mu_x(A) = 1$ then an element x fully belongs to a set A, if $\mu_x(A) = 0$ then x is not a member of A, and for $\mu_x(A) \in (0,1)$ an element x is a member of A to some extent given by the value of $\mu_x(A)$. By analogy with crisp sets and crisp relations, a fuzzy relation R of sets X_i is denoted $R(X_i)$ and it is a subset of Cartesian product of X_i: $X_1 \times X_2 \times ....$.

Definition 6. Let Ag be a set of agents. A fuzzy conflict (FC) is a binary fuzzy relation $\mu_R : A_g \times A_g \to [0,1]$.

A fuzzy conflict among n agents can be represented by a square symmetric $n \times n$ matrix A with elements $a_{ij} = a_{ji}$, where a_{ij} denotes a degree of conflict between agents i and j, $a_{ii} = 0$ for all i (for an example see Table 3). The matrix A is a *representation* of a fuzzy conflict.

$$A = \begin{pmatrix} 0 & 0.3 & 0.8 \\ 0.3 & 0 & 1 \\ 0.8 & 1 & 0 \end{pmatrix}$$

Table 3. An example of a fuzzy conflict matrix

Clearly, when all agents are in absolute agreement, A is the zero matrix. On the other hand, when all agents are in absolute conflict, then $a_{ij} = 1; \forall i \neq j$ and $a_{ii} = 0$. Now, elements a_{ij} are not just labels, they express real intensity of an agreement/disagreement (or a 'distance') between all pairs of agents i and j on the scale from 0 to 1. The matrix representation of a conflict enables to evaluate a conflict degree as a distance between the fuzzy conflict matrix and the absolute agreement matrix, where the standard matrix distance function d between square matrices A and B of order n is used: $d(A,B) = \sum_{i=1}^{n}\sum_{j=1}^{n}\left|a_{ij} - b_{ij}\right|$. Because the absolute agreement matrix is the zero matrix and $a_{ij} \geq 0, \forall i, j$, it suffices to evaluate $\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}$:

Definition 7. Let A be a matrix representation of a fuzzy conflict. The degree of a fuzzy conflict is given as:

$$Deg(FC) = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}}{n^2 - n} \tag{4}$$

Clearly, $Deg(FC) \in [0,1]$. With formula (4) we can evaluate the degree of the fuzzy conflict given in Table 3:

$$Deg(FC) = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}}{n^2 - n} = \frac{4.2}{6} = 0.7$$

The matrix representation of a fuzzy conflict also allows evaluating a degree of a conflict of each individual with all other individuals as a mean value of the corresponding row of the matrix A.

Definiton 8. The individual degree of a conflict $IDC(j)$ for j-th agent is given as:

$$IDC(j) = \frac{\sum_{i=1}^{n} a_{ji}}{n-1} \tag{5}$$

The most conflicting individual from Table 3 is the third agent with *IDC* (3) = 0.9.

Extension to conflicts with a real-valued voting function

The fuzzy conflict is similar (but not equivalent) to conflicts, where the voting function *v* assigns real values ($I \subset R$) to each agent. The main difference rests in the fact that in a fuzzy conflict there can be all non-diagonal elements $a_{ij} \in A$ equal to 1, while this is not possible in a conflict situation with real-valued voting function due to the standardization by relation (6).

Definition 9. Let *Ag* be a set of *n* agents *ag*, $ag \in Ag$, and let *v* be a voting function $v : Ag \to I$, where I is a (real) value set of *v*. Let *a* = min *I* and *b* = max *I*. Then the conflict between two agents *i* and *j* is given as:

$$Con(i, j) = \frac{|v(i) - v(j)|}{b - a} \tag{6}$$

From (6) it's clear that $Con(i, j) \in [0,1]$, $Con(i,i) = 0$ and $Con(i, j) = Con(j,i)$, so one can put $a_{ij} = Con(i, j)$, where matrix $A(a_{ij})$ is a representation of a fuzzy conflict, and proceed in the conflict evaluation by formulas (4) and (5). The approach is illustrated by Example 2.

Example 3. Consider situation, where 6 members of a municipal council (x_1 to x_6) decide about a sum (in million $) to be invested into a repair of a town hall. Members' opinions are shown in Table 4. Using formula (6), where *a* = 10 and *b* = 20, we get a fuzzy conflict represented by a matrix *A* (see Table 5).

The degree of the conflict from (4): $Deg(FC) = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}}{n^2 - n} = \frac{14}{30} = 0.47$.

The most conflicting individual is x_2 with $IDC(2) = 0.64$.

Agents	***v***
x_1	12
x_2	20
x_3	18

x_4	13
x_5	10
x_6	15

Table 4. Opinions of municipal council members

$$A = \begin{pmatrix} 0 & 0.8 & 0.6 & 0.1 & 0.2 & 0.3 \\ 0.8 & 0 & 0.2 & 0.7 & 1 & 0.5 \\ 0.6 & 0.2 & 0 & 0.5 & 0.8 & 0.3 \\ 0.1 & 0.7 & 0.5 & 0 & 0.3 & 0.2 \\ 0.2 & 1 & 0.8 & 0.3 & 0 & 0.5 \\ 0.3 & 0.5 & 0.3 & 0.2 & 0.5 & 0 \end{pmatrix}$$

Table 5. The representation of the fuzzy conflict in Table 4

The evaluation of a conflict dynamics

Degree of a conflict can be used to the evaluation of a *conflict dynamics* too. Through negotiations, one conflict situation transforms into another conflict situations with generally different conflict's degree. Thus, conflict resolution can be considered an alternating sequence of situations and negotiations. For a conflict resolution the conflict degree has to decline to zero or under some (small) threshold value denoting conflict solution.

Example 4. Suppose, that in the situation described in Example 3, agents x_3 and x_4 agree on the amount of 15 mil. $ (*ceteris paribus*). Does this negotiation lead to a lesser conflict?

With the use of (6) we rebuild the fuzzy conflict matrix A from Table 5 into Table 6. Then using formula (4) we get the degree of the conflict $Deg(FC) = 0.39$.

Indeed, the negotiation lead to the lesser conflict. However, if both agents agreed on the amount of 18 mil. $ (*ceteris paribus*), it wouldn't help the conflict resolution, as the conflict degree would rise.

$$A = \begin{pmatrix} 0 & 0.8 & 0.3 & 0.3 & 0.2 & 0.3 \\ 0.8 & 0 & 0.5 & 0.5 & 1 & 0.5 \\ 0.3 & 0.5 & 0 & 0 & 0.5 & 0 \\ 0.3 & 0.5 & 0 & 0 & 0.5 & 0 \\ 0.2 & 1 & 0.5 & 0.5 & 0 & 0.5 \\ 0.3 & 0.5 & 0 & 0 & 0.5 & 0 \end{pmatrix}$$

Table 6. Opinions of council members.after negotiations

Summary

The article provides some generalizations to the conflict analysis proposed in Pawlak and Skowron (2007) and Skowron et al. (2006). It introduces measures for the quantitative assessment of conflicts' degree (or intensity) in situations, where conflict participants can express their attitudes as labels or real numbers. Such a setting might be relevant for a wide range of conflicts in economics, politics, law, medicine or military, where group decision making is involved. With the proposed measures both a conflict situation as a whole and each conflict participant can be evaluated. Moreover, the evaluation of conflicts' degree can be used to describe conflicts' evolution in time: it provides information whether a negotiation leads to a lesser conflict than before or not. Hence, these extensions to the original framework bring the theory closer to real-world conflict situations with more complex relationships between conflict participants and their feasible options, and they provide useful tool for conflicts potential resolution.

References

[1] Carpenter, S. L., Kennedy, W. J. D. Analyzing the Conflict. In: *Managing Public Disputes: A Practical Guide for Professionals in Government, Business and Citizen's Groups*. John Wiley & Sons, 2001, p. 71-91.
[2] Klir, G., Y. Bo. *Fuzzy sets and fuzzy logic: Theory and Applications*. Prentice Hall, 1995.
[3] Myerson, R. B. *Game Theory: Analysis of Conflict*, Harvard University Press, 1997.
[4] Owen G. *Game theory*. The third revised edition, UK: Academic Press, 1995.
[5] Pawlak, Z. Rough sets. *International Journal of Computer and Information Sciences*, Vol. 11, Nr. 5, 1982, pp. 341-356.
[6] Pawlak, Z. Rough set theory and its applications. *Journal of Telecommunications and Information Technology*, Nr. 3, 2002, pp. 7-10.
[7] Pawlak, Z., Skowron, A. Rough Sets and Conflict Analysis, In: *E-Service Intelligence,* 2007, pp. 35-74.
[8] Peterson, L. J. Petri Nets. *ACM Computing Surveys* 9 (3), 1977, p. 223–252.

[9] Skowron, A., Ramanna, S., Peters, J. F. Conflict analysis and Information Systems: A Rough Set Approach. In: *Proceedings of RSKT*, Springer-Verlag, 2006, pp. 233-240.
[10] Susskind, L., Thomas-Larmer, J. Conducting a Conflict Assessment. In: L. Susskind, S. McKearnan and J. Thomas-Larmer. *The Consensus Building Handbook: A Comprehensive Guide to Reaching Agreement*, Thousand Oaks, Calif.: Sage Publications, 2002, pp. 99-135.
[11] Zadeh, L. A. Fuzzy sets. *Information and control* 8, 1965, p. 338-353.

OCENA STOPNIA KONFLIKTÓW W GRUPOWYM PODEJMOWANIU DECYZJI

Abstrakt: W decyzjach grupowych konflikty wynikają z faktu że osoby lub grupy mają często różne opinie na temat rozwiązań problemów. Konflikty te mogą mieć różne stopnie, począwszy od pełnej zgody aż do absolutnego sprzeciwu obu stron. Celem niniejszego artykułu jest rozszerzenie oceny stopnia konfliktów wprowadzonego przez Z. Pawlaka i innych, pierwotnie w trudnym kontekście teorii mnogości. Zakres tego uogólnienia obejmuje dowolną liczbę grup lub osób zaangażowanych w konflikt, dowolną liczbę ich postaw a także różne stopnie zgodności lub niezgodności w kwestii „rozmytych konfliktów". Ocena stopnia konfliktów dzieli się na dwóch poziomach, zarówno sytuacja konfliktu jako całości jak i ocena poszczególnych osób konfliktu. Proponowane środki pomiaru stopnia konfliktu mogą być stosowane w początkowych etapach decyzyjnych do ilościowej oceny intensywności konfliktu, dzięki czemu mogą one dostarczyć użytecznych informacji dotyczących potencjalnego konfliktu a także mogą pomóc opisać dynamikę konfliktu. Zawarte zostały również numeryczne przykłady oceny stopnia konfliktu.

对群体决策中的冲突程度的评价

摘要：在群体决策中，冲突起源于个人或者团体对问题的解决方案有不同的意见。这些冲突可能

有不同程度。从意见几乎完全一致到完全对立的两个势均力敌的群体。这篇文章的目的旨

在进一步研究Z. Pawlak引进的冲突程度的评价，以及其他在粗糙集合理论的背景下的冲突

程度的评价。本文的讨论范围内，冲突程度的评价被推广至涵盖有任意数量的团体或个人

的冲突，有任意数量的态度，以及任意程度的一致意见或者不同意见的情况（模糊冲突）。

冲突程度的评估分为两个层次，作为一个整体的冲突情形，以及每个人在冲突中的情形都

降进行评估。本文提出的冲突评估的方法可以用在初始阶段的决策过程，以定量评估冲突

的强度。因此它们可以为冲突提供有效的信息以解决冲突。并且这些方法可以动态的描述

冲突。本文还列举一些冲突评估的算例。