

# APPLICATION OF VARIATIONAL DATA ASSIMILATION ALGORITHMS IN THE ECOLOGICAL MONITORING SYSTEM

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**Abstract:** The article deals with a question of application of variation algorithms in the system of ecological monitoring for the detection of sources of pollution with unknown location.

**Keywords:** modeling of atmospheric processes, environment protection, anthropogenic impact

## WYKORZYSTANIE ALGORYTMÓW WIARIANCYJNEJ ASYMLACJI DANYCH W SYSTEMIE MONITORINGU EKOLOGICZNEGO

**Streszczenie:** Artykuł poświęcony jest zadaniu wykorzystania algorytmu wariacyjnej asymilacji danych w systemie monitoringu ekologicznego do wyznaczenia funkcji stanu oceny intensywności znanego źródła oraz detekcji źródeł zakłóceń o nieznanym lokalizacjach.

**Słowa kluczowe:** modelowanie procesów atmosferycznych, ochrona środowiska, oddziaływanie antropogenne

### Introduction

The automated system of the air quality monitoring, meteorological parameters, radiation background, and water level works in the city of Ust-Kamenogorsk (Kazakhstan) since 2011 [1]. Implementation of the system allows obtaining up-to-date information on a condition of atmospheric air and using a database of observations taken in the past.

However, the system is still unable to detect location as well as parameters of atmospheric pollution sources. Therefore, an expansion of the present system is proposed regarding to detection of pollution sources and identification of polluting substances emission parameters, based on the data of ecological monitoring.

It is obvious that due to state regulations, possibility of making full-scale ecological and environmental experiments are considerably limited. Therefore, mathematical modelling based on both models and observations data is the only method for situational risks estimation, study of dynamics of natural and anthropogenic accidents, forecasting their impact on the ecosystem.

In this paper, an approach to the atmospheric pollution sources detection, based on the variation algorithms is proposed. Application of variation data assimilation algorithm in ecological monitoring system allows to recover state functions and to determine pollution sources using model of substances propagation in atmosphere, their circulation in the atmosphere model, and the observation data.

### 1. Systems of ecological monitoring

Computational capacity of modern computer systems as well as increased urgency of environmental problems have enabled application of up-to-date information technologies in environmental monitoring. It is defined as a system of observation, estimation, and forecasting of a state of environment based on measurements, modeling, expert estimations and other methods of ecological objects condition indicators changes detection [2]. The functions describing concentration of polluting substances are the main quantitative indicators of the atmospheric air quality.

According to the used techniques of the impact on the nature environment estimation, the information technologies can be classified into two groups [3]:

- standard engineering techniques, which realize calculations using concentration area, based on properties of emissions source;
- software products, which are based on mathematical modeling of the process of impurities propagation.

There are two basic approaches to solve estimations problems of the atmospheric air quality and its pollution forecasting from the man-made emissions, using mathematical modelling [4].

The first, traditional approach consists of the direct modeling methods, which are based on the initial boundary value problems solving with the different input parameters specification, such as initial concentration distribution, of the characteristic pollution sources, etc. The direct modeling methods are effective mainly for short-term, real time forecasting of the atmospheric air quality.

The second approach is a return modeling, which is used for the estimation of long-term ecological prospects and nature protection under conditions of changing climate.

### 2. Variation algorithms

The variational principles are used for the system organization of computing technologies and construction of direct and return connections between models. According to them, along with differential statement of the model, the variation formulation is used. For the inclusion into the modeling system of observation data, the functional dependence between the data of measurements and state functions in a mode of direct and return connections is formulated.

The approach based on a classical Lagrangh variational principle with the use of adjoint equations is the most promising for the estimation and forecasting problems of natural processes solving [5].

Methods of data assimilation using variation algorithms were developed by researchers of the Institute of Computational Technologies and Mathematical Geophysics of the Siberian Branch of the Russian Academy of Sciences (Novosibirsk, Russia) [4 - 7].

According to this method, by means of integral identities models can be rewritten in the variation form. The choice of the metrics and functionals for the identification of these identities was carried out in order to coordinate description of processes of various space-time scales and to unite various models in an one uniform manner.

The variation principles are used for the computing technologies system organization and construction of both direct and return links between models. According to them, along with differential statement of the model, the variation formulation is used. In order to include observation data into the modeling system, the functional dependence between the data of measurements and state functions is formulated in a mode of direct and return links.

The construction of the core, functional for the modeling organization considered: the models; the available data on sources; the results of observations of various types (direct, indirect, contact, remote, land, space basing, etc.), minimization of the influence of models uncertainty and data included in the functional.

In the adjoint problems the gradients of functionals with respect to the state function components in nodes of the net domain represent the sources. Therefore, under the functional content and structure, these problems involve all internal links between various elements of the modeling system included in the basic functionality. The discretization of functionals is carried out using weak approximation methods, splitting and decomposition. The final discrete models approximations and modeling algorithms are obtained from the models stationary conditions with respect to variations of the state function components.

Thus, the adjoint problems allow receiving valuable information on functional sensitivity to the deviation of some parameters from normal values. The role of the adjoint problems in variation modeling consists of the links formation between the disturbances of the objects defined in the space of state functions with the disturbances of characteristics from the space of parameters. The technology of forecasting is carried out based upon the sensitivity relations.

### 3. Determining the pollution sources

It is important to consider the problem of pollution sources determining, according to available measurements. The diffusive form of the equation of the propagation of substances in atmosphere is described by the following system [8]:

$$\begin{aligned} \frac{\partial \varphi}{\partial t} + \text{div} \bar{u} \varphi &= f + \frac{\partial}{\partial z} (\mu_z \frac{\partial \varphi}{\partial z}) + \bar{\Delta} \varphi, \\ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \omega}{\partial z} &= 0, \end{aligned} \quad (1)$$

where

$$\text{div} \bar{u} \varphi = u \frac{\partial \varphi}{\partial x} + v \frac{\partial \varphi}{\partial y} + \omega \frac{\partial \varphi}{\partial z}; \quad \bar{\Delta} = \frac{\partial}{\partial x} \mu_x \frac{\partial}{\partial x} + \frac{\partial}{\partial y} \mu_y \frac{\partial}{\partial y},$$

$t$  is time;  $f$  is a function, which describes sources of impurities;  $\mu_x, \mu_y$  are horizontal coefficients of turbulence;  $\mu_z$  is the vertical coefficient of turbulence;  $u, v, \omega$  are the components of a vector of a wind velocity in the directions of the Cartesian coordinates;  $\varphi$  is the concentration of a pollutant.

Equation (1) is solved in the domain  $D = \{0 \leq x \leq X, 0 \leq y \leq Y, h \leq z \leq H, 0 \leq t \leq T\}$  under the following initial and boundary value conditions.

The initial value condition can be described:

$$\varphi = \underline{\varphi} \text{ when } t = 0;$$

but the considered boundary conditions represent equations:

$$\begin{aligned} \mu_x \frac{\partial \varphi}{\partial x} &= 0, \text{ when } x = 0, X; \\ \mu_y \frac{\partial \varphi}{\partial y} &= 0, \text{ when } y = 0, Y; \\ -\mu_z \frac{\partial \varphi}{\partial z} &= q_i - V_a \varphi \text{ when } z = h, \\ \mu_z \frac{\partial \varphi}{\partial z} &= 0 \text{ when } z = H. \end{aligned}$$

The sources impurity on the lower boundary is determined by  $q_i$ , and  $V_a$  is the speed of the sedimentation of impurities.

The hydrodynamic background formation for the calculation of concentration area of polluting substance is conducted using the model of atmosphere circulation. [8].

The first stage of the numerical solution is approximation in time domain.

The differential form of the equation (1) can be written:

$$\frac{\partial \varphi}{\partial t} + A_x \varphi + A_y \varphi + A_z \varphi = f,$$

$$\varphi|_{t=0} = \underline{\varphi},$$

where  $A_x, A_y, A_z$  are differential operators.

The approximation in time domain considers purely implicit scheme:

$$\frac{\varphi^n - \varphi^{n-1}}{\tau} + (\Lambda_x + \Lambda_y + \Lambda_z) \varphi^n = f^n, \quad (2)$$

$$\varphi^0 = \underline{\varphi},$$

where  $\Lambda_x, \Lambda_y, \Lambda_z$  are the difference operators, which approximate  $A_x, A_y, A_z$  respectively;  $\tau$  is time step;  $n$  is time index,  $0 \leq n \leq N_t$ .

Evolutionary problem (2) using splitting on spatial variables is reduced to the consecutive solving of the following equations [9, 10]:

$$\begin{aligned} (E + \tau \Lambda_z) \varphi_1^n - (\varphi_3^{n-1} + \mathcal{T}^n) &= 0; \\ (E + \tau \Lambda_x) \varphi_2^n - \varphi_1^n &= 0; \\ (E + \tau \Lambda_y) \varphi_3^n - \varphi_2^n &= 0; \end{aligned}$$

where  $\varphi_1, \varphi_2, \varphi_3$  are the values of  $\varphi$  on the fractional steps of splitting,  $\varphi_3^n = \varphi^n$ ,  $E$  is a unit matrix.

According to the variation approach, the relation among unknown function and measurement data takes the form [10]:

$$\begin{aligned} \varphi^{*N_t} &= \varphi_1^{*N_t} = 0; \\ (E + \tau \Lambda_y)^* \varphi_3^{*n} &= \varphi_1^{*n+1} + \mathcal{T}_m^n; \\ (E + \tau \Lambda_x)^* \varphi_2^{*n} &= \varphi_3^{*n}; \\ (E + \tau \Lambda_z)^* \varphi_1^{*n} &= \varphi_2^{*n}; \end{aligned} \quad (3)$$

where  $\varphi^*$  is the conjugate function;  $p_m$  is a function of measurement in point  $m$ . In the system of ecological monitoring, this is a place of dislocation of the gas analyzer. If the conjugate function satisfies equation (3), then a Lagrange identity is hold.

$$\langle \varphi, p_m \rangle = \langle \varphi_m^*, f \rangle. \quad (4)$$

From the Lagrange identity for various measurements, the following system can be obtained:

$$\Gamma f = \psi,$$

where  $\Gamma$  is the matrix of adjoint functions corresponding to the set of functional of observations,  $\psi$  are the measurement data,  $f$  is a sought function of a source.

### 4. Results of calculations

The two-dimensional case was considered and the results of the source coordinates determination are presented in the Figure 1. The dot source, located in the point  $x=30, y=30$ , is represented as a red circle. The source position was obtained according to the observation data. The isolines in the figure define obtained source.

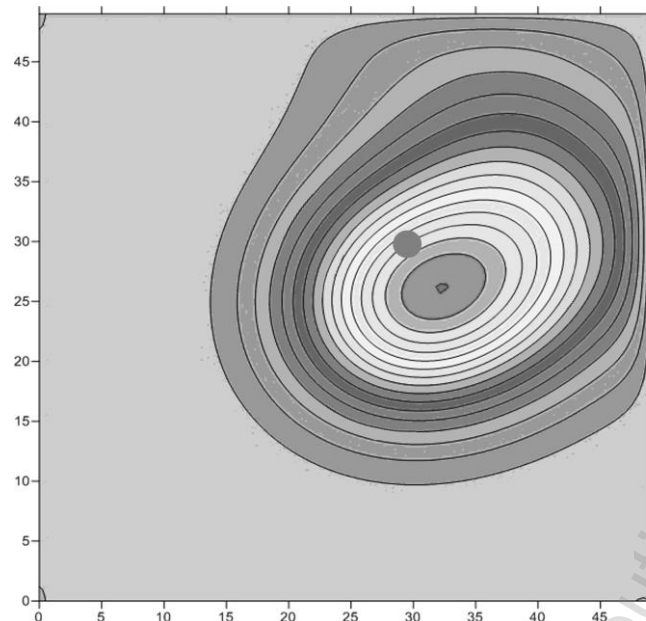


Fig. 1. Result of the identification of the source of pollution

The coordinates of observation points are presented in Table 1.

Table 1. Coordinates of observation points

No.	X coordinate	Y coordinate
1	30	40
2	10	30
3	20	25
4	10	10
5	20	15

The comparison of coordinates of the given source and the restored source allowed to achieve the satisfactory result.

## 5. Conclusions

Nowadays, it is extremely important to monitor natural environment against environmental hazards. To achieve this the automated early warning system of the air quality monitoring, meteorological parameters, radiation background, and water level function can be very useful.

The ability to identify the source of contamination ought to be one of its essential features. Both the algorithm and application of atmospheric pollution sources detection, were proposed.

They included the model of substances propagation in atmosphere, their circulation in the atmosphere model as well as the observation data.

The conducted numerical experiment has shown that the use of the algorithm of the detection of the instant dot source based on variation principles, returned satisfactory results. This fact suggests that it may be used in the ecological monitoring system.

The further research directions include the algorithm implementation for permanent and plural sources.

The program module, which realises described the pollution source identification algorithm has been introduced into the automated air quality monitoring system of the Ust Kamenogorsk city.

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