ANALITICAL METHOD OF PID CONTROLLER TUNING FOR A CLASS OF UNSTABLE PLANT

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Abstract: The aim of the paper is to present the synthesis method of classic PID controller for a class of unstable plant. The proposed method based directly on Skogestad paper, where analitical synthesis of PID controllers in described. This paper is generalization of that approach on a class of unstable plants with delay. Analitical method for synthesis of fractional controllers is given. The considerations are illustrated By numerical example and results of computer simulation with MATLAB/Simulink.

Key words: PID Controller, Skogestad, Stability, Delay

1. INTRODUCTION

PID controllers are so far widely used in practice, because of well known simple structure. Many methods of tuning PID controllers for satisfactory behavior have been proposed in the literature (Johnson andMoradi, 2005). The methods are based on knowledge of mathematical descriptionof process (O'Dwyer, 2003).

This paper is generalization of Skogestad method for a class of unstable plant. The starting point has been the IMC PID tuning rules of Rivera (1986). Furthermore Skogestad start by approximating the process by afirst-order plus delay processes. He proposed analytic tuning rules, simply but still result in a good closed-loop behavior.

2. METHOD

Consider the feedback control system show in Fig.1:

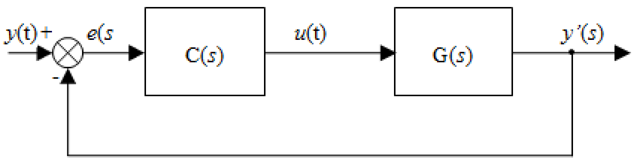

Fig. 1. Feedback control system structure

The process to be controlled is described by (1):

$$G(s)=\frac{k}{1+T_1 s \quad 1+T_2 s}e^{-sh} \tag{1}$$

where: $T_1 \geq 0$, $T_2 \geq 0$, $h \geq 0$, and $C(s)$ – transfer function of the controller.

The method proposed by Skogestad is based on approximating the process by a first- or second-order plus delay model, where: k – plant gain, τ_1 – dominant time constant, h –effective time delay.

Skogestad method is based directly on analytical set of the controller parameters (Skogestad, 2001):

$$C(s)=K_c\frac{(1+\tau_I s)}{\tau_I s}(1+\tau_D s) \tag{2}$$

with fallowing rules:

– Controller gain K_c:

$$K_c=\frac{1}{k}\frac{T_1}{\tau_C+h} \tag{3}$$

– Time constansτ_I:

$$\tau_I=\min\{T_1,4(\tau_C+h)\} \tag{4}$$

– Time constansτ_D:

$$\tau_D=T_2 \tag{5}$$

In equation (3) parameter τ_c is recomended as follows:

$$\tau_C \geq h \tag{6}$$

In the paper the synthesis method of PID controller for an unstable plant described with transfer function (7) is proposed.

$$G(s)=\frac{k}{1 \quad T_{,1} s \quad 1+T_2 s}e^{\ sh}, \tag{7}$$

where$T_1 \geq 0$, $T_2 \geq 0$, $h \geq 0$.

To make generalization of Skogestad method for a class of unstable plant (7), the controller gain $K_c$is described with:

$$K_c=\frac{1}{k}\frac{-T_1}{\tau_C+h} \tag{8}$$

Equations (4)-(6) take their form unchanged.

Compare proposed method with synthesis method of fractional order controller proposed in Nartowicz (2011).

In that paper transfer function of the controller follows directly from the use of Bode's ideal transfer function as a reference transfer function for the open loop system:

$$K(s)=\left(\frac{\omega_c}{s}\right)^{\beta} \tag{9}$$

To obtain an open loop system in the form (9) the controller transfer function should have a structure:

$$C(s)=-k_c\frac{s(1+sT)}{s^{\alpha}}=-k_c s^{1-\alpha}(1+sT) \tag{10}$$

where α is real number, and $\tau_2 = T$.

Open loop transfer function is given by:

$$K(s)=C(s)G(s)=\frac{kk_c}{\tau}\frac{e^{-sh}}{s^{\alpha}} \tag{11}$$

Knowing$(j\omega)^{\alpha}=|\omega|^{\alpha}e^{j\alpha\pi/2}$ gain and phase for a transfer function (7) was written:

$$|K(j\omega)|=\frac{kk_c}{\tau}\frac{1}{\omega^{\alpha}} \qquad \phi(\omega)=\arg K(j\omega)=-h\omega-\alpha\frac{\pi}{2} \tag{12}$$

For a gain ω_g and ω_p phase crossover frequency terms was given:

$$|K(j\omega_g)|=1 \qquad \phi(\omega_p)=\arg K(j\omega_p)=-\pi. \tag{13}$$

Using (12) the equations (13) was rewritten as:

$$\frac{kk_c}{\tau\omega_g^{\alpha}}=1 \qquad -h\omega_p-\alpha\frac{\pi}{2}=\pi \tag{14}$$

By solving equations (14) gain ω_g and ω_p phase crossover frequency:

$$\omega_g^{\alpha}=\frac{kk_c}{\tau} \qquad \omega_p=\frac{(2-\alpha)\frac{\pi}{2}}{h} \tag{15}$$

For a given $A_m$gain and ϕ_m phase margins:

$$\frac{kk_c}{\tau\omega_p^{\alpha}}=\frac{1}{A_m} \qquad \phi_m=\pi-h\omega_g-\alpha\frac{\pi}{2} \tag{16}$$

Transformed equation (16) were written in a form:

$$\omega_p=\left(\frac{A_m kk_c}{\tau}\right)^{1/\alpha} \qquad \omega_g=\frac{(2-\alpha)\frac{\pi}{2}-\phi_m}{h} \tag{17}$$

Using given equationsgain margin $A_m$was given by:

$$A_m=\left(\frac{(2-\alpha)\frac{\pi}{2}}{(2-\alpha)\frac{\pi}{2}\quad\phi_m}\right)^{\alpha} \tag{18}$$

Nonlinear equation (18) is handling$A_m$gainand ϕ_m phase margins and fractional order of the considerable controller (4)α.

Parameter α can be determined by solving equation (18) using computer methods.

By solving one of the first equations of (14) or (16):

$$k_c=\frac{\tau\omega_g^{\alpha}}{k}=\frac{\tau\omega_p^{\alpha}}{kA_m} \tag{19}$$

where:k – gain of the transfer function (1).

3. RESULTS

Example 1: $T_1 \gg h$

Consider the feedback control system shown in Fig. 1 in which the process to be controlled is described by transfer function:

$$G(s)=\frac{4}{(1-6s)(1+1.2s)}e^{-0.25s} \tag{20}$$

Determine controller parameters for a given transfer function (20) to stabilize the closed loop control system. In that case:$k=5, T_1=6, T_2=1.2; h=0.25.$

Using equations (4)-(6) and (8) transfer function of the controller is given:

$$c(s)=-3\frac{(1+2s)}{2s}(1+1.2s) \tag{21}$$

Fig. 2 shows step response for control system presented in Fig.1, where the process to be controlled is described by (20), controller (21), and fractional controller proposed using Bode transfer function (Nartowicz, 2011), where:

$C(s)=-2.56/s^{0.13385}-3.07s^{0.8661}$.

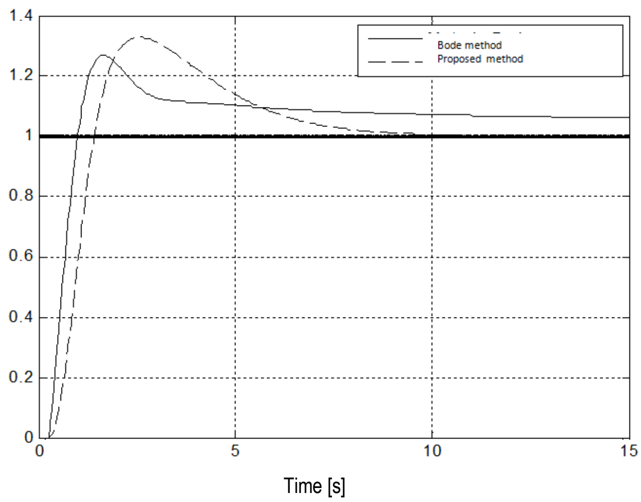

Fig. 2. Step response of the closed loop system with object (20) and controller (21), and fractional order controller synthesis with Bode method

Overshoot of step response drawn with method proposed in the paper is 10% bigger than drawn using Bode's method. Settling time is 8 sec. The time is much longer while Bode method using. Fig 3 shows step response for control system with controller (21), fractional order controller proposed with Bode method for a few values of parameter k (Fig.3a), and few values of delay h for a controlled process described with transfer function (20)

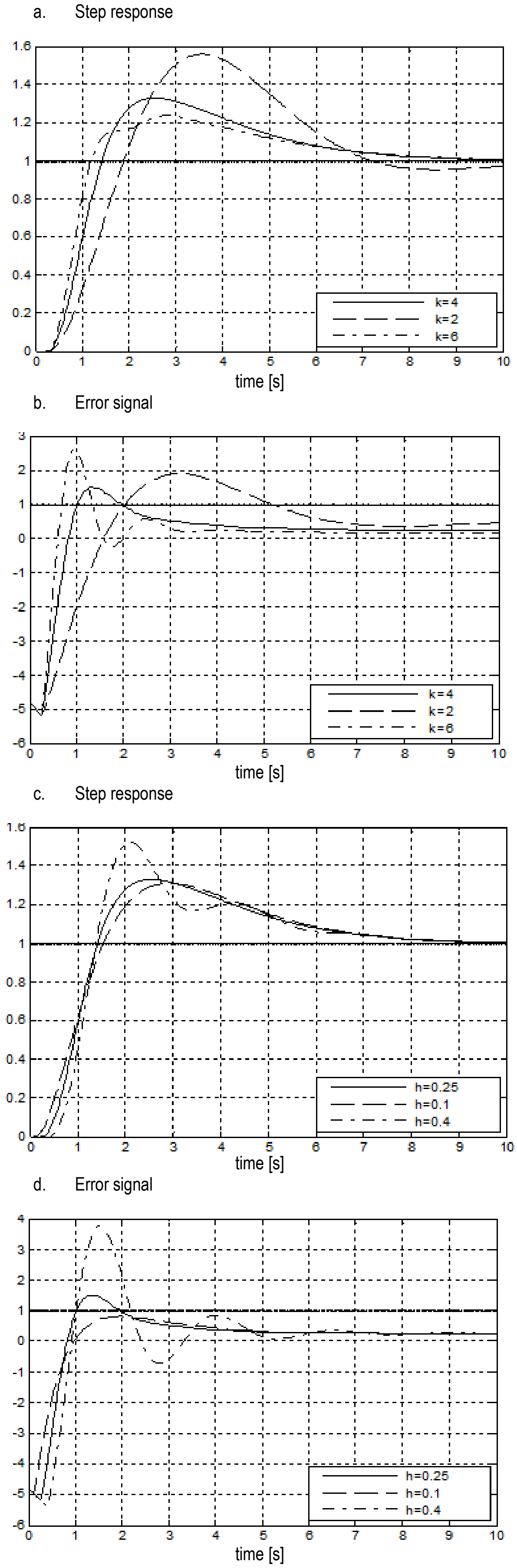

Fig. 3. Step response of the closed loop system with object (20) and controller (21) for a few different values of parameters *k* and *h*: a), b) step response; c), d) error signal

Fig 3b and 3d shows error signal of the controlled system. Overshoot for $k = 2$ is 55% bigger than for original value ($k = 4$), and moreover settling time is growing. There is no overshoot change while k is bigger, settling time also the same.

For a few values of time delay h settling time is const, but when the time is growing, the overshoot is also growing, for $h = 0.4$ even to 50%.

Example 2: $\mathbf{T_1 > h}$

Consider the feedback control system shown in Fig.1 in which the process to be controlled is described by transfer function:

$$G(s) = \frac{0.55}{(1\text{-}62s)(1+36s)} e^{\ 10s} \tag{22}$$

Determine controller parameters for a given transfer function (22) to stabilize the closed loop control system.

In that case: $k = 0.55, T_1 = 62, T_2 = 36; h = 10.$

Using equations (4)-(6) and (8) transfer function of the controller is given:

$$c(s) = \text{-}5.64 \frac{(1+62s)}{62s}(1+36s) \tag{23}$$

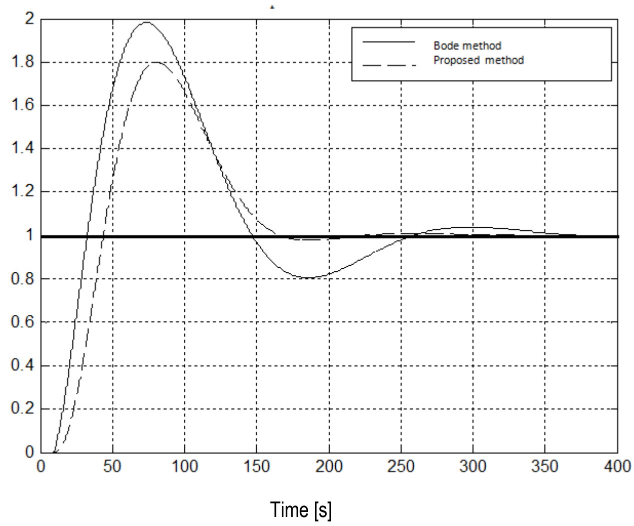

Fig. 4. Step response of the closed loop system with object (20) and controller (21), and fractional order controller syntheses with Bode method

Fig. 4 shows step response for control system presented in Fig. 1, where the process to be controlled is described by (22), controller (23), and fractional controller proposed using Bode transfer function:

$$C(s) = -2.9358/s^{0.13385} - 105.6888s^{0.8661}.$$

Overshoot of step response drawn with method proposed in the paper is 20% bigger than drawn using Bode method. Fig. 5 shows step response for control system with controller (23), fractional order controller proposed with Bode method for a few values of parameter k (Fig. 5a), and few values of delay h for a controlled process described with transfer function (22). Settling time is ab. 160 sec, while the time for Bode method is 250 sec.

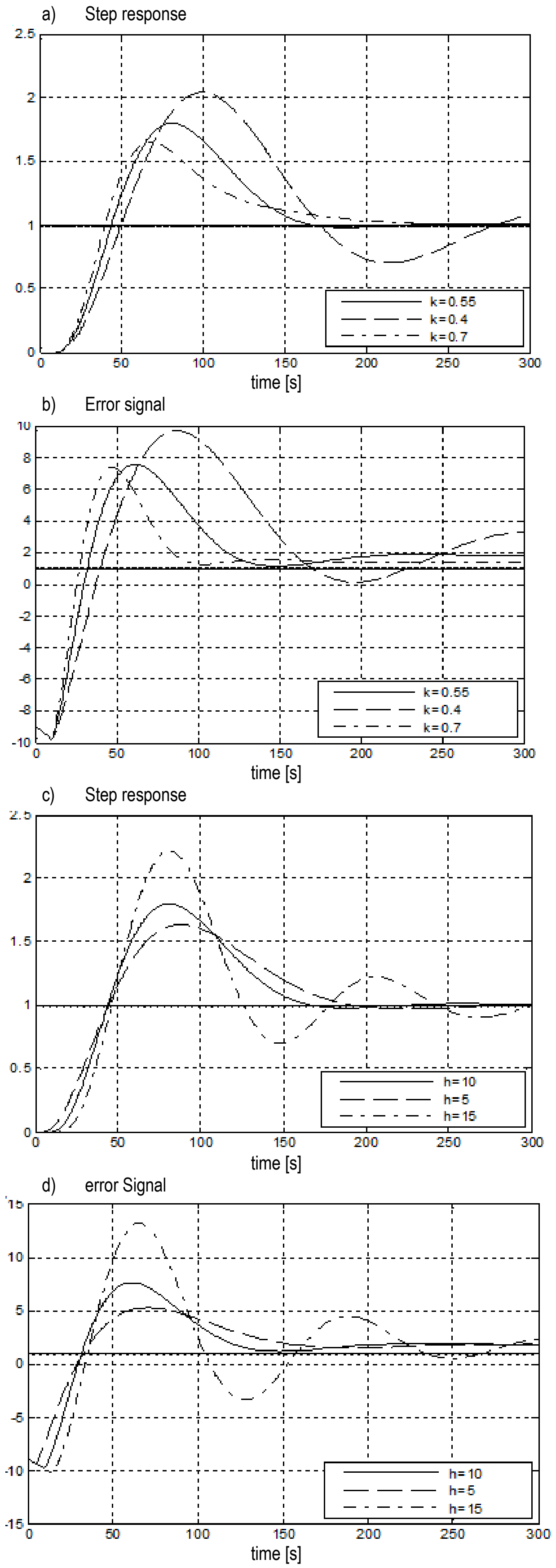

Fig. 5. Step response of the closed loop system with object (22) and controller (23) for a few different values of parameters *k* and *h*: a), b) step response; c), d) error signal

Fig. 5b and 5d show error signal of the controlled system. Overshoot for $k = 0.4$ is 100%, and moreover settling time is growing. For bigger value k overshoot value is decreasing, for $k = 0.7$even to 60%. For a few values of time delay h settling time is const when time delay is decreasing.

4. CONCLUSIONS

Skogestad method is well knows in the literature. That paper is generalization of Skogestad method for a class of unstable plant described with transfer function (7). For an unstable plant Skogestadanalytical method is extended for an negative gain of the controller, what makes positive feedback while simulating.As in Skogestad method, in that paper simulation for different time T_1, T_2, and time delayh was drawing. Note that for simulation $\tau_C$was posited as:$\tau_C = h$ In the paper two cases was presented: $T_1 \gg h$, and$T_1 > h$. Proposed method was compared with synthesis method of fractional order controller proposed in other author papers.Computer method for synthesis of fractional controllers is given. The considerations are illustrated by numerical exampleandresults of computer simulation with MATLAB/ Simulink.

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