A NEW STRESS CRITERION FOR HOT-TEARING EVALUATION IN SOLIDIFYING CASTING

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Abstract: This work concerns a new criterion for hot-tearing evaluation in castings. Algorithm describing the conduction of computer simulations of phenomena accompanying the casting formation, which performing is the preparation stage for using of this criterion, is also described. According to the low recurrence of phenomena occurring during solidification (e.g. grained structure parameters, stresses distribution) the casting's hot-tearing inclination can be estimated only in approximated manner. Because of still following at present rapid computer processors development, and techniques of its programming, enables to suppose that in short time the efficiency of computer simulations will arise so much, the problem of hot-tearing evaluation newly became interesting for the team working on computer simulations at the Institute for Computer and Information Sciences at Częstochowa University of Technology.

Key words: Casting, Computer Simulation, Hot Tearing, Solidification Processing

1. INTRODUCTION

The production of casts is a technology that involves many significant factors which have an impact on the end result, that is to say, on the quality of the cast.

In shape casting, an equiaxial structure is formed (Fig. 1). During solidification processes various types of defects may appear in the solid-liquid areas. These areas solidify as the last batches of material. The shrinkage leads to micro-porosity effect. Looking at it the other way, the stress effect reveals the commonly named hot tearing in the casting.

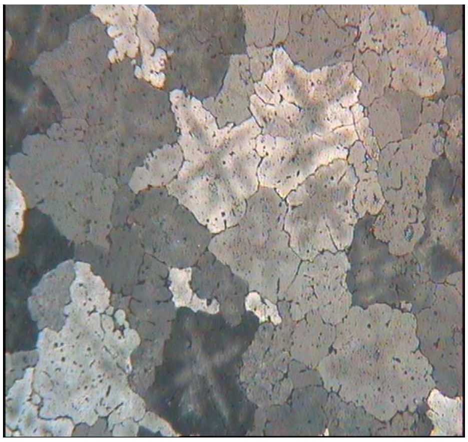

Fig. 1. Equiaxial grains in Al-2% Cu alloy

Hot tearing of solid-liquid areas occurs when the stresses acting on them are able to break the backbone of solid phase, filed with the liquid phase (Sczygiol and Szwarc, 2003b).

Founders and scientists were and still are interested in the problem of hot tearing of castings.

At the beginning, the problem of hot tearing formations was solved by experimental estimation of the hot tearing susceptibility of foundry alloy. Then, the mathematical models describing hot tearing have been developed. The review and analysis of this work can be found in paper Parkitny and Sczygiol (1987). Research on this field focused mostly on the formation of a single crack and were not relevant to industrial practice. The next step in the development of methods for testing susceptibility to hot tearing was the use of advanced numerical methods, through the computer simulation (Rappaz et al., 1999).

There are two groups of works which uses computer simulations. The first group concentrates on the analysis of a single crack development. The second group involves a comprehensive analysis of thermo-mechanical phenomena, accompanying the production process of castings.

On the basis of this analysis, the degree of risk of the appearance of defects in continuity in the entire casting or in its selected parts, is attempted to be drawn (Szwarc, 2003; Sczygiol and Szwarc, 2005). The use of such approach is also possible while performing simulations with use of commercial engineering programs. Usually, such programs do not provide any criteria for hot tearing evaluation in castings. Users of such kind of software have to choose which of the available values characterizing the state of stress and/or deformation should be used to the rupture-susceptibility assessment. It should be mentioned, that such an analysis requires good knowledge of the phenomena in casting formation and skills in simulation of these phenomena. Specialized engineering software, usually based on a finite element method is also required. In turn, simulations performed with use of such software are very time consuming.

This paper concerns research on the analysis of the susceptibility to hot tearing during an equiaxial structure casting. A new stress criterion to evaluate the level of risk of rupture in selected fragments of the casting is proposed. According to the algorithm described in this paper, the assessment of castings hot tearing susceptibility with the use of this criterion is possible only after conducting a series of simulation calculations. The information about the degree of rupture risk in selected areas of the casting is obtained as a result of such evaluation. Studying the susceptibility to hot tearing by using the method proposed here is time-consuming. However, continuous development of computational processors, such as GPUs, as well as effective methods of programming such processors, let us believe that accelerations reached nowadays (Michalski and Sczygiol, 2010; Michalski, 2011) give us ability to use proposed solutions in foundry practice.

2. THE CRITERION FOR EVALUATION OF SUSCEPTIBILITY TO HOT TEARING

Metal alloys often solidify by increasing equiaxial dendrites. It can be assumed that initially each dendrite grows individually. As the dendrites are in contact with each other they form the backbone of the solid phase. Dendrite arms are intertwined with the arms of their neighbors. From this moment, in the solidifying solid-liquid area appears tension. It is carried by each entangled dendrite arm. Dendrites are separated by layers largely filled with the liquid phase.

The two-phase region described in this way consists of growing equiaxial grains and layers of the liquid phase which separate these grains. Such area in the numerical modeling is represented by hexagonal solid phase grain and the surrounding layer of the liquid phase. These hexagonal grains can be also divided into smaller hexagons. The solid phase is presented using regular hexagons and the liquid phase using flattened. The corresponding parts of these two types of hexagons are placed on the border area, see Fig. 1. The size of both areas (solid and liquid phases) is characterized by participation of the solid phase, calculated at the stage of solidification simulation.

This way of modeling used for area of solidifying cast enables operating at the micro level of analysis separately for growing grains and for narrowing layers of the liquid phase. Finite elements used in calculations and in the macroscopic stress analysis are almost always much bigger than solidifying metal grains. In the macroscopic analysis, two-phase area is treated as isotropic. The grain nature of the casting construction is ignored. Nevertheless, because the solidification simulation is conducted on the basis of the coupled model, i.e. macro-microscopic, so after the simulation of solidification the accumulation of grains in two-phase areas can be easily reconstructed. In combination with the analysis of stress at a microscopic level, it enables to analyze the phenomena leading to hot tearing.

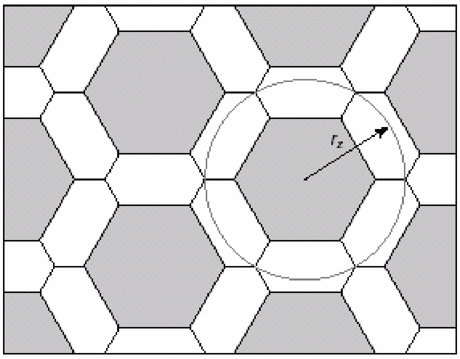

Fig. 2. Model of two-phase area for the alloy solidifying in the form of equiaxial grains

Different temperature gradients and the resistance posed by the wall of the mold to the shrinking casting are the most significant causes of stress in the cast. Conditions of heat evacuation from the casting to the mold and to the environment determine the speed of the alloy solidification, i.e. the equiaxial grains growth speed, but also the speed of the stress generated in the casting.

A new stress criterion is proposed to assess the hot tearing susceptibility of solidifying cast. The criterion takes into account the stress-speed ratio of effective stress in the layers separating the congealed particles to the speed of effective strain in these grains. The proposed criterion is expressed by the local coefficient of susceptibility to hot tearing and it is marked as Θ. This criterion assumed, that stress states are considered in micro scale, but these conditions are obtained under the stress states in a macro scale. During the solidification, the changes in geometry (size) of grains and separating layers is obtained from microscopic analysis conducted on the basis of macroscopic modeling. This is possible because in the macroscopic modeling the growth of equiaxial grains is represented by the connection of diffusion phenomena (micro scale) with thermal phenomena (macro scale).

The calculation of the local coefficient of susceptibility to hot tearing proceeds in the following time steps, beginning with the participation of the solid phase, in which the backbone of solid phase is formed, until complete solidification.

Effective strain rate can be written as:

$$\dot{\bar{\sigma}} = \frac{|\Delta\bar{\sigma}|}{\Delta t} \qquad (1)$$

where: $\Delta\bar{\sigma}$ is the effective stress increment in the time step Δt Δt.

Nevertheless, conducted research show, that much better results are obtained if the relative effective stress is introduced to the criterion. Consequently, the criterion can be described as follows:

$$\Theta = \frac{|\Delta\bar{\sigma}_l|}{|\Delta\bar{\sigma}_g|}\frac{\bar{\sigma}_g}{\bar{\sigma}_l} \qquad (2)$$

where: l is a sub-layer separation, while g denotes the sub-grain-solidified parts. Because the quotient of relative increment of effective stress in the layers and the grains tends to zero with increasing share of the solid phase, in the sake of clarity, the hot tearing criterion can be transformed to the following form:

$$\Theta = -\ln\left(\frac{|\Delta\bar{\sigma}_l|}{|\Delta\bar{\sigma}_g|}\frac{\bar{\sigma}_g}{\bar{\sigma}_l}\right) \qquad (3)$$

Application of the criterion (3) requires a computer simulation in the macro scale, and afterwards in the micro scale. At the macro level standard macroscopic finite elements are used. Conversely, at the micro level – microscopic elements are used. These microscopic elements cover the macroscopic element area. The finite element method, for both types of simulation, is formulated in slightly different way. A traditional formulation e.g., based on the method of weighted residuals is used at the macro level. At the micro level was used a hybrid formulation (Ghosh and Moorthy, 1995; Parkitny et al., 2001).

The equation (3) describes the local susceptibility to hot tearing of a small macroscopic area, corresponding to one finite macroscopic element, subdivided into two areas, i.e. grains and layers separating them. Stress values and their increments used in equation (3) are determined for the subdivisions of layers and grains, receiving two tensors which describe the resultant state of stress in all the grains and the resultant state of stress in all the layers of separation, which belong to the analyzed area. Tensors are obtained as a result of the so-called homogenization, based on the integration of the stress function in the above-mentioned subdivision, and then dividing the resulting value by the area of integrated subdivision Θ.

High susceptibility to hot tearing is indicated by large values of factor Θ. Nevertheless, the criterion Θ does not indicate a specific limit value, above which the casting will crack. This follows from the fact that, that the value of Θ increases with increasing equiaxial grain, as a result of stress growing with an increasing solidification area. Θ factor values are used to indicate the areas of analyzed casting, where most likely appears a damage, i.e. the rupture. Θ criterion can also be used to determine the conditions most conducive to the production of a given type of cast.

2.1. Description of algorithm used for preparatory calculations

Computation of factor Θ is possible after a complex computer simulations. Data provided by these simulations are used to estimate the susceptibility to hot tearing of the cast. A number of preparatory tasks must be performed at this stage. The steps leading to determining the casting susceptibility to hot tearing cover the following:

1. Simulation of solidification. For succeeding time steps the temperature field, the distribution of the solid phase participation and the mean radii of equiaxial grains, are determined.
2. Calculating distributions of stress in consecutive time steps.
3. Identification (selection) of subdivided areas for the hot tearing analysis.
4. Division of the macroscopic finite elements into the microscopic-hexagon-hybrid-finite elements in order to obtain the solid-liquid areas. These solid-liquid areas are part of subdivisions which are to be analyzed if terms of susceptibility to hot tearing
5. Calculation of the stress in all solid-liquid areas corresponding to the macroscopic finite elements,
6. Calculation of the value of coefficient Θ for each macroscopic finite element in the selected areas
7. Preparation of the scale of susceptibility to hot-tearing based on the simulations and calculations carried out for all the analyzed variants of the task.
8. Execution of a local distribution coefficient diagrams for susceptibility to hot tearing for different variants of the task.
9. Drawing conclusions.

Since the first two steps are standard and well described in literature (Desbiolles et al., 1987; Sczygiol, 2000), only the remaining steps will be shortly described below.

2.2. Identification of subdivisions which are to be analyzed

It makes no sense to carry out the analysis of the susceptibility to hot tearing for all the macroscopic finite elements of the casting, because experience (the practice) shows that, the cracks appear only in selected, fairly easy to identify parts of the cast. The identification of such fragments requires selecting a group of finite elements and the area around them. This selection should be done on the basis of the probability of the hot tearing localization. The group of selected macroscopic elements should be slightly greater than the area of the analysis.

The second group of macroscopic elements (of a similar number of the elements) selected for analysis should be chosen in the area least subjected to hot tearing. If there is a suspicion about the possibility of appearance of hot tearing in other parts of the casting, then another group of elements should be created and analyzed.

Fig. 3 shows the cast and three groups of elements selected for analysis. The main group is placed in the central part of the cast and consists of elements collected under infusion and forming a notch around the bottom of the casting. This group is most at high risk of hot tearing. The second group is located on the left, in the casting arm. The risk of occurring hot tearing in this area is very low. Rupture should not occur either, by design, in the third group, comprising the area around a 'notch' connecting the right shoulder with the casting 'head' located at its end.

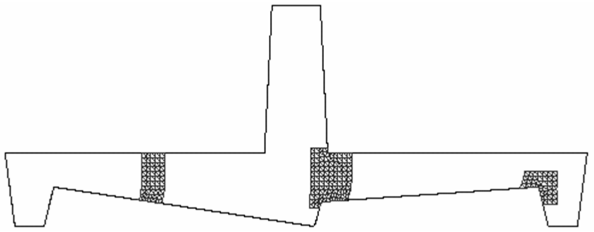

Fig. 3. Location of selected groups of macroscopic elements for the analysis of hot tearing

Comparision of the sizes of areas selected for the analysis with the size of the entire area of the casting (Fig. 3) it can be easily remarked, that the number of elements selected for analysis is relatively small in comparison with the number of finite elements in the whole casting.

2.3. Division of the macroscopic finite elements into the microscopic – hybrid elements

Each of macroscopic finite elements, which belong to the group of the elements analyzed from the point of view of hot tearing susceptibility, is divided into hybrid, microscopic finite elements (Parkitny et al., 2002; Szwarc and Sczygiol, 2002). The mesh of hybrid finite element is generated on the basis of the characteristic dimension of the grains (grain radius). This value is obtained during the simulation of solidification. The mesh created in this way ca be also taken into account in further analysis of two areas of material properties: densely tangled dendrites (solid phase) and the layers separating them in a solid-liquid state (Sczygiol and Szwarc, 2003a).

The surface area of a macroscopic finite element determines the number of microscopic finite elements that belongs to the given macro element. Regardless of the original shape of this element, the hybrid elements mesh is always built on a rectangular plan (similar to a square) with an area equal to or close to the macro element area.

Such an approximation of projecting a macroscopic element to microscopic elements is dictated by the polygonal shape of hybrid elements.

In the areas of separating layers there is not only the liquid phase, but also the solid phase in the form of dendrite arms. Therefore, the participation of the grains area in the region of the whole solid-liquid area can be written as: Szwarc and Sczygiol, (2004):

$$q = \frac{A_g}{A} = \frac{f_s(1-u)}{1-uf_s} \quad \text{where } u \in \langle 0,1) \tag{4}$$

where: u is the part of the solid phase share in the separating layers, while f_s is the share of the solid phase. The course of the function q, depending on the participation of the solid phase, including a displacement of half participation of the solid phase to the area of layers ($u = 0.5$) and assuming that all of the solid phase is in the grain area ($u = 0$), is shown in Fig. 4.

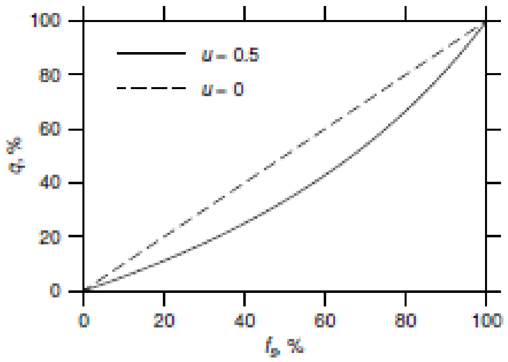

Fig. 4. The course of the function q for different u values

$$x = x_c - \sqrt{\frac{q}{q'}}(x_c - x') \tag{5}$$

where: x is the coordinate of the node, x_c is the coordinate of the so-called measure of the solid phase increase, while the symbol' denotes the current location of the node and the output share of the grain area.

2.4. Stress calculation in microscopic areas

As a result of the macroscopic calculation a number of instantaneous fields is obtained. These are: the temperature profile, liquid phase participation, stress, strain and deformation. The temperature of the end of solidification, the characteristic grain's size and the stress field are relevant for further simulations.

In the selected area the macroscopic finite elements are isolated from the rest of the elements mesh of the casting. The parameters describing the state of the macro elements are used as input for further calculations leading to the determination of the susceptibility to hot tearing. The dimension of the hybrid finite elements is determined on the basis of the equiaxial grain radius assigned to the macro element. The growth of the grains area in successive time steps is controlled with use of the solid phase participation function $f_S(t)$. The temperature profile $T(t)$ is used to control the change in material properties.

The stress tensor $\sigma(t)$ constitute the basis for the formulation of the appropriate boundary conditions, see Fig. 5.

Because of there is no symmetry in loading the system, the stress tensor is converted into an equivalent tensor of main stresses. As the result of this approach it is possible to analyze only a quarter of the system, suitably mounted on symmetry axes and charged by the main stress.

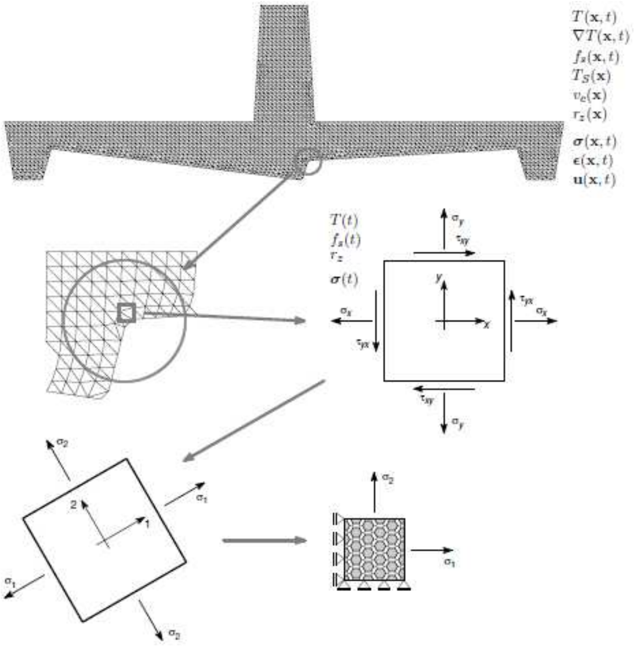

Fig. 5. The division of macroscopic finite element into hybrid microscopic finite elements and the way of support and load of the analyzed area

For the purpose of numerical modeling of the solid-liquid center cracking it is necessary to separate the macroscopic properties as the properties of the two subdivisions. These properties are determined in an experimental way. The participation of the area surface q in the whole solid-liquid area determines the "amount" of the subdivision. Therefore, the value of material property W can be described as:

$$W = qW_g + (1-q)W \tag{6}$$

where: W_g is the value of material properties for the solid phase area (grains), while W_l is the value of material properties for the area of separating layers. Furthermore, it has been assumed that the material properties in the subdivisions are in a relationship expressed as:

$$\frac{W_l}{W_g} = p \quad \text{where} \quad p \in (0,1\rangle \tag{7}$$

Equation (7) describes the distribution of material properties related from temperature ($p = p(T)$). Substituting (7) to (6) we obtain the relationship describing material properties for the solid phase:

$$W_g = \frac{W}{p + (1-p)q} \tag{8}$$

One of the possibilities of determining function p consists in making it relevant of the range of the solidification temperatures:

$$p(T) = \frac{T_L - T}{T_L - T_S} \tag{9}$$

where: T_L and T_S are the liquidus and the solidus temperatures, respectively. Function of participation of solid phase (Sczygiol, 2000) can be also used as the function of distribution of material properties.

The analyzed solid-liquid area is covered by the microscopic finite element mesh. This mesh is charged by the macroscopic state of stress. The boundary conditions are updated on grains arising from the simulation at the macro level.

On the basis of the current temperature values material properties of sub-grains and separating layers are determined. The calculations are carried out from the 'appearance' of stress, i.e., when the share of the solid phase exceeds a critical value (e.g. 25%) until complete solidification.

2.5. Calculation of the value of Θ for the macroscopic finite element

The equation (3) allows for calculation of the values of the local coefficient of susceptibility to hot-tearing Θ. Since different areas of the casting solidify at different time intervals it is convenient, due to further analysis, to present the course of Θ in the function of the solid phase share. Fig. 6 presents sample graph representing such a course.

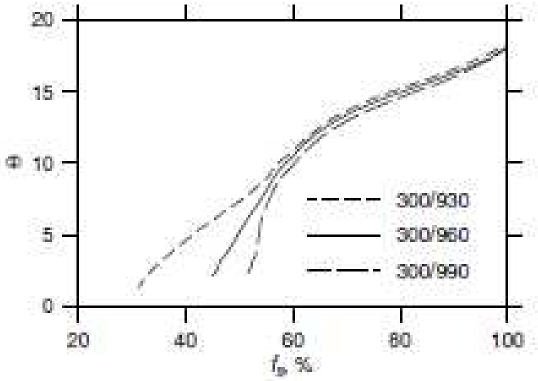

Fig. 6. Sample courses of Θ for given casting conditions (temperature of the mold/casting temperature)

Presentation of results in the function of the solid phase share enables a direct comparison of the coefficient value of all the solved task variants.

2.6. Drawing up the scale of Θ

In order to compare values Θ for different tasks, different conditions for pouring and solidification have been drawn up the scale of susceptibility to hot tearing, based on the critical value Θ_{cr}. It was assumed that the scale is dependent on the participation of the solid phase. The critical value Θ_{cr} is determined from the maximum values Θ for all the variants of the simulation for the solid phase participation, ranging from 50 to 95%, in steps of 5%. On the basis of the received values the function determining critical values of Θ in the function of the solid phase may be constructed. This function is the basis for determining the degrees of the susceptibility to hot tearing.

Thus one should decide whether further analysis of susceptibility to hot tearing will run for four degrees. As high (the highest) degree adopted values Θ larger and equal to Θ_{cr}, as the average – values from $0.9\Theta_{cr}$ to Θ_{cr}, as low degree – values from $0.8\Theta_{cr}$ to $0.9\Theta_{cr}$. For values Θ below $0.8\Theta_{cr}$ the lack of susceptibility to hot tearing is accepted.

2.7. Execution of diagram Θ distribution

Proposed in the previous section, the scale is the basis for drawing up diagrams (maps) of the coefficient Θ distribution for the main group of elements and for the control groups (Fig. 3). The maps are drawn up for certain selected values of the solid phase participation (Fig. 7).

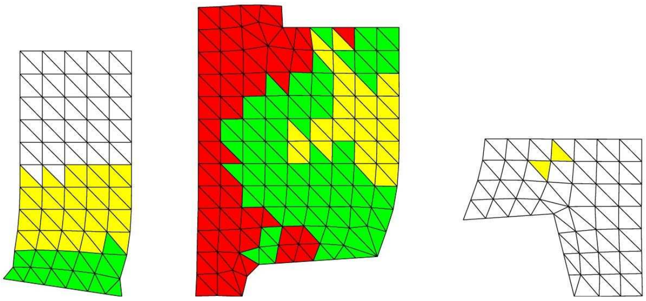

Fig. 7. Sample map of the coefficient Θ distribution of susceptibility to hot-tearing (a darker color means a greater susceptibility to hot tearing)

Since the coefficient Θ distribution maps are only comparative, there are compared elements with the same share of the solid phase in a single casting. So they do not represent any real situation, i.e. those which may occur in the solidifying casting. Such maps are made to indicate that while the main group values Θ indicate the possibility of hot tearing, in the control groups the coefficient values Θ are so small, that they are not at risk from cracking.

2.8. Conclusions from the simulations

After preparing maps of the coefficient Θ distribution some relevant, for the casting practice, conclusions arise. These conclusions may involve the casting hot tearing at different stages of solidification. What is also important, is the evaluation of infusion conditions, which determine the temperature of the mold or flooding temperature, to ensure obtaining sound castings.

3. THE CRITERION FOR EVALUATION OF SUSCEPTIBILITY TO HOT TEARING

Application of the proposed criterion for the hot tearing evaluation has been illustrated by the simulations and analysis of the casting made of Al-2% Cu alloy, solidifying in a metal form. A shape of the metal form and cast are presented in Fig. 8.

The material properties are taken from the work Sczygiol, 2000. Three groups of the macroscopic finite elements (indicated in Fig. 3) were selected for analysis of hot tearing susceptibility. Three series of computer simulations of solidifications were performed. For all simulations, the initial mold temperature was set to 300 K.

The variable parameter was the pouring temperature, that was equal to: 930 K, 960 K and 990 K, respectively.

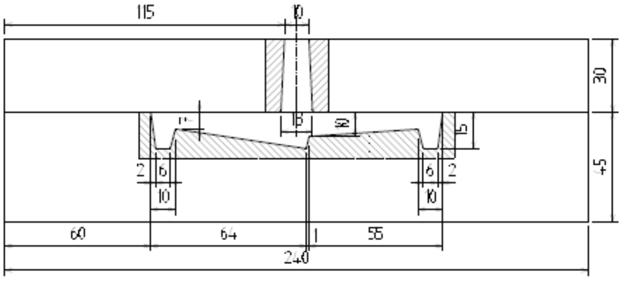

Fig. 8. The analyzed cast in the casting mold

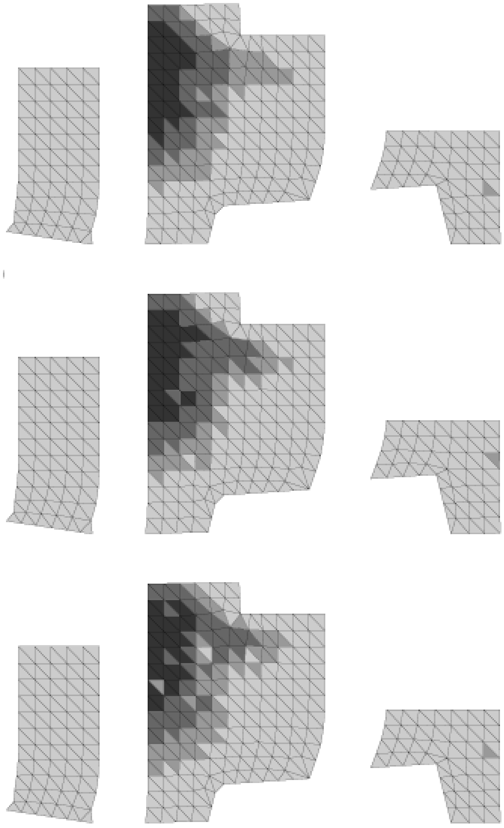

Fig. 9. The distribution of Θ for the solid phase share of 60% in each macroscopic finite element

Distributions of the local coefficient of susceptibility to hot tearing for the major group and control groups are presented in Fig. 9, 10, 11. The upper distributions were made for the pouring temperature 930 K, the middle – for 960 K and the lower distributions for 990 K.

The analysis shows that in all the cases, there is a high risk of the rupture of hot casting. It is therefore concluded that the initial mold temperature is too low. The obtained results were confirmed by experimental research. The hot tearing occurred for an initial mold temperature of 300 K (Fig. 12), while raising the temperature to 600 K guaranteed to receive a sound casting.

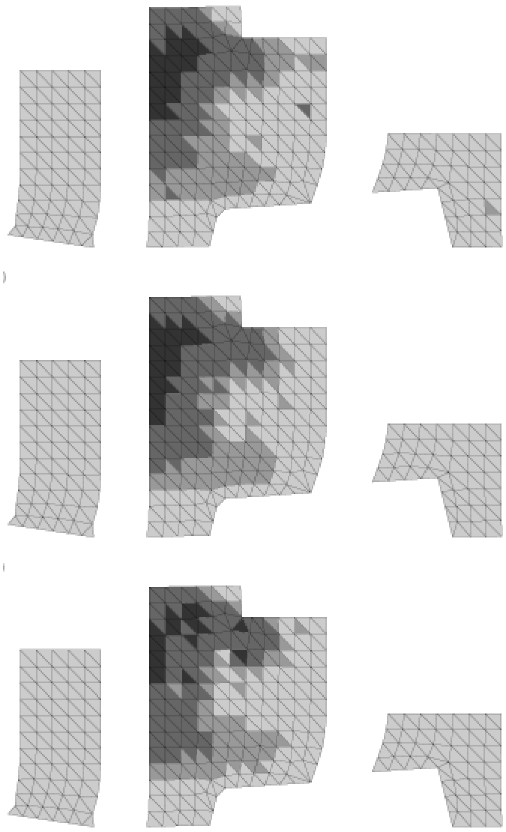

Fig. 10. The distribution of Θ for the solid phase share of 75% in each macroscopic finite element

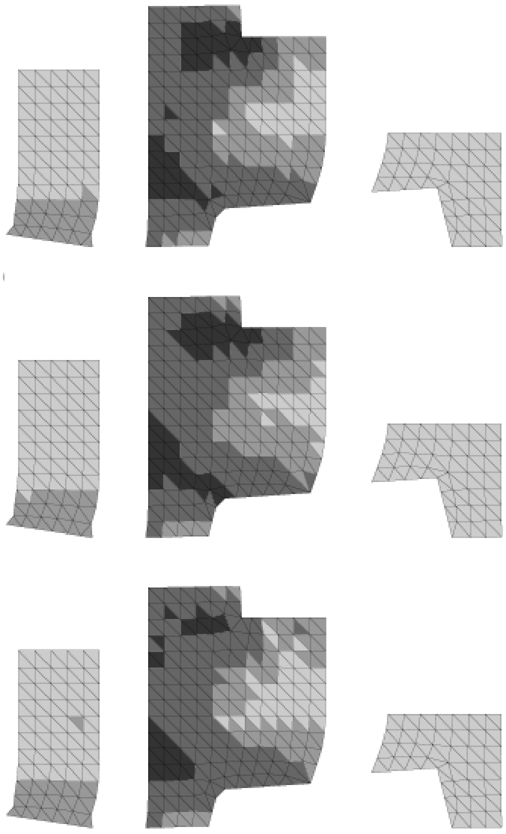

Fig. 11. The distribution of Θ for the solid phase share of 95% in each macroscopic finite element

Fig. 12. Hot tearing in Al-2% Cu alloy

4. SUMMARY

The new stress criterion, proposed in this paper, is a local criterion used for the evaluation of hot tearing susceptibility in casts. It covers the area of a single macroscopic finite element.

A global evaluation of the casting susceptibility to hot tearing is possible with application of this criterion for compact groups of finite elements, in selected casting areas. The analysis can be carried out jointly for several ranges of the initial and the boundary conditions. As a result of computer simulations and the analysis of the susceptibility to hot tearing, the most advantageous (from the point of view of the rupture risk) variant of the casting can be chosen.

Application of proposed criterion requires a lot of preparatory work and computer simulations. However, due to the increasing computing performance of new processors, the use proposed solutions in foundry practice becomes more real.

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