# SYNTHESIS OF PROGRAM TRAJECTORIES BY MOVEMENT DEGREE OF MANIPULATING ROBOTS

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**Abstract:** The work addresses the issue of program trajectory synthesis for the robot manipulator degrees of mobility. Each mobility degree considers the acceleration options, motion at a given speed and braking action. These options depend on generalized coordinates value changes in powers of the mobility of the robot manipulator.

Keywords: handling robot, degree of mobility, generalized coordinate, program trajectory

# SYNTEZA PROGRAMU TRAJEKTORII W STOPNIACH SWOBODY MANIPULATORA ROBOTA

Streszczenie: Praca dotyczy problemu syntezy programowania trajektorii dla stopni swobody manipulatora robota. Dla każdego stopnia swobody rozważa opcje przyspieszenia, ruchu jednostajnego i hamowania. Opcje te zależą od ogólnych zmian wartości wspólrzednych w ramach zakresu mobilności manipulatora robota.

Słowa kluczowe: obsługa robota, stopień swobody, współrzędne uogólnione, program trajektorii

### Introduction

The problem of trajectory program synthesis is to determine generalized coordinates law of change in degrees of the mobility of the robot manipulator for movement along a given trajectory of the gripper.

Synthesis of the program trajectories by movement degree of manipulating robots (MR) is based on solving the inverse kinematics problems [6, 7, 9].

The main aim of inverse kinematics is to determine the generalized coordinates values  $q_{i,j}$ , where  $i = \{1,...,n\}$  – degree of mobility,  $j = \{1,...,m\}$  – characteristic point of the MR gripper movement of the given trajectory, n – MR number of mobility degrees, m - number of characteristic point defining the trajectory of the MR gripper [5, 8].

The results of solving the inverse kinematics problems by the provisions can be represented analogously to the table 1.

Table 1. The results of the inverse kinematics problems solving by the provisions

Trajectory characterized points	Generalized coordinates values by movement degrees					
	$q_1$	$q_2$		$q_i$		$q_n$
$A_1(x_1, y_1, z_1)$	$q_{11}$	$q_{21}$		$q_{i1}$		$q_{n1}$
$A_2(x_2, y_2, z_2)$	$q_{12}$	$q_{22}$		$q_{i2}$		$q_{n1}$
:			:			
$A_m(x_m, y_m, z_m)$	$q_{1m}$	$q_{2m}$		$q_{im}$	N	$q_{nm}$

As it is shown in the table, each value of a given point of positioning the gripper, characterized point of the trajectory  $A_j(x_j, y_j, z_j)$ , is corresponded to its own vector of generalized coordinates  $Q_j = \{q_{1j}, q_{2j}, ..., q_{nj}\}$  [10].

To make the motion of MR gripper from point  $A_j(x_j, y_j, z_j)$ to point  $A_{j+1}(x_{j+1}, y_{j+1}, z_{j+1})$ , it is necessary to change the generalized coordinates vector value  $Q_j = \{q_{1j}, q_{2j}, ..., q_{nj}\}$  to value  $Q_{j+1} = \{q_{1j+1}, q_{2j+1}, ..., q_{nj+1}\}$ . To determine the generalized coordinates law of change in powers of MR mobility it is necessary to consider not only the values of these two vectors, but also the values of the following vector  $Q_{j+2} = \{q_{1j+2}, q_{2j+2}, ..., q_{nj+2}\}$ .

This task should be solved also by taking into account the constraints on the values of generalized coordinates, growing momentum and accelerations by MR mobility degree. These values are limited by constructional features of manipulator, the energy capacity of degree mobility drives. These restrictions define the following inequalities [2]:

$$q_i'' \le q_i \le q_i^{\varepsilon}, \tag{1}$$

$$0 \le \dot{q}_i \le \dot{q}_i^{\,\vartheta} \,, \tag{2}$$

$$\ddot{q}_i^m \le \ddot{q}_i \le \ddot{q}_i^p \,. \tag{3}$$

there are:

 $q_i^i$ ,  $q_i^a$  – minimum and maximum values of the generalized coordinates *i* - the degree of mobility MR;

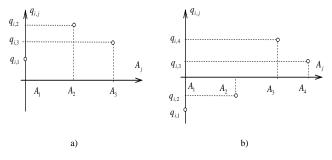
 $\dot{q}_i^{a}$  – nominal rate on *i* - the degree of mobility MR;

 $\ddot{q}_i^{\delta}$ ,  $q_i^{\delta}$  - values of acceleration and deceleration ramp for *i* - the degree of mobility MR.

## 1. Simulation results

Let us consider the options for changing the vectors values of generalized coordinates. Let the initial position of the gripper is in the point  $A_1(x_1, y_1, z_1)$ , which is corresponded by vector  $Q_1 = \{q_{11}, q_{2j1}, ..., q_{n1}\}$ . The growing momentum and accelerations at the initial point is zero, that are the vectors  $\dot{Q}_1 = \{0, 0, ..., 0\}$ ,  $\ddot{Q}_1 = \{0, 0, ..., 0\}$ . It is necessary to remove the gripper to the point with coordinates  $A_2(x_2, y_2, z_2)$ , which corresponds to the vector  $Q_2 = \{q_{12}, q_{22}, ..., q_{n2}\}$  and the vectors of velocities and accelerations  $\dot{Q}_2 = \{\dot{q}_{12}, \dot{q}_{22}, ..., \dot{q}_{n2}\}$ ,  $\ddot{Q}_2 = \{\ddot{q}_{12}, \ddot{q}_{22}, ..., \ddot{q}_{n2}\}$ .

At this point, initially the given degree of mobility should be increased to the best possible acceleration of the nominal speed value, further to move up with the same speed to the point, from which need to start the braking action with maximum negative acceleration, and stop at the point with a given value of the generalized coordinates. The same case is possible to be used at intermediate section graph of behavior of generalized coordinates by the MR mobility degree (Figure 1a).





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Figure 1b illustrates second case of behavior graph of the generalized coordinates values for a given MR mobility degree, when from point 1 to point 2 it is necessary to be proceeded with a nominal speed, and at point 3 to execute braking action. Figure 2 shows the third case, when there is need to start the acceleration from zero speed, further at the point 3 to keep continuing to move with the nominal speed. This allows the unrestricted behavior graph of the generalized coordinates by MR mobility degree to devide into 3 sections corresponding to the above cases.

Let us consider the 1 case, the process of performing the movement in the trajectory section from the point 1 to the point 2 (Figure 1a). Behavior graphs of the value of the generalized coordinates  $q_i$ , velocity  $\dot{q}_i$  and acceleration  $\ddot{q}_i$ are illustrated in Figure 3a

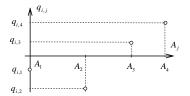


Fig. 2. Behavior graph of generalized coordinates value for 1 case

The starting section of mobility is divided into three subsections. Hence the total time interval  $\Delta t_{i,1}$  and magnitude of changes in the generalized coordinates values  $\Delta q_{i,1} = q_{i,2} - q_{i,1}$  are defined as the sum at these three subsections:

 $\Delta t_{i,1} = \Delta t_1 + \Delta t_2 + \Delta t_3 \quad \Delta q_{i,1} = \Delta q_1^1 + \Delta q_1^2 + \Delta q_1^3$ 

there are:  $\Delta t_1$ ,  $\Delta t_2$ ,  $\Delta t_3$  – interval values acceleration, motion at the rated speed and braking to *i* - the degree of mobility MR;  $\Delta q_1^1$ ,  $\Delta q_1^2$ ,  $\Delta q_1^3$  – value of the maximum change of generalized

 $\Delta q_1$ ,  $\Delta q_1$ ,  $\Delta q_1$  – value of the maximum charge of generalized coordinates required for acceleration, motion at the rated speed and braking to *i* - the degree of mobility MR.

At the first subsection  $\Delta t_1$ , the acceleration of zero speed  $\dot{q}_i = 0$  to the nominal speed  $\dot{q}_i^{\rho}$  with a given acceleration  $\ddot{q}_i^{\rho}$  is realized. The time interval value  $\Delta t_1$  and magnitude of the generalized coordinates changes  $\Delta q_1^1$  are defined by the formula (4)

$$\Delta t_1 = \frac{\dot{q}_i^{\vartheta}}{\ddot{q}_i^p}, \ \Delta q_1^1 = \frac{\ddot{q}_i^p (\Delta t_1)^2}{2}.$$
(4)

Further on the second subsection  $\Delta t_2\,$  realizes the movement

of a given nominal rate  $\dot{q}_i^{\delta}$ . In this case, the time interval  $\Delta t_2$  and the magnitude of the generalized coordinates

changes  $\Delta q_1^2$  are defined by formula (5):

$$\Delta t_{2} = \frac{\Delta q_{1}^{2}}{\dot{q}_{i}^{o}}, \ \Delta q_{1}^{2} = \Delta q_{i,1} - \Delta q_{1}^{1} - \Delta q_{1}^{3}.$$
(5)

An unknown value  $\Delta q_1^3$  determines the interval required for the implementation of the braking action to zero speed. This is the third part of the trajectory of the motion, where the desired time interval  $\Delta t_3$  and the magnitude  $\Delta q_1^3$  are determined from formula:

$$\Delta t_{3} = \frac{\dot{q}_{i}^{\,\,\partial}}{\ddot{q}_{i}^{\,\,m}}, \ \Delta q_{1}^{\,3} = \dot{q}_{i}^{\,\,\partial} \Delta t_{3} - \frac{\ddot{q}_{i}^{\,\,m} (\Delta t_{3})^{2}}{2} \,. \tag{6}$$

Let's consider the next option of the generalized coordinates value changes by the degree of MRPS mobility, which correspond to Figure 1b (case 2) and has appearance of the Figure 3b version.

This case differs from the above case, that it does not have first subsection acceleration. It is possible for the case when  $q_{i,1} \ge q_{i,2} \ge q_{i,3}$ , or  $q_{i,1} \le q_{i,2} \le q_{i,3}$ . The formulas of time intervals and the generalized coordinates value changes calculating remains the same (5) (6), but in this case there is only the second and third subsections  $\Delta t_1 = 0$ . In some cases, there might be cases of brake action not from nominal speed to zero speed, but to the set speed  $q_i^3$ , in this case

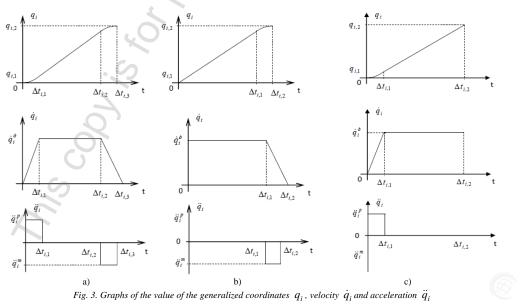
$$\Delta t_{3} = \frac{\dot{q}_{i}^{\delta} - \dot{q}_{i}^{3}}{\ddot{q}_{i}^{m}}, \ \Delta q_{1}^{3} = \dot{q}_{i}^{\delta} \Delta t_{3} - \frac{\ddot{q}_{i}^{m} (\Delta t_{3})^{2}}{2}.$$
(7)

In some cases, there is realized the braking action from one velocity magnitude  $q_i^{3,1}$  to the magnitude  $q_i^{3,2}$ 

$$\Delta t_{3} = \frac{\dot{q}_{i}^{3,1} - \dot{q}_{i}^{3,2}}{\ddot{q}_{i}^{m}}, \ \Delta q_{1}^{3} = \dot{q}_{i}^{3,1} \Delta t_{3} - \frac{\ddot{q}_{i}^{m} (\Delta t_{3})^{2}}{2}.$$
(8)

The next possible option is the case presented in Figure 3c, which corresponds to the behavior graph of generalized coordinates changes of 3 case (Figure 2). In this case, the acceleration goes to rated speed, further the movement goes with a rated speed, and the third subsection of braking action – stay out, that is  $\Delta t_3 = 0$ . It is possible that case of acceleration from the zero rate to a given rate  $a_i^3$ 

$$\Delta t_{1} = \frac{\dot{q}_{i}^{\,3}}{\ddot{q}_{i}^{\,p}}, \ \Delta q_{1}^{1} = \frac{\ddot{q}_{i}^{\,p} (\Delta t_{1})^{2}}{2}.$$
<sup>(9)</sup>



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In some cases, the acceleration goes from one magnitude of given rate  $q_i^{3,1}$  to magnitude  $q_i^{3,2}$ 

$$\Delta t_1 = \frac{\dot{q}_i^{3,2} - \dot{q}_i^{3,1}}{\ddot{q}_i^p}, \ \Delta q_1^1 = \dot{q}_i^{3,1} + \frac{\ddot{q}_i^p (\Delta t_1)^2}{2}.$$
(10)

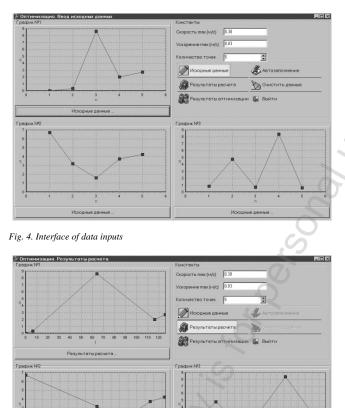
Further, based on analysis of changes in the values of generalized coordinates (see Figure 1) and formulas (4) - (10), the calculation of the required values  $\Delta t_{i,j}$  is calculated.

On the basis of *n* defined time intervals  $\Delta t_{i,j}$ , the time intervals needed to perform the movement from point  $A_j(x_j, y_j, z_j)$  to point  $A_{j+1}(x_{j+1}, y_{j+1}, z_{j+1})$  is determined:

 $\Delta t_{j} = \max_{i=1,2,\dots,n} \Delta t_{i,j} \cdot$ 

To determine the intervals  $\Delta t_{i,j}$ , the program on the algorithmic language Delphy 7.0 was developed [4]. This program allows calculating the time intervals, spent to the movement of the MR gripper from the given point  $A_j(x_j, y_j, z_j)$  to the point  $A_{j+1}(x_{j+1}, y_{j+1}, z_{j+1})$  and etc., as well as to calculate the velocity and acceleration, required for the passage of different distances for each degree of mobility of the *i* step at the same time [1, 3].

The Figure 4 illustrates the general view of the interface data input, and also where it is possible to set the maximum speed of the manipulator and the maximum acceleration, and the number of data points.



#### Fig. 5. Calculation results

Figure 5 shows the results of the program trajectory calculation window by MR movement degree. This window presents the following information:

- time spent to passage of the segment from point n-1 to point n,
- the maximum speed achieved on the illustrated interval,
- acceleration and deceleration, distance traveled,

- the way in which the manipulator gripper gaining speed,
- the way in which the gripper moving with constant velocity,
- the graph of the velocity (V) from the distance traveled (S).

### 2. Conclusion

Thus, on the basis of analyses of the generalized coordinates values, there can be determined the period of required time for acceleration, the motion at a given speed and braking changes in degree of the generalized coordinates of the robot manipulator mobility. Further, comparing the values of the necessary period of time, there can be determined period of time to mobility from one point to another given point of manipulation robot gripper trajectory.

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