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USING THE CONCEPT OF ENTROPY IN THE DISPLACEMENT RESEARCH OF THE POINTS OF A HORIZONTAL GEODETIC NETWORK

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Abstract. The aim of objects monitoring, carried out by a geodetic method, is to get any information about the casual and effect relationships which occur around the examined object. Application of a suitable measuring technique requires setting an advantageous network structure, which has to meet both the condition of measuring economy and assure the suitable characteristics of the determined parameters accuracy. The advantageous network structure may be obtained on the basis of selecting observations which contain the largest information content. The measurement results of the chosen network elements and the application of suitable methods of results processing enables to apply a correct identification of points of a reference system, and as a consequence, it also enables to define a correct model of the displacement. The process of setting an advantageous linear network structure on the basis of entropy of an observational system was presented in the paper.

Key words: geodetic network, entropy, information, reference system, displacement

INTRODUCTION

The geodetic networks structure which is set to determine displacements should be defined in the aspect of the advantageous characteristics of estimated parameters accuracy and the minimalization of incurred costs should also be taken into consideration. The advantageous network structure may be obtained on the basis of selecting an observation that contains the largest information content. The determined number of observations is to come up to the required expectations, because measurement of all possible geometrical elements of the network is not an optimal solution according to the entropy definition. The paper presents the author's deliberations on an advantageous structure of a linear measuring-controlling network which has been determined on the basis of the

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entropy of an observational system consisting of observations which contain the largest information content.

THE ENTROPY OF A GEODETIC OBSERVATIONAL SYSTEM

According to the accessible literature analysed by the author of this paper the first attempts of deliberations on the entropy, referring to geodetic observational systems, were made by Neuman [Neuman 1965], and they were next continued by Gil [1997, 1999].

The random vector of an observation $L = (l_1, l_2, ..., l_m)$ can be burdened to some extent with indeterminacy which concerns its real values. As a result of an observational system adjustment the indeterminacy of observations vector $L = (l_1, l_2, ..., l_m)$ generates the indeterminacy of estimator $\hat{X} = (\hat{x}_1, \hat{x}_2, ..., \hat{x}_n)$ parameters vector $X = (x_1, x_2, ..., x_n)$ [Gil 1999]. The observational system is represented by the matrix of coefficients of observation equations in the configuration:

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix} \qquad m \geq n \tag{1}$$

where: m – the number of observations (testing area),
n – the number of unknowns.

The content of information of a single supernumerary observation written in the line *b* the matrix *A* containing the following elements

$$b(a_{m+1,1}, a_{m+1,2}, ..., a_{m+1,n}) \tag{2}$$

to the general information of the whole system was described by the formula [Neuman 1965]:

$$\Delta h = log_2 \frac{g(m,n)\left\{1 + trace[(A^T A)^{-1}(b^T b)]\right\}}{g(m+1,n)\prod_{j=1}^{n}\sqrt{1 + \frac{a_{m+1,j}^2}{\sum_{i=1}^{m} a_{ij}^2}}} \tag{3}$$

while

$$g(m,n) = \frac{(m-n)^{\frac{n}{2}} B\left(\frac{m-n}{2}, \frac{n}{2}\right) exp\left\{\frac{m}{2}\left[\Psi\left(\frac{m}{2}\right) - \Psi\left(\frac{m-n}{2}\right)\right]\right\}\pi^{\frac{n}{2}}}{\Gamma\left(\frac{n}{2}\right)}. \tag{4}$$

The symbols in the formulas (3) and (4) signify:
$a_{i,j}$ – matrix *A* element,

$B\left(\frac{m-n}{2},\frac{n}{2}\right)$ – beta function,
$\Gamma\left(\frac{n}{2}\right)$ – gamma function,
$\left[\Psi\left(\frac{m}{2}\right),\Psi\left(\frac{m-n}{2}\right)\right]$ – Ψ Eulera function (logarithmic derivative of gamma function).

As a result of an observation vector L adjustment, the component of parameters vector estimator is to be also burdened to some extent with uncertainty. The entropy of parameters vector X assuming that the estimator $\hat{X}$ is known can be calculated on the basis of the following formula [Neuman 1965]:

$$H_{\hat{X}}[X]=\log_2\left[\frac{g(m,n)m_0^n\prod_{j=1}^{n}\sqrt{\sum_{i=1}^{m}a_{ij}^2}}{\prod_{j=1}^{n}\varepsilon_j\det\left(A^TA\right)}\right] \tag{5}$$

where: m_0 – means a mean error a priori of a singular observation [mm],
$\varepsilon_j=m_0\sqrt{\frac{\delta_0}{\lambda_j}}$ – system sensitivity along the given axis of coordinates,
δ_0 – decentralization parameter,
λ_j – eigenvalue matrix $\left(\boldsymbol{A}^{\boldsymbol{T}}\boldsymbol{A}\right)$.

The base of logarithm in the formulas (3) and (4) which equals 2, determines the unit of information content which equals one bit.

THE CHARACTERISTICS OF A TESTING GEODETIC NETWORK

The geodetic network was set to carry out a research on the stability of the earth dam crone of an accumulation lake. The 252 segment lengths among the points in the network, consisting of 30 controlling points located on the earth dam crone according to the project recommendation, were measured by a distance measuring instrument having a nominal accuracy $\mathrm{m}=\pm(3\mathrm{mm}+2\mathrm{ppm}\times D)$. During the measurement, the examined points located on the object were signalled by a prism placed on a tripod.

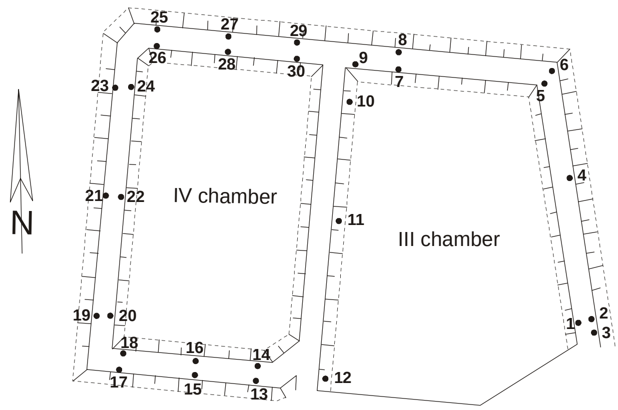

Fig. 1. The sketch of the arrangement of the points of a measuring-controlling network
Rys. 1. Sieć rozmieszczenia punktów sieci pomiarowo-kontrolnej

THE NETWORK STRUCTURE IN THE ASPECT OF ENTROPY

In the beginning of the research 252 length changes were used to be calculated. These length changes were defined by periodic measurement results carried out during the following years: 2002, 2003 and 2004. The obtained length differences resulted from two measuring periods 2002-2003 and 2002-2004 were adjusted on the assumption of the minimal limitation of the degrees of freedom and constancy of the consecutive points and accurate directions which were observed. These adjustments were carried out to get the minimal value of the trace of variance-covariance matrix.

The minimal value of the trace of variance-covariance matrix was claimed to be the base for the further research, the final network adjustment within the minimal limitation of the degrees of freedom, was carried out on the assumption of constancy of the point 23 and the direction 23-6. As a result of this adjustment, the following values of mean errors of a singular observation were obtained: $m_{0(02\text{-}03)}$ = 0,59 *mm*, $m_{0(02\text{-}04)}$ = 0,61 *mm*. The observations adjustment done by the least square method was to be applied on the assumption that the observation errors were in the inconformity with a normal distribution. This was to be proved by applying a test of the compatibility of corrections vector distribution $V = (v_1, v_2, ..., v_m)$ to a normal distribution. The statistics χ^2 was used in the normality of distribution research assuming that the classification of random variables was done on the basis of an optional selection of the classifying cell length (the length of class range). This selection enabled to get the information maximum in the cell according to the following formula [Brillouin 1969]:

$$s = \left(t + \ln \frac{tT}{s(s-1)} \right) \tag{6}$$

where: T – range of examined feature,
t – length of class range,
s – value of a classified variable.

The value $k \approx t / s$ was a solution to this equation meant as an optimal number proportion of the length of class range to the classified variables value. As a result of this, the set of testing data was divided into 63 class ranges. The calculated values of statistics χ^2 for both periods, on the assumption of confidence level p=0,95 equalled : $\chi^2_{(02-03)} = 64{,}82$; $\chi^2_{(02-04)} = 71{,}99$, while the quantile of a distribution equalled: $\chi^2 = 79{,}05$. As a consequence, the tested theory on compatibility of the distribution of coefficient of correction vector V with a normal distribution was not rejected for both periods. The sum of information content $[\Delta h]$ of an observational system and the entropy of parameters vector $H_{\hat{X}}(X)$ were calculated later in the research. The following results were obtained: $[\Delta h] = 52{,}75$, $H_{\hat{X}}(X) = 40{,}20$ bit. The information content in the individual observations was different and it depended on the assumptions which eliminated the external network defect but the sum of information content of the whole observational system was constant while the minimal limitations of the degrees of freedom were chosen optionally. The information content, in each individual 252 observations with a selecting procedure taken into account, was presented in the Figure 2.

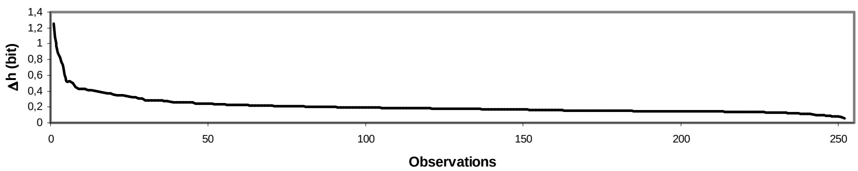

Fig. 2. The chart of information content in the individual observations
Rys. 2. Wykres zawartości informacji w poszczególnych obserwacjach

The optimal network structure was determined on the basis of the calculated results of entropy of parameters vector *X* and the trace of variance-covariance matrix trace $[A^TA]$ with the increasing number of eliminated observations. The changes of entropy of parameters vector *X* and the trace of variance-covariance matrix and the sum of information content in the process of elimination of the successive observations which contain the smallest information content for both periods, were presented in the Figure 3.

The calculated results of entropy changes $H_{\hat{X}}(X)$ of parameters vector *X* and the value changes of the trace of variance-covariance matrix were approximated by a quadratic polynomial in order to evaluate their minimal values. These minimal values were corresponding with the optimal network structure according to the taken assumption. The compatibility of matching a polynomial with the obtained data was characterised on the basis of a curvilinear correlation coefficient R^2. This coefficient was forming on the level of $R^2 = 0{,}98$ for the entropy changes but for the changes of traces of variance-covariance matrix, the described coefficient equalled appropriately for the individual periods $R^2_{(02-03)} = 0{,}97$ and $R^2_{(02-04)} = 0{,}95$. As a result of the analysis 35 observations were not classified. As a consequence, the sum of information content $[\Delta h] = 54{,}65\,bit$ and the entropy of parameters vector $H_{\hat{X}}(X) = 39{,}81\,bit$ were determined for the observational system which consisted of 217 observations.

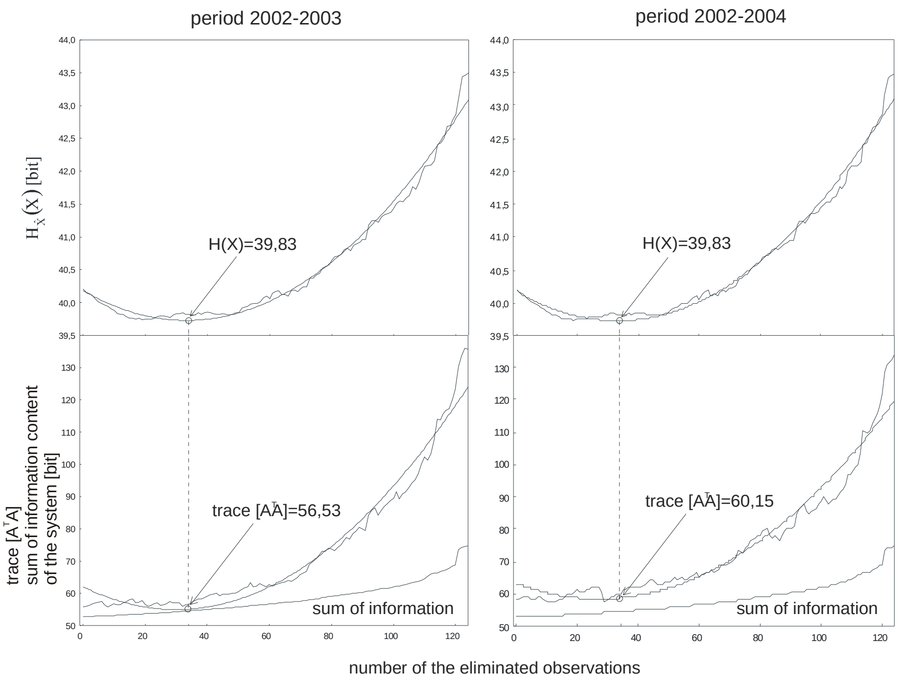

Fig. 3. The charts of entropy changes and the traces of variance-covariance matrix depending on the number of supernumerary observations

Rys. 3. Wykresy zmian entropii I śladów macierzy wariacyjno-kowariacyjnej w zależności od liczby obserwacji nadliczbowych

The reduced number of observations was again adjusted with the minimal limitation of the degrees of freedom used earlier and as a result of this the following characteristics of accuracy were obtained: $m_{0(02-03)} = 0{,}57$ mm and $m_{0(02-04)} = 0{,}59$ mm. It is worth pointing out that the limitation of an observation which contained the relative diminutive information content had an influence on the rise of information content of the whole observational system, which could be proven by the smaller value of entropy of parameters vector, furthermore, the change of the characteristics of the individual observation accuracy, obtained from the adjustment, was not observed either.

Entropy could be also defined as an average velocity of information which in the case of a geodetic observational system meant the entropy decrease which was to fell on the given supernumerary observation. Therefore, the determination of the system for the assumed confidence level α_0 and significancy level β_0 was to be realized on the condition that the entropy rise $\Delta H_{\hat{X}}[X]$ calculated from the formula:

$$-\alpha_0 \, log \, \alpha_0 - \beta_0 \, log \, \beta_0 \leq \Delta H_{\hat{X}}[X] \tag{7}$$

was to take the value:

– for $\alpha_0 = 0{,}90 \quad (\beta_0 = 0{,}10) \quad \Delta H_{\hat{X}}[X] \leq 0{,}4690\ [bit]$,
– for $\alpha_0 = 0{,}95 \quad (\beta_0 = 0{,}05) \quad \Delta H_{\hat{X}}[X] \leq 0{,}2864\ [bit]$,
– for $\alpha_0 = 0{,}99 \quad (\beta_0 = 0{,}01) \quad \Delta H_{\hat{X}}[X] \leq 0{,}0808\ [bit]$,
– for $\alpha_0 = 0{,}999 \quad (\beta_0 = 0{,}001) \quad \Delta H_{\hat{X}}[X] \leq 0{,}0114\ [bit]$.

While analyzing a testing network, which consisted of 217 observations, the number ‘n’of measurements was as followed:

– for $\alpha_0 = 0{,}95 \quad (\beta_0 = 0{,}05) \quad \Delta H_{\hat{X}}[X] \leq 0{,}2864\ [bit]$, n=138
– for $\alpha_0 = 0{,}99 \quad (\beta_0 = 0{,}01) \quad \Delta H_{\hat{X}}[X] \leq 0{,}0808\ [bit]$, n=195
– for $\alpha_0 = 0{,}999 \quad (\beta_0 = 0{,}001) \quad \Delta H_{\hat{X}}[X] \leq 0{,}0114\ [bit]$, n=217

THE ALGORITHM OF DEFINING OF A REFERENCE SYSTEM

In order to define a reference system the next network adjustment was done using the full collected data and Moore-Penrose’s inverse was applied, for the sake of non-random character of a displacement, to set the state of displacements of all the network points with the defined approximation. The geodetic system of observational equations having a defined redundancy was to take the following form:

$$\boldsymbol{AX} = \boldsymbol{L} \tag{8}$$

where: $\boldsymbol{A} \in \boldsymbol{R}^{\boldsymbol{m,n}}$ – matrix of coefficients of the system of equations correction,
$\boldsymbol{L} \in \boldsymbol{R}^{\boldsymbol{m}}$ – vector of absolute terms,
$\boldsymbol{X} \in \boldsymbol{R}^{\boldsymbol{n}}$ – vector of unknowns.

The decomposition SVD of matrix *A* to the singular values was used to solve a system (8) in the following form: [Kielbasinski, Schwetlick 1994]:

$$\boldsymbol{A} = \boldsymbol{U}\Sigma\boldsymbol{V}^{\boldsymbol{T}} \tag{9}$$

where: $\boldsymbol{U} \in \boldsymbol{R}^{\boldsymbol{m,m}}$ – matrix of the column vectors,
$\boldsymbol{V} \in \boldsymbol{R}^{\boldsymbol{n,n}}$ – matrixes of ortonormal vectors of eigen matrix $\boldsymbol{A}^{\boldsymbol{T}}\boldsymbol{A}$ corresponding to eigenvalues,
$\Sigma \in \boldsymbol{R}^{\boldsymbol{m,n}}$ – matrix of eigenvalues.

The shortest solution $\hat{\boldsymbol{X}}$ of a system $\boldsymbol{AX} = \boldsymbol{L}$ was being looked for among the whole solutions $\boldsymbol{X} \in \boldsymbol{R}^{\boldsymbol{n}}$ and the following dependence was applied:

$$\hat{\boldsymbol{X}} = \boldsymbol{A}^{+}\boldsymbol{L} \tag{10}$$

where:

$\boldsymbol{A}^{+} = \boldsymbol{V}\Sigma\boldsymbol{U}^{\boldsymbol{T}} \in \boldsymbol{R}^{\boldsymbol{n,m}}$ – pseudo inverse of matrix $\boldsymbol{A}$ (Moore-Penrose’s inverse)

The sorted lengths of apparent displacement vectors were presented in the Figure 4.

Fig. 4. The sorted values of apparent displacements
Rys. 4. Uszeregowanie wartości przemieszczeń pozornych

The designation of a reference system for the individual periods was carried out on the basis of the square increment of a norm of corrections vector V taken from the successive adjustments with the assumption of constancy of the rising number of network points. The succession of fixed points was done according to its rising value of the apparent displacement. The criterion, of an identification of a set of reference points, was such an increment of a square norm of corrections vector, which did not overflow the critical value ΔE_k. This critical value was to take the following formula [Gil 1995]:

$$\Delta E_k = -3\left(m^2 + \frac{m^2}{2r}\right) ln\left(1 - 0{,}9973^{\frac{1}{k}}\right) \tag{11}$$

where: m – means an error of a typical observation,
r – number of supernumerary observations,
k – number of fixed points.

The results of the identification of the points of a reference system were shown in the Figure 5. In this given case a reference system meant as a set of points characterized by reciprocal constancy was defined with the probability p = 0,9973.

According to the taken actions the following points were classified into a set of reference points:

– the period 2002-2003: 8, 3, 26, 11, 30, 27, 1, 19, 24, 22,
– the period 2002-2004: 9, 8, 15, 22, 3, 4, 24, 1.

Therefore, the reference system in the intervals of time was defined on the following points: 1, 3, 8, 22 and 24. The model of a displacement of the geodetic network points, shown in the figure 6, was determined on the basis of a defined reference system. The displacements of network points were incorporating in the range 0,2-8,6 mm for the period 2002-2003 and 0,8-5,9 mm for the period 2002-2004. The 13 points were displaced in the period 2002-2003 and their displacement value overflowed 2,5 multiplicity of a mean error of the determination. In the period 2002-2004 the 19 points were also displaced.

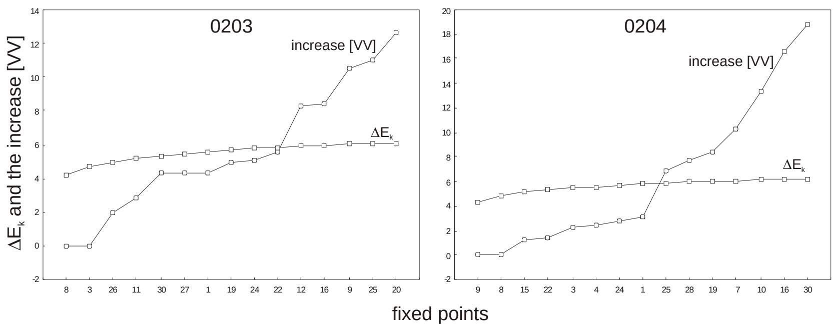

Fig. 5. Identification of the points of a reference system based on the critical value ΔE_k
Rys. 5. Identyfikacja punktów układu odniesienia na podstawie wartości krytycznej ΔE_k

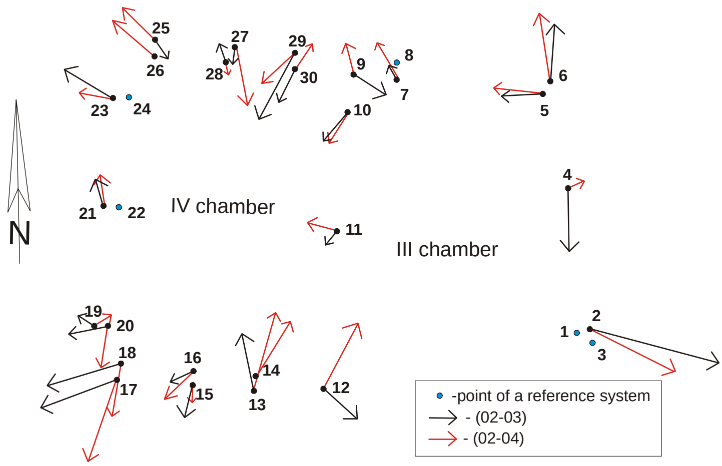

Fig. 6. Displacements of the measuring-controlling network points
Rys. 6. Przemieszczenia punktów sieci pomiarowo-kontrolnej

CONCLUSIONS

The issue of entropy in the displacement research is one of the variant of optimalization of the geodetic network structure. According to the assumptions, observations with the largest information content, are the most essential in the process of defining a displacement model. These observations have the largest influence on the entropy decrease of the system. The observations system, seen as information carriers, is the most advantageous when their mutual position in the measurement space does not show a large deflection from ortogonality. Such a system makes getting advantageous characteristics of the accuracy of estimated parameters possible and mineralizes the incurred costs in the technological process of a measuring-controlling network.

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WYKORZYSTANIE POJĘCIA ENTROPII W BADANIACH PRZEMIESZCZEŃ PUNKTÓW SIECI GEODEZYJNEJ POZIOMEJ

Streszczenie. Celem monitoringu obiektów prowadzonych metodą geodezyjną jest uzyskanie informacji o związkach przyczynowo-skutkowych zachodzących w obrębie badanego obiektu. Zastosowanie określonej techniki pomiarowej wymaga ustalenia korzystnej struktury sieci spełniającej warunek ekonomiki pomiaru przy jednoczesnym zapewnieniu odpowiedniej charakterystyki dokładności wyznaczanych parametrów. Korzystną strukturę sieci można uzyskać na podstawie doboru obserwacji o największej zawartości informacji. Wyniki pomiarów wybranych elementów sieci oraz zastosowanie odpowiednich metod ich przetwarzania pozwala na prawidłową identyfikację zbioru punktów odniesienia i sformułowanie poprawnego modelu przemieszczeń. W artykule został opisany proces ustalenia korzystnej struktury sieci liniowej na podstawie entropii układu obserwacyjnego.

Słowa kluczowe. sieć geodezyjna, entropia, informacja, układ odniesienia, przemieszczenia

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