MODELLING FUZZY BELIEFS OF AGENTS

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Abstract: Ordered fuzzy numbers (OFN) were introduced by Kosiński, Prokopowicz and Ślęzak in 2002. The definition of OFN uses the extension of the parametric representation of convex fuzzy numbers. So far, they were applied to deal with optimization problems when data are fuzzy. In 2011 Kacprzak and Kosiński observed that a subspace of OFN called step ordered fuzzy numbers (SOFN) may be equipped with a lattice structure. In consequence, a Boolean operations like conjunction, disjunction and, what is more important, diverse types of implications can be defined on SOFN. In this paper we show how OFN can be applied in multi-agent systems for modelling agents' beliefs about fuzzy expressions. Then we present preliminary version of a logic based on SOFN and study how this logic can be helpful in evaluating features of multi-agent systems concerning agents' fuzzy beliefs.

Keywords: ordered fuzzy numbers, step ordered fuzzy numbers, fuzzy beliefs, multi-agent systems

1. Introduction

In real life we often use notions like bad weather, high temperature, small women, high humidity, obese man, or a firm which does well. Let us focus on these expressions. When we say that somebody is obese, when we talk about obesity? As a criterion we may consider body mass index (BMI) - a measurement which compares weight and height. However, in every day chatting, nobody calculates this index and then such an assessment deeply depends on a performer of the claim. Expressions which are not clear-cut and for which it is difficult to assign one from the values *true* or *false*, occur not only in human communication but also in software engineering, e.g., in rules exploited in fuzzy controllers. In the current work we are going to

study and analyze a problem of representation and evaluation of fuzzy expressions in communication performed in multi-agent systems. To capture diversity of approaches concerning expressions like "obesity", in literature are considered multi-valued logics [24,25] or fuzzy logics [31].

The good example of application of fuzzy technique in the context of Multi-Agent Systems (MAS) technology was done by Maione and Naso in [26] in manufacturing control systems. Namely, it is known that agents derive inspiration from communities of intelligent decision makers in uncertain and extremely dynamic environments, and that fuzzy techniques are suited to model human decision-making. Therefore, in [26] the authors discuss the potentialities of the challenging combination of Soft Computing, namely fuzzy logic techniques, and Multi-Agent paradigms in task contracting problems for manufacturing control. In particular, the paper examines if and how much agents' decision schemes benefit from the application of fuzzy methodologies.

The Fuzzy Set Theory gives effective tools to model the degreases of satisfaction of decision objectives and to combine them in a unique criterion of evaluation. In particular, each decision objective can be described with a fuzzy membership function where degree zero (one) expresses the minimum (maximum) satisfaction of the objective, while all the intermediate values represent degrees of partial satisfaction ([3,33]). Then the global objective is the fuzzy aggregation of the weighted goals when t-norm may be used or, parameterized operator providing a more realistic tradeoff between the conflicting objectives (suggested in [33], p. 37) called the "Compensatory AND" operator with some free parameter to be fixed.

In our paper we propose new approach in which **ordered fuzzy numbers** (OFN) are applied. We limit our considerations to multi-agent systems and concentrate on agents' beliefs. Our study is twofold. On the one hand we want to make agents able to use ordered fuzzy numbers in their "thinking" and making decisions. On the other hand we plan to use ordered fuzzy numbers for evaluating agents' beliefs about their beliefs.

The theory of fuzzy numbers [6] is that set up by Dubois and Prade [7], who proposed a restricted class of membership functions, called (L, R)–numbers with shape functions L and R. However, approximations of fuzzy functions and operations are needed if one wants to follow Zadeh's [31] extension principle. It leads to some drawbacks that concern properties of fuzzy algebraic operations, as well as to unexpected and uncontrollable results of repeatedly applied operations. These problems are resolved in ordered fuzzy numbers. OFN were invented by Kosiński, Prokopowicz and Ślęzak in the previous decade [17,18,19,20,21]. The definition of OFN uses the extension of the parametric representation of convex fuzzy numbers. **Step ordered fuzzy**

numbers (SOFN) form a subspace of ordered fuzzy numbers and may be equipped with a lattice structure. Then fuzzy implications can be defined on OFN and SOFN with the help of algebraic operations defined on OFN.

Fuzzy logic, as the originator of this idea Lotfi A. Zadeh noticed (c.f. [32]), has two main directions. Fuzzy logic in the *broad* sense is one of the techniques of soft-computing and serves mainly as apparatus for fuzzy control, analysis of vagueness in natural language and several other application domains. In this field the methods of fuzzification, approximate reasoning and defuzzification are often exploited. Here, we give three examples of works which join the paradigm of multi-agent systems with fuzzy control. One of them is a fuzzy-based approach for partner selection in MAS [30]. In this work, agents, using fuzzy reasoning, can adapt their individual behaviors for partner selection in negotiation. By employing fuzzy logic, the proposed approach can be applied in open and dynamic environments easily and flexibly. The next example is a multi-agent system for knowledge-based access to distributed databases [29]. In this work, the KQML (Knowledge Query and Manipulation Language) is extended with fuzzy linguistic variables to deal with the human style of decision processing and support fuzzy decision making. The last example is a multi-agent systems for environmental control and intelligent buildings [12]. This work refers to the intelligent home project in which home environment is fitted with distributed intelligent home-control agents like WaterHeater, CofeeMaker, DishWasher, etc. All of them exploit in their work fuzzy inferencing.

Fuzzy logic in the *narrow* sense is a branch of many-valued logic based on the paradigm of inference under vagueness. Here, the focus of the research is on a logical system and its metamathematical properties. A basic monograph in this field is written by Hajek [13]. One of the main operations in fuzzy logic are fuzzy implications. Deep study on analytical and algebraic aspects of fuzzy implications are presented by Baczynski and Jayaram [1].

Our current paper refers to both meanings of the term of fuzzy logic. On the one hand, our scientific interests concern multi-agent systems where agents are fuzzy controllers. Our main idea is to enrich such systems with much more sophisticated dialogues than are offered by KQML. On the other hand, we make a first step for introducing a new logic where ordered fuzzy numbers play a crucial role. The main goal of this research is to offer a formal tool adequate for evaluating properties concerning dialogues between fuzzy controllers and other agents.

The paper is organized as follows. Section 1 gives a brief overview of ordered fuzzy numbers. In Section 2, step ordered fuzzy numbers are presented. Section 3 defines a lattice structure on OFN. In section 4 application of OFN in reasoning about agents' beliefs is discussed. Section 5 gives some conclusions.

2. Ordered fuzzy numbers

Proposed recently by the second author and his two coworkers: P.Prokopowicz and D. Ślęzak [17,18,19,20,21] an extended model of convex fuzzy numbers [27] (CFN), called ordered fuzzy numbers (OFN), does not require any existence of membership functions. In this model an ordered fuzzy number is a pair of continuous functions, f and g, say, defined on the interval $[0,1]$ with values in $\mathbf{R}$. To see OFN as an extension of CFN - model, take a look on a parametric representation know since 1986, [9] of convex fuzzy numbers.

The continuity of both parts implies their images are bounded intervals, say UP and $DOWN$, respectively. We may used symbols to mark boundaries for $UP = [l_A, 1_A^-]$ and for $DOWN = [1_A^+, p_A]$. In general, the functions f, g need not to be invertible, only continuity is required. If we add the constant function on the interval $[1_A^-, 1_A^+]$ with its value equal to 1, we might define the membership function

$$\mu(x) = \mu_{up}(x), \text{ if } x \in [l_A, 1_A^-] = [f(0), f(1)], \tag{1}$$
$$\mu(x) = \mu_{down}(x), \text{ if } x \in [1_A^+, p_A] = [g(1), g(0)] \text{ and}$$
$$\mu(x) = 1 \text{ when } x \in [1_A^-, 1_A^+]$$

if $f \leq g$ are both invertible, i.e. inverse functions $f^{-1} =: \mu_{up}$ and $g^{-1} =: \mu_{down}$ exist, and f is increasing, and g is decreasing. Obtained in this way the membership function $\mu(x), x \in \mathbf{R}$ represents a mathematical object which reminds a convex fuzzy number in the classical sense [5,15].

On OFN four algebraic operations have been proposed between fuzzy numbers and crisp (real) numbers, in which componentwise operations are present. In particular if $A = (f_A, g_A), B = (f_B, g_B)$ and $C = (f_C, g_C)$ are mathematical objects called ordered fuzzy numbers, then the sum $C = A + B$, product $C = A \cdot B$, division $C = A \div B$ and scalar multiplication by real $r \in \mathbf{R}$, are defined in natural way: $r \cdot A = (rf_A, rg_A)$ and $f_C(y) = f_A(y) \star f_B(y)$, $g_C(y) = g_A(y) \star g_B(y)$ where "$\star$" works for "+", "·", and "$\div$", respectively, and where $A \div B$ is defined, if the functions $|f_B|$ and $|g_B|$ are bigger than zero. Notice that the subtraction of B is the same as the addition of the opposite of B, i.e. the number $(-1) \cdot B$, and consequently $B - B = 0$. From this follows that any fuzzy algebraic equation $A + X = C$ with given A and C as OFN possesses a solution, that is OFN, as well. Moreover, to any convex and continuous fuzzy number correspond two OFNs, they differ by the orientation: one has positive, say (f, g), another (g, f) has negative.

A relation of partial ordering in the space of all OFN, denoted by $\mathcal{R}$, can be introduced by defining the subset of 'positive' ordered fuzzy numbers: a number $A =$

(f,g) is not less than zero, and by writing

$$A \geq 0 \quad \text{iff} \quad f \geq 0, g \geq 0 . \tag{2}$$

In this way the set $\mathcal{R}$ becomes a partially ordered ring. Notice, that for each two fuzzy numbers $A = (f_A, g_A), B = (f_B, g_B)$ as above, we may define $A \wedge B =: F$ and $A \vee B =: G$, both from $\mathcal{R}$, by the relations:

$$F = (f_F, g_F), \text{if } f_F = \inf\{f_A, f_B\}, g_F = \inf\{g_A, g_B\} . \tag{3}$$

Similarly, we define $G = A \vee B$.

Notice that in the definition of OFN it is not required that two continuous functions f and g are (partial) inverses of some membership function. Moreover, it may happen that the membership function corresponding to A does not exist; such numbers are called improper. In any case for $A = (f, g)$ we call f - the up-part and g - the down-part of the fuzzy number A. To be in agreement with further and classical denotations of fuzzy sets (numbers), the independent variable of the both functions f and g is denoted by y (or some times by s), and their values by x.

3. Step ordered fuzzy numbers

It is worthwhile to point out that a class of ordered fuzzy numbers (OFNs) represents the whole class of convex fuzzy numbers that possess continuous membership functions. To include all CFN (with discoontiuous meembership functions) some generalization of functions f and g is needed. This has been already done by the second author who in [16] assumed they are functions of bounded variation. i.e. they belong to BV. Then all convex fuzzy numbers are contained in this new space $\mathcal{R}_{BV} \supset \mathcal{R}$ of OFN. Then operations are defined $\mathcal{R}_{BV}$ in the similar way, the norm, however, will change into the norm of the cartesian product of the space of functions of bounded variations. Then all convex fuzzy numbers are contained in this new space $\mathcal{R}_{BV}$ of OFN. Notice that functions from BV [23] are continuous except for a countable numbers of points.

Important consequence of this generalization is the possibility of introducing a subspace of OFN composed of pairs of step functions. If we fix a natural number K and split $[0,1)$ into $K-1$ subintervals $[a_i, a_{i+1})$, i.e. $\bigcup_{i=1}^{K-1} [a_i, a_{i+1}) = [0,1)$, where $0 = a_1 < a_2 < ... < a_K = 1$, and define a step function f of resolution K by putting u_i on each subinterval $[a_i, a_{i+1})$, then each such function f is identified with a K-dimensional vector $f \sim u = (u_1, u_2 ... u_K) \in \mathbf{R}^K$, the K-th value u_K corresponds to
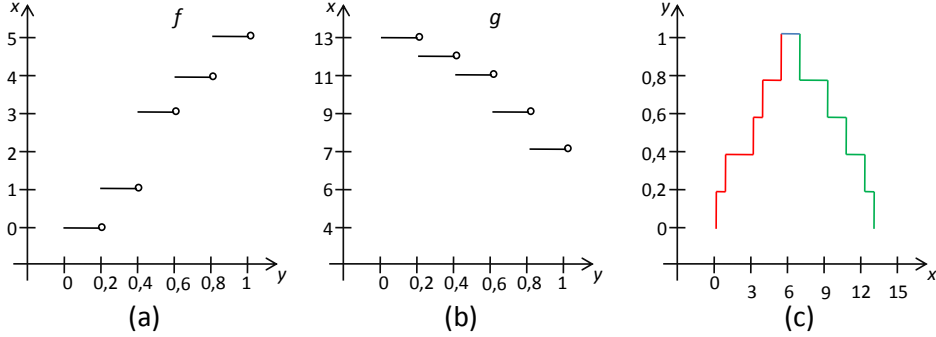
Fig. 1. Example of a step ordered fuzzy number $A = (f,g) \in \mathcal{R}_K$, (a) function f, (b) function g, (c) membership function.

$s = 1$, i.e. $f(1) = u_K$. Taking a pair of such functions we have an ordered fuzzy number from $\mathcal{R}_{BV}$. Now we introduce

Definition 2. By a step ordered fuzzy number A of resolution K we mean an ordered pair (f,g) of functions such that $f,g : [0,1] \rightarrow \mathbf{R}$ are K-step functions.

We use $\mathcal{R}_K$ for denotation the set of elements satisfying Def. 2. The example of a step ordered fuzzy number and its membership function are shown in Fig. 1. The set $\mathcal{R}_K \subset \mathcal{R}_{BV}$ has been extensively elaborated by our students in [10] and [22]. We can identify $\mathcal{R}_K$ with the Cartesian product of $\mathbf{R}^K \times \mathbf{R}^K$ since each K-step function is represented by its K values. It is obvious that each element of the space $\mathcal{R}_K$ may be regarded as an approximation of elements from $\mathcal{R}_{BV}$, by increasing the number K of steps we are getting the better approximation. The norm of $\mathcal{R}_K$ is assumed to be the Euclidean one of $\mathbf{R}^{2K}$, then we have a inner-product structure for our disposal.

4. Lattice structure on $\mathcal{R}_K$

Let us consider the set $\mathcal{R}_K$ of step ordered fuzzy numbers with operations $\vee$ and $\wedge$ such that for $A = (f_A, g_A)$ and $B = (f_B, g_B)$

$$A \vee B = (sup\{f_A, f_B\}, sup\{g_A, g_B\}), \quad A \wedge B = (inf\{f_A, f_B\}, inf\{g_A, g_B\}).$$

In [14] we have shown that the algebra $(\mathcal{R}_K, \vee, \wedge)$ defines a lattice structure and proved the following theorem.

Theorem 1. The algebra $(\mathcal{R}_K, \vee, \wedge)$ is a lattice.

Let $\mathcal{B}$ be the set of two binary values: $0, 1$ and let us introduce the particular subset $\mathcal{N}$ of $\mathcal{R}_K$ $\mathcal{N} = \{A = (\underline{u}, \underline{v}) \in \mathcal{R}_K : \underline{u} \in \mathcal{B}^K, \underline{v} \in \mathcal{B}^K\}$. It means such that each component of the vector $\underline{u}$ as well as of $\underline{v}$ has value 1 or 0. It is easy to observe that all subsets of $\mathcal{N}$ have both a join and a meet in $\mathcal{N}$. In fact, for every pair of numbers from the set $\{0, 1\}$ we can determine *max* and *min* and it is always 0 or 1. Therefore $\mathcal{N}$ creates a *complete lattice*. In such a lattice we can distinguish the greatest element $\underline{1}$ represented by $(1, ..., 1)$ and the least element $\underline{0}$ represented by $(0, ..., 0)$.

Theorem 2. The algebra $(\mathcal{N}, \vee, \wedge)$ is a complete lattice.

We say that two elements A and B are *complements* of each other if and only if $A \vee B = \underline{1}$ and $A \wedge B = \underline{0}$. The complement of a number A will be marked with $\neg A$ and is defined as follows:

Definition 3. Let $A \in \mathcal{N}$ be a step ordered fuzzy number represented by a binary vector $(a_1, a_2, \ldots, a_{2K})$. By the complement of A we understand

$$\neg A = (1 - a_1, 1 - a_2, \ldots, 1 - a_{2K}).$$

A bounded lattice for which every element has a complement is called a *complemented lattice*. Moreover, the structure of step ordered fuzzy numbers $\{\mathcal{N}, \vee, \wedge\}$ forms a complete and complemented lattice in which complements are unique. In fact it is a *Boolean algebra*. In the example with $K = 2$ a set of universe is created by vectors

$$\mathcal{N} = \{(a_1, a_2, a_3, a_4) \in \mathbf{R}^4 : a_i \in \{0, 1\}, \text{for } i = 1, 2, 3, 4\}.$$

The complements of elements are $\neg(0, 1, 0, 0) = (1, 0, 1, 1)$, $\neg(1, 1, 0, 0) = (0, 0, 1, 1)$ etc. Now we can rewrite the definition of the complement in terms of a new mapping.

Definition 4. For any $A \in \mathcal{N}$ we define its negation as

$$N(A) := (1 - a_1, 1 - a_2, \ldots, 1 - a_{2K}), \text{if } A = (a_1, a_2, \ldots, a_{2K}).$$

It is obvious, from Definitions 3 and 4, that the negation of given number A is its complement. Moreover, the operator N is a strong negation, because is involutive, i.e.

$$N(N(A)) = A \text{ for any } A \in \mathcal{N}.$$

One can refer here to known facts from the theory of fuzzy implications (cf. [1,2,8]) and to write the strong negation N in terms of the standard strong negation N_I on the unit interval $I = [0,1]$ defined by $N_I(x) = 1 - x, x \in I$, namely $N((a_1, a_2, \ldots, a_{2K})) = ((N_I(a_1), N_I(a_2), \ldots, N_I(a_{2K}))$.

In the classical Zadeh's fuzzy logic the definition of a fuzzy implication on an abstract lattice $\mathcal{L} = (L, \leq_L)$ is based on the notation from the fuzzy set theory introduced in [8].

Definition 5. Let $\mathcal{L} = (L, \leq_L, 0_L, 1_L)$ be a complete lattice. A mapping $I : L^2 \to L$ is called a fuzzy implication on $\mathcal{L}$ if it is decreasing with respect to the first variable, increasing with respect to the second variable and fulfills the border conditions

$$I(0_L, 0_L) = I(1_L, 1_L) = 1_L, I(1_L, 0_L) = 0_L. \tag{4}$$

Now, possessing the lattice structure of $\mathcal{R}_\mathcal{K}$ (SOFN) and the Boolean structure of our lattice $\mathcal{N}$, we can repeat most of the definitions know in the Zadeh's fuzzy set theory. The first one is the Kleene–Dienes operation, called a binary implication, already introduced in our previous paper [14] as the new implication (cf. Definition 4 in [14])

$$I_b(A, B) = N(A) \vee B, \text{ for any } A, B \in \mathcal{N}. \tag{5}$$

In other words, the result of the binary implication $I_b(A, B)$, denoted in [14] by $A \to B$, is equal to the result of operation *sup* for the number B and the complement of A:

$$A \to B = sup\{\neg A, B\}.$$

Next we may introduce the Zadeh implication by

$$I_Z(A, B) = (A \wedge B) \vee N(A), \text{ for any } A, B \in \mathcal{N}. \tag{6}$$

Since in our lattice $\mathcal{R}_K$ the arithmetic operations are well defined we may introduce the counterpart of the Lukasiewicz implication by

$$I_L(A, B) = C, \text{where } C = 1 \wedge (1 + B - A). \tag{7}$$

In the calculating the RHS of (7) we have to regard all numbers as elements of $\mathcal{R}_K$, since by adding the ordered fuzzy number A from $\mathcal{N}$ to the crisp number 1 we may leave the subset $\mathcal{N} \subset \mathcal{R}_\mathcal{K}$. However, the operation $\wedge$ will take us back to the lattice $\mathcal{N}$. It is obvious that in our notation $1_N = 1$. The explicit calculation will be: if $C = (c_1, c_2, \ldots, c_{2K}), A = (a_1, a_2, \ldots, a_{2K}), B = (b_1, b_2, \ldots, b_{2K})$, then $c_i = min\{1, 1 - a_i + b_i\}$, where $1 \leq i \leq 2K$.

5. Application of OFN in reasoning about fuzzy beliefs

In this section we show how agents' attitudes can be modelled by means of ordered fuzzy numbers. Assume a model of a multi-agent system which is consistent with the formalism used in the software tool Perseus [4]. In the future this verifier may be extended to analyze also distributed systems where agents have fuzzy beliefs. In this formalism, a model of a multi-agent system is assumed to be an enriched Kripke structure $\mathcal{M} = (Agt, S, RB, I, v)$ where

- $Agt = \{1, 2, 3, \ldots, n\}$ is a set of names of agents,
- S is a non-empty set of states (the universe of a structure),
- RB is a (doxastic) function which assigns to every agent a binary relation, $RB : Agt \longrightarrow 2^{S \times S}$ - this function gives an interpretation for agents' beliefs,
- I is an interpretation of actions, $I : \Pi_0 \longrightarrow (Agt \longrightarrow 2^{S \times S})$ where Π_0 is a set of atomic actions,
- v is a valuation function, $v : S \longrightarrow \{\mathbf{0}, \mathbf{1}\}^{V_0}$ where V_0 is a set of propositions.

This model should be extended with a set L of **linguistic variables**. By linguistic variables we mean variables which values are from the set of words or sentences of a natural (or artificial) language. Formally it is a foursome $l = (Z, T, U, m)$ where

- Z is a name of variable l,
- T is a set of fuzzy terms which can be assigned to l,
- U is a digital interval of values of l,
- m is a rule which assigns ordered fuzzy numbers to terms from the set T.

For example, let l describes *speed* of a car, then fuzzy terms which can be assigned to l are *back*, *very slowly*, *slowly*, *fast*, digital values for *speed* are assumed to be from the interval $[-50, 100]$. Ordered fuzzy numbers assigned for *speed* are figured in Fig. 4a.

Since a current state of a multi-agent system changes dynamically it is reasonable to expect that a rule m of a linguistic variable l will be different in different states. Therefore we propose to extend definition of l and assume that $l = (Z, T, U, m_{AS})$ where $m_{AS} : T \times Agt \times S \rightarrow OFN$ is a function which for every agent and every state of a system assigns an ordered fuzzy number being an interpretation for elements from T. In other words, in various situations for various agents different rule m may be accepted.

Now, consider a system with three agents A, B, C. The aim of the system is to simulate a movement of a point laying on a segment (cf. [22]). Agent A observes speed of the point. Agent B observes a distance from the point to the given stop point.
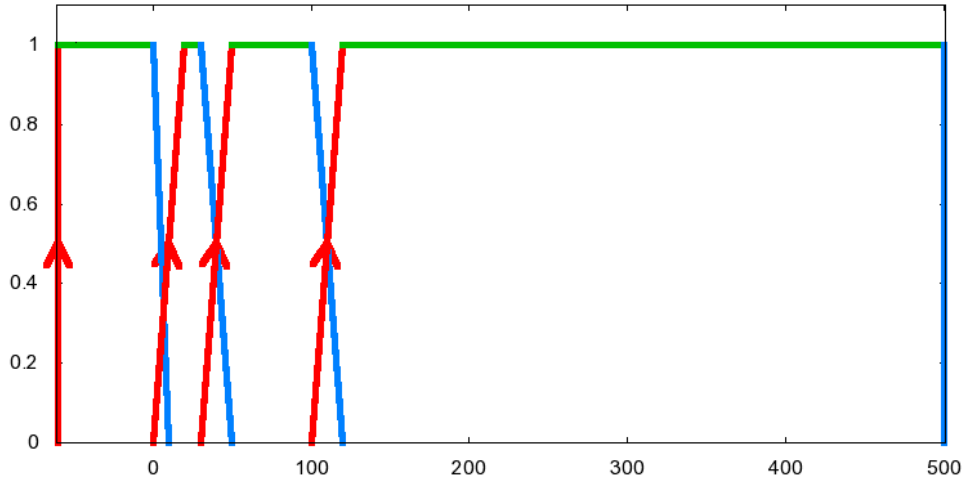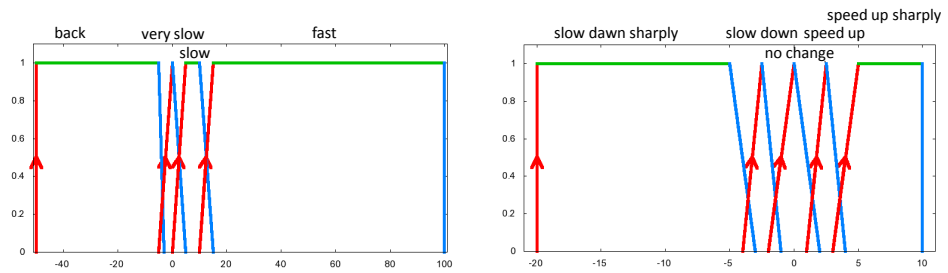
Fig. 2. Linguistic variable: *distance*

Fig. 3. Linguistic variables: *speed* and *acceleration*

Agent *C* is a fuzzy controller. Agents *A* and *B* provide to the controller data about the speed and the distance of the point. We assume that *speed* is a linguistic variable with values from the interval [-50,100] and terms "back, very slowly, slowly, fast". Similarly, *distance* is a linguistic variable with values from the interval [-50,300] and terms "too close, close, moderately close, far". Rules assigned ordered fuzzy numbers to the above terms at the initial state are presented in Fig. 3 and 4a. The tasks of agents *A* and *B* are to measure speed and the distance of the point (respectively), exchange digital values for fuzzy expressions and then provide these data to the controller. The task of the controller is to stop the point before the given stop point. It operates
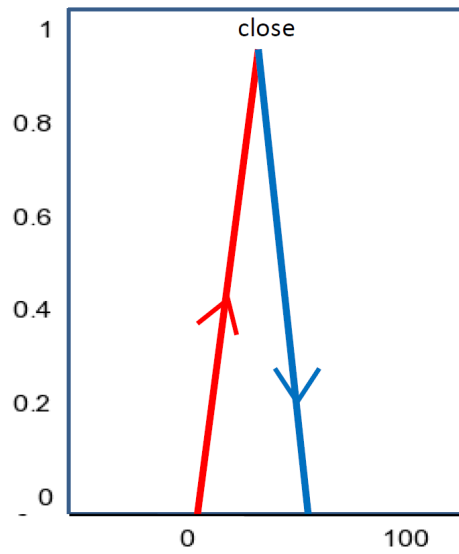
Fig. 4. Linguistic variable: *distance*

on the third linguistic variable, i.e., *acceleration*. Values of this variable are form the interval [-20,10] and are described by terms "slow dawn sharply, slow down, no change, speed up, speed up sharply". The rule which assigns ordered fuzzy numbers to these terms are pictured in Fig. 4b. The controller given fuzzy terms concerning *speed* and *distance* uses special rules to control the movement of the point. A base of rules for this example is given in Table 1.

Table 1. Rules of the fuzzy controller.

	too close	close	moderately close	far
back	speed up	speed up sharply	speed up sharply	speed up sharply
very slow	slow down sharply	no change	speed up	speed up sharply
slow	slow down sharply	slow down	no change	speed up
fast	slow down sharply	slow down sharply	slow down	no change

Finally the controller determines the output fuzzy set describing acceleration and transforms it into real value.

Application of OFN in modelling agents' beliefs has great advantage since allows for manipulating fuzzy expressions rather then strict digital values. It makes agents' communication easier and faster and definitely simplifies knowledge bases and rules which agents use in their decision process. In the example, agent C affects the speed of the point but it does not care when it is to close of too far. It is a role of agent B. In other words, agent B is a specialist which has abilities to evaluate the distance and current scenario and then judge, e.g., whether the distance is to small. For instance, if a point simulates a truck 1 meter to a wall means "close", but if this point simulates an ant 1 meter can be treated as very log distance. It is a specialist B job to assessment this.

An extension of a model of a multi-agent system to linguistic variables and exchanging a rule m with a set of rules m_{AS} causes that agents can manipulate fuzzy expressions and what is more can have different point of view on criteria which determine their subjective interpretations of this expressions.

The most important problem when we consider fuzzy beliefs of agents is how to check properties of such defined systems. The question is about a language in which we can evaluate whether some property is true or not. Leu us discus it now. Assume that in the above example is also agent D. It tries to guess the behavior of agent C, i.e., agent D needs to learn what action C decided to perform. D beliefs that if A says that the point is *close* and B says that the point moves *slowly* then C decides to *slow up*. To create a formula describing this property use here a commonly accepted language of epistemic logic based on Kripke structure [11]. In this formalism we can write

$$B_D(A\_close) \wedge B_D(B\_slow) \rightarrow B_D(C\_slow\_up)$$

where $B_D(T)$ informally means that agent D beliefs that T holds. Our aim is to verify whether this formula is true in a model of the system from the example.

Furthermore, we know that agent D is not sure about beliefs of agents A and B and assumes that terms *close* and *slow* are interpreted by ordered fuzzy numbers depicted in Fig. 5. Notice that D departs from the truth (cf. Fig. 3) but not so much. However, if we take into account classical two-valued logic then at the initial state of the system formula $B_D(A\_close) \wedge B_D(B\_slow)$ is not true. It stems from the fact that beliefs of D about beliefs of A and B are not true. For some digital values they agree but for another not. Although the OFN representations are not the same they are very similar. In two-valued logic we lose this important information. Therefore, we propose to use new, innovative approach in which step ordered fuzzy numbers are applied. In Section 3 we showed that SOFN creates a lattice with Boolean operations of conjunction, disjunction and implication. Therefore it is possible to employ these numbers as a logical values for OFN. Let v_l be a valuation function which for every

formula assigns an ordered fuzzy number and assume that (111111) means absolutely true and (000000) means absolutely false. Values between (111111) and (000000), like e.g. (10100) express different kinds of half-truth. Below is given a hypothetical assignment:

(a) $v_l(B_D(A_-close)) = (101101)$
(b) $v_l(B_D(B_-slow)) = (100111)$
(c) $v_l(B_D(C_-slow_-up)) = (000000)$

Analyze intuitions concerning these values. In (a) it is assumed that agent D does not know exactly for which digital values from [-50,500] term "*close*" is ascribed since the assigned value does not equal to (11111). However, if the interval [-50,500] is divided into 3 parts then in parts one and three agent D agrees with agent B. Similar interpretation is for value (100111) assigned to formula $B_D(B_-slow)$. In (c) it is assume that agent D has no idea what and when agent C says about acceleration of the point. Based on these values we can determine value of the whole formula:

$$v_l(B_D(A_-close) \wedge B_D(B_-slow) \rightarrow (B_D(C_-slow_-up)) = (011010)$$

It means that agent D guesses faultlessly the kind of activity of the controller when considered digital values of speed and distance are very small, middle or very high. Such information surely can not be expressed by classical logical values *true* and *false*. Although multi-valued and fuzzy logics can deal with more than two values such a precise knowledge can be captured only by ordered fuzzy numbers.

6. Conclusion

The paper lays the foundations of new logic based on step ordered fuzzy numbers which will be very helpful in capturing how agents can reason about fuzzy expressions. This is innovative approach to modelling agents' beliefs and their uncertainty about beliefs of other agents. We show motivation for introducing such a new logic. The application of it we mainly find in analyzing agents' communication when knowledge base of agents is represented by a set of ordered fuzzy numbers expressing diverse agents' attitudes. Furthermore, step ordered fuzzy numbers, when are applied as logical values for propositions and other formulas of the applied language, give much more information than that something is *true* or *false*. We hope that this innovative approach is very promising in specification and verification of multi-agent systems where some software engineering ideas are applied, e.g. where fuzzy control is suitable. It could be also very useful in reasoning about software agents which are

decision support systems. For example, we can analyze activity of agents which assist clients with their decisions in e-shops, i.e., agents which support users of a system in choosing a right product.

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MODELOWANIE ROZMYTYCH PRZEKONAŃ AGENTÓW

Streszczenie: Skierowane liczby rozmyte (SLR) zostały wprowadzone przez W. Kosińskiego, P. Prokopowicza i D. Ślęzaka w 2002 roku. Definicja skierowanych liczb rozmytych wykorzystuje rozszerzenie parametrycznej reprezentacji wypukłych liczb rozmytych. SLR do tej pory były wykorzystywane do rozwiązywania problemów optymalizacyjnych dla rozmytych danych. W 2011 roku M. Kacprzak i W. Kosiński zaobserwowali, że schodkowe skierowane liczby rozmyte (SSLR) stanowiące podprzestrzeń SLR tworzą kratę. W konsekwencji, Boolowskie operacje takie jak koniunkcja, alternatywa oraz różne rodzaje (rozmytych) implikacji mogą być zdefiniowane w zbiorze schodkowych skierowanych liczb rozmytych. Celem niniejszej pracy jest pokazanie nowego zastosowania SLR jakim jest modelowanie przekonań agentów w środowisku systemów wieloagentowych, gdy przekonania te dotyczą rozmytych pojęć i danych. Jest to pierwszy krok w kierunku stworzenia pełnej logiki opartej na wartościach ze zbioru SSLR. Logika ta umożliwi analizę własności systemów, w których agenci mają rozmyte przekonania.

Słowa kluczowe: skierowane liczby rozmyte, schodkowe skierowane liczby rozmyte, rozmyte przekonania, systemy wieloagentowe