Dariusz Skorupka, Artur Duchaczek, Artur Szleszyński
General Tadeusz Kosciuszko Military Academy of Land Forces in Wroclaw, Management Department

Application of multi-criteria optimization methods to the process of transportation means selection in warehouse logistics

In engineering practice, most of the decision-making problems can be described with means of three variables, namely: goal, decision-making variants and utility of the given variant.[1] In simple decision-making problems goal functions are determined by their utility, understood as the optimal result, assessed on the basis of the adopted evaluation criteria.[2] When the number of the criteria for the evaluation of the decision variant is greater than one, it means that an optimal decision is excluded and the only one we can obtain is a sub-optimal decision. Sub-optimality of decisions results from the fact that none of the decisive variants does not meet all the evaluation criteria better than others.

In case of complex decision problems preferences of a decision maker plays an important role, which sometimes explicitly indicates an acceptable solution.[3] In the problems of multi-criteria support of decisions rigid restrictions may result in failure to obtain a solution to a given task, despite the existence of partial solutions.

A. Sobotka discussed the existence of problems related to logistics in the construction field and described ways to solve these problems in practice. In addition, the author suggested the possibility of using scientific methods to solve the logistical problems associated with decision making.[4]

In the field of logistics we can often meet multi-criteria decision tasks. One example of this kind can be a decision to purchase forklifts operating in companies' warehouses or in logistics centers. The purpose of decision analysis is the efficient use of storage space and ensuring smooth transportation in the area. Efficient transport is ensured by forklifts moving materials stored in the warehouse. Therefore, decision variants are different types of forklifts available on the Polish market. Utility of the analyzed task will be optimal (sub-optimal) efficient movement of goods in the warehouse. The decision analysis should take into account rigid requirements which, among many other issues, include: distance between the shelves, shelf height, maximum weight of goods transported with a forklift, costs of purchasing and operating forklift trucks and warehouse operational safety.

On the basis of rigid restrictions we can see that some of them are mutually contradictory. Restrictions, which are required, often stay in a mutual conflict. In this case, it may be useful to take Pareto-optimal decision that indicates the preferences of the decision-maker.

The presented example shows that the multi-criteria decision-making requires an examination of the relationships between requirements which, as shown above, can be contradictory to each other. This paper presents and compares applicability of rarely used in engineering practice Bellinger and Electre III methods in the area of the above described human activity.

Description of applied optimization methods

Electre methods, whose precursor was Bernard Roy, were derived from the French school of the Université Paris-Dauphine.[5] According to M. Wolny ,in the above mentioned methods a relational system of preferences, which is built there, is created on the basis of the so-called outranking method. We assume that between a pair of decision variants there is an outranking relation, created on the basis of the outranking tests, that rely on finding the concordance condition between a pair of decision variants and on lack of discordance between certain variants.[6]

[1] *Badania operacyjne,* W. Sikora (ed.), PWE, Warszawa 2008, p. 11.

[2] T. Trzaskalik, *Wprowadzenie do badań operacyjnych z komputerem*, PWE, Warszawa 2003, p. 20.

[3] *Ibidem*, p. 20.

[4] A. Sobotka, *Logistyka przedsiębiorstw i przedsięwzięć budowlanych*, Wyd. AGH, Kraków 2010.

[5] B. Roy, *Wielokryterialne wspomaganie decyzji*, WNT, Warszawa 1990, p. 250.

[6] M. Wolny, *Wspomaganie decyzji kierowniczych w przedsiębiorstwie przemysłowym. Wieloatrybutowe wspomaganie organizacji przestrzennej komórek produkcyjnych z zastosowaniem teorii gier*, Wyd. Politechniki Śląskiej, Gliwice 2007, p. 38.

All solutions should be evaluated by use of each of the criteria resulting in creation of directed graphs corresponding to different criteria. Overall rating is represented by a synthetic graph that represents a compromise between the evaluation of all the criteria.[7]

Electre method involves the use of so-called concordance and discordance indices, taking values from the range of <0; 1>.[8] However, as mentioned by T. Kasprowicz, when variant *a* equals variant *b*, then the value of concordance and discordance indices is undetermined.[9]

Electre I method introduced the concept of the concordance threshold *pz* and discordance threshold *pn*. These are numbers from the range <0; 1>[10] which are independent from the concordance and discordance indices but the concordance threshold *pz* value is close to 1 and discordance threshold *pn* close to 0.[11] It is assumed that variant *a* is greater than variant *b* if and only if the pair (a, b) obtained the concordance index $z(a, b) \leq pz$ and discordance index $n(a, b) \leq pn$.[12]

According to P. Górny, the drawback of Electre method results from the adopted outranking property that allows the formation of cycles in the graph whose interpretation at low values of concordance threshold *pz* and large values of discordance threshold *pn* raises some concerns. In order to eliminate the disadvantages of the above described Electre I method, there were its modifications called Electre II and Electre III developed.[13]

In Electre II method, which is focused mainly on the embedded outranking variants, it is important to determine the outranking of weak and strong relations. This approach is a more accurate expression of opinions of decision-makers and helps to prioritize or rank the variants.[14]

Electre III method introduces fuzzy outranking relation which means that each pair of variants (a, b) is attributed the number of $\rho(a, b) \in [0, 1]$ expressing the degree of the decision maker conviction of the outranking of variant *b* by variant *a*.[15] Credibility index value $\rho(a, b)$, defining fuzzy relation is the basis for the construction of two pre-orders (descending and ascending) based on the classification algorithm on which the whole decision variants ranking is built.[16] According to K. Stachowiak, distillation process fully exploits the credibility matrix, creating an outranking matrix.[17]

An important element of the method is determining the appropriate threshold values, similarly as in Electre III method, model of preferences is defined precisely in the form of threshold values, such as the equivalence threshold *q*, the preferences threshold *p* and veto threshold *v* and importance coefficient criterion *c*, in other words the weight of the criterion *c*.[18]

The most important difference between the first and the third version of the discussed methods is the possibility to evaluate decision variants that will be in the range between the equivalence threshold *q* and preferences threshold *p*.[19]

P. Sawicki presented a clear algorithm of Electre III method's application, describing the used computational procedure, which was divided into three stages.[20] Stage I is construction of an evaluation matrix and definition of decision-maker's preferences, whereas stage II includes construction of the valued outranking relation *S*, and stage III is the application of the valued outranking relation.[21]

Bellinger method, which took its name from the name of its founder — Bernhard Bellinger — is one of the methods of multi-criteria analysis, which ranks objects on the basis of aggregate evaluation value set on the basis of adopted partial criteria.[22] This method was originally used either to evaluate the credibility of bank customers who applied for loans or in determination of the production range in industrial enterprises.[23]

Bellinger method relies on bringing of decision variant assessment in reference to all criteria first to comparability and then to their subsequent aggregation.[24] It must therefore be determined, for each of the analyzed criterion of available decision-making variant, what is the most and least desirable condition. Then, we determine whether the given evaluation criterion is a stimulant or destimulant for this decision variant.[25]

We assume that the difference between conditions represents the total way we need to take from one condition to another. Therefore, for each available decision variant we must specify the evaluation of every criterion as a fraction of the way. It is assumed that the sub-optimal variant is the one for which the aggregate

[7] E. Ignasiak, *Badania operacyjne*, PWE, Warszawa 2001, p. 223.

[8] O. Kapliński *et al*., *Metody i modele badań w inżynierii przedsięwzięć budowlanych*, Wyd. PAN KILiW IPPT, Warszawa 2007, p. 325.

[9] T. Kasprowicz, *Inżynieria przedsięwzięć budowlanych. ITE Radom*, Warszawa-Radom 2002, p. 220.

[10] *Ibidem*.

[11] E. Ignasiak, Badania operacyjne…, *op. cit*., p. 225.

[12] *Ibidem*.

[13] P. Górny, *Elementy analizy decyzyjnej*, Akademia Obrony Narodowej, Warszawa 2004, pp. 87–88.

[14] *Ibidem*, p. 88.

[15] *Ibidem*.

[16] M. Wolny, *Wspomaganie…*, *op. cit*., p. 41.

[17] K. Stachowiak, *Wielokryterialna analiza decyzyjna w badaniach ekonomiczno-przestrzennych.* http://www.zprie.amu.edu.pl/pliki/stachowiak_td.--pdf, 10.01.2013.

[18] P. Sawicki, *Zastosowanie WWD w logistyce. Metoda ELECTRE III*. http: //etacar.put.poznan.pl/piotr.sawicki/Dydaktyka/Etel/prezentacje/-Etel_08_Wielokryt_new. pdf, 10.01.2013.

[19] M. Wolny, *Wspomaganie …*, *op. cit*., p. 40.

[20] P. Sawicki, *Zastosowanie …*, *op. cit*.

[21] *Ibidem*.

[2] P. Górny, *Elementy…*, *op. cit*., p. 75.

[23] M. Wolny, *Wspomaganie…*, *op. cit*., p. 38.

[24] *Ibidem*.

[25] *Ibidem*.

way is the longest, i. e. the analyzed variant of decision-making is given the evaluation of the highest value.[26]

P. Górny described the algorithm applied in this method in eight stages.[27] In stage I there appears the determining of definition of the requirements and restrictions for future hypothetical variant solutions to analyzed problem. Stage II relies on definition of the decision-making variants available in a given situation. In stage III there appears determining of the adopted evaluation criteria in detail, selection of measurement units and the desired direction of change within a given criterion (stimulant or destimulants), as well as upper and lower bound of changes for the analyzed partial criteria.[28]

Stage IV is designed to determine the hierarchy of the separate criteria, by determining the weights given by the decision-maker to evaluation criteria. Stage V is to create a matrix that contains the actual values of the analyzed criteria for the different variants. Stage VI includes presentation of each number from table created in phase V as a percentage of the way from the least to the most desirable condition. In step VII we multiply the numbers obtained in step VI by weight adopted in stage IV. The final stage (VIII) is to determine the best variant based on aggregate evaluations awarded to individual variants, taking into account all the analyzed criteria.[29]

Example of application of multiobjective optimization methods to selection of means of transportation

The paper presents the applicability of Bellinger and Electre III methods in material logistics on the example of optimization of forklifts selection for warehouses of products and building materials.

The presented work was based on the data available on the website of *Firma Lemarpol — Wózki Widłowe Sp. z o.o.* (*Lemarpol Company — Forklift Trucks Ltd.*).[30] The analyzes included only diesel engine powered forklifts having a lifting capacity of 2000 kg. All of the above mentioned trucks had a center of gravity located at a height of 500 mm. Optimization was carried out primarily for forklift in terms of driving characteristics which have the greatest impact on the smooth operation of the warehouse.

Table 1 shows the parameters adopted for the forklift analysis. There were six possible variants (alternatives) founded marked with symbols W1–W6, these were trucks produced by Nissan, Toyota, Lyson and three Komatsu trucks. There were five criteria analysis adopted marked with K1–K5 which includes width, height or length of the truck, its turning radius and engine power.

Table 1
List of forklift trucks parameters

Criterion	Forklift name — variant designation					
	Nissan DX-20 W1	Toyota 8FD20 W2	Lyson FD20T W3	Komatsu FD20NT-16 W4	Komatsu FD20T-16 W5	Komatsu FD25NT-16 W6
K1 — width (mm)	1157	1150	1155	1090	1150	1090
K2 — height (mm)	2130	2110	2120	2025	2110	2025
K3 — lenght (mm)	2530	2560	2530	2535	2535	2405
K4 — turning radius (mm)	2190	2200	2175	1980	2190	2050
K5 — engine power (kW)	38	39	33	46	46	46

Source: http: //www.wozki.biz/pl, wozki-nowe.html.

First, the calculations were performed using Electre III method.[31] In this case, the calculation used a demo version of Electre III/IV (Demo version 3.1b) available at http://www.lamsade.dauphine.fr. Using the software application one must set the parameters entered in the appropriate tabs of the program, which among many other issues include *criteria edition* (criteria, criterion validity coefficient *c*, the direction of preference), *solutions edition* (available solutions), *evaluation table edition* (evaluation matrix of each of the solutions in terms of all the criteria) and *thresholds edition*.

In case of Electre III method the difference between evaluated variants *a* and *b* can indicate three potential situations, namely, the equivalence of variants, weak preference of one of the variants and a strong preference.[32]

Table 2 presents adopted in the first variant calculations of the thresholds values (q, p, v), and the im-

[26] *Ibidem*.
[27] P. Górny, *Elementy…*, *op. cit*., p. 75.
[28] *Ibidem*, pp. 75–76.
[29] *Ibidem*, p. 76.
[30] http://www.wozki.biz/pl, wozki-nowe.html
[31] D. Skorupka, A. Duchaczek, A. Szleszyński, *Zastosowanie metody Electre w optymalizacji doboru środków transportu w magazynie wyrobów budowlanych*, „Zeszyty Naukowe Politechniki Rzeszowskiej, Budownictwo i Inżynieria Środowiska" 2012/III, Vol. 59, No. 3, pp. 105–112.
[32] P. Sawicki, *Zastosowanie…*, *op. cit*.

portance of separate criteria (c). The thresholds are set in the program using a linear function with the use of directional coefficient α and slope-intercept β. In the analyzed case, a constant value of the thresholds was assumed, i. e., they were set only by parameter β.

According to M. Wolny, the equivalence threshold q is the value, due to which it can be stated if there occurs a situation of equivalence or preference between the decision variants.[33] The preference threshold p, however, determines the value, which sets the limit of the difference of the two decision variants to which there appears a possibility of preference and a situation from which there is a strong preference situation.[34] The two values (p and q) can be defined either as a constant for each criterion or as a function of dependency on the evaluation values comparable to decision variants. The equivalence threshold q and preferences threshold p therefore create some barriers to concordance and discordance indices.[35] Veto threshold v determines which pairs of compatible decision variants are characterized with incomparability. It applies only to those pairs of decision variants (a, b) which met the concordance condition, whereas the difference in the evaluation of variants (b, a) with respect to a appropriate criterion excludes the situation of outranking.[36]

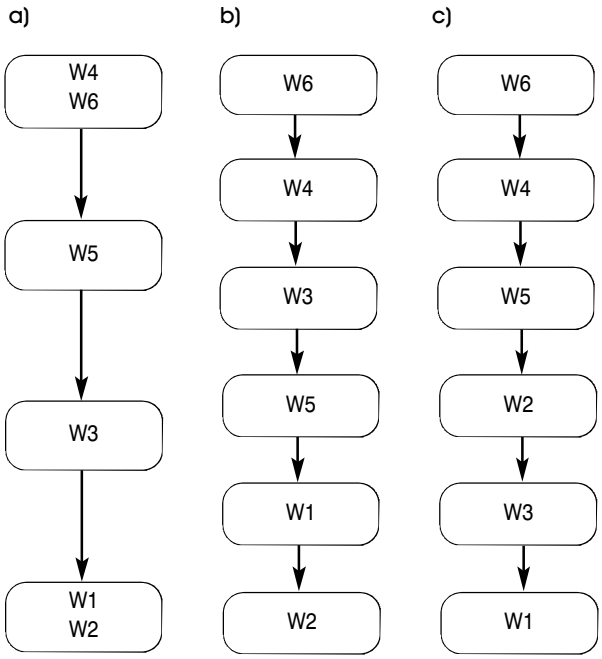
Table 2
Threshold values adopted for separate criteria

Specification	Threshold values			Importance criterion c
	of equivalence q	of preference p	of veto v	
K1 — width (mm)	2	30	100	3
K2 — height (mm)	10	50	200	1
K3 — lenght (mm)	5	50	200	2
K4 — turning radius (mm)	10	100	200	3
K5 — engine power (kW)	1	5	15	1

Source: authors' own elaboration.

Credibility coefficients $\rho\,(a, b)$ were the basis for the construction of two preorders based on classification algorithm. On the basis of these preorders there was a ranking of alternative decision variants build, which is presented in a graphical way in Figure 1a.

Figure 1
Final graphs for variants: a) I Electre III method, b) II Electre III method, c) Bellinger method



Source: authors' own elaboration.

The carried out analysis showed that with the assumed computational parameters the following variants should be selected: these are W4 and W6 (i. e. forklift Komatsu FD20NT-16 and Komatsu FD25N-T-16), which are better than all the others, and their advantage was small width and a small turning radius. In the analyzed example, however, the worst appeared to be W1 and W2 variants, i. e. forklift Nissan DX-20 and Toyota 8FD20.

The accepted threshold values (p, q, v) resulted in the fact that there was not clear hierarchy of possible alternative variants of solutions established. The results presented in this form prevented an immediate decision as to the choice of one option. Therefore, in the second variant the calculations were performed with the assumption that for all the five criteria the equivalence threshold q, preference threshold p and veto threshold v all equal zero.

The calculation results were presented in Figure 1b. Analyzing the data presented in this figure, one can conclude that regardless of the adopted variant the computing solution W6 is optimal, whereas W2 variant is the worst option.

The next stage of research carried out over the optimization of the forklift selection in warehouses of construction materials was made with the use of Bellinger methods.[37] In this case, similarly as in the

[33] M. Wolny, *Wspomaganie…*, *op. cit*., p. 38.
[34] *Ibidem*, p. 39
[35] *Ibidem*.
[36] *Ibidem*.

[37] D. Skorupka, A. Duchaczek, A. Szleszyński, *Optymalizacja doboru środków transportowych w logistyce magazynowej materiałów budowlanych*, „Zeszyty Naukowe Wyższej Szkoły Oficerskiej Wojsk Lądowych we Wrocławiu" 2012, Vol. 4, passed to the editors.

previous analysis, there were six possible options of trucks W1–W6 available, described in Table 1, and the analysis was conducted for the five criteria K1–K5, also described in Table 1.

The calculations were started with determination of desired direction of numeric changes and adoption of lower and upper bound for each criterion, as shown in Table 3 (Case I).

Table 3
Desired direction of numerical change numbers and upper and lower bound of changes for each criterion

Criterion value	Case	Cr 1	Cr 2	Cr 3	Cr 4	Cr 5
Desired (stimulant)	I	1000	2000	2300	1900	50
Undesired (destimulant)		1200	2500	2800	2400	30
Desired (stimulant)	II	1090	2025	2405	1980	46
Undesired (destimulant)		1157	2130	2560	2200	33

Source: authors' own elaboration.

Then there were weight for each criterion assumed, which were identical to those presented in the case of Electre III method (see Table 2).

The analysis shows that from the point of view of decision-maker's preferences (weight values), and adopted criteria the highest overall rating, namely 70.80 was achieved by W6 variant (Table 4 and Figure 1c). This means that, with the assumed boundary conditions, Komatsu forklift FD25NT-16 (variant W6) is an optimal solution.

In the analyzed example, the lower and upper bound of change for each criterion was quite broad and differed greatly from the extreme values of the individual criteria. In the following example, it was assumed that these limits are equal to the extreme value of the criteria, which are presented in Table 3 (Case II). Table 4 also shows the results of the calculations for this variant (Case II), on whose basis the new aggregate evaluations were determined and the best variant was found again.

Table 4
Aggregate evaluations obtained in the result of calculations

Variant	Aggregate evaluation	
	Case I	Case II
W1	41,25	9,08
W2	41,40	9,65
W3	40,15	9,13
W4	69,80	83,23
W5	46,50	19,63
W6	70,80	90,45

Source: authors' own elaboration.

The analysis shows that also in this case the variant W6 received the highest aggregated value, which means that with the assumed boundary conditions Komatsu forklift FD25NT-16 (variant W6) is an optimal solution (Fig. 1c), and, in fact, as previously mentioned — a suboptimal solution. It is worth to notice, however, that in the latter case, the results obtained with the variant W6 and W4 are much larger than for the other, which suggests that other options should not be taken into consideration at all (see also Figure 1a and 1b).

Analyzing the data presented in Figure 1 one can state that, regardless of the adopted calculation method, W6 is an optimal variant. The calculation results obtained by the two methods are comparable, especially in case of Electre III method in which there appears adoption of computing thresholds (p, q, v) equal to zero.

The presented research results carried out under the theme No WZ/002/DzS were financed from own resources of General Tadeusz Kosciuszko Military Academy of Land Forces.

Streszczenie

Zastosowanie metod optymalizacji wielokryterialnej w procesie doboru środków transportowych w logistyce magazynowej

W pracy przedstawiono propozycję zastosowania metod optymalizacji wielokryterialnej w procesie doboru środków transportowych w logistyce magazynowej (magazyny wyrobów i materiałów budowlanych). Optymalizacja wielokryterialna została pokazana na przykładzie doboru wózków widłowych dostępnych na rynku krajowym. Wykorzystano do tego celu metody Bellingera i Electre III, opisując ich podstawowe założenia oraz dokonując oceny ich wad i zalet. Ponadto porównano wyniki rankingu wariantów decyzyjnych otrzymanych obiema metodami.