FREQUENCY ANALYSIS WITH CROSS-CORRELATION ENVELOPE APPROACH

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Abstract: A new approach for frequency analysis of recorded signals and readout the frequency of harmonics is presented in the paper. The main purpose has been achieved by the cross-correlation function and Hilbert transform. Using the method presented in the paper, there is another possibility to observe and finally to identify single harmonic apart from commonly used Fourier transform. Identification of the harmonic is based on the effect of a straight line of the envelope of the cross-correlation function when reference and signal harmonic have the same frequency. This particular case is the basis for pointing the value of the frequency of harmonic component detected.

Key words: Frequency Analysis, Cross-Correlation, Hilbert Transform, Envelope

1. INTRODUCTION

It is common knowledge that spectrum analysis using fast Fourier transform (FFT) presents the amplitudes of all harmonics the fast way. This method of showing the frequency profile of the signal is applied both during the post-processing and as real-time processing.

There is many engineering applications of correlation function (Bendat and Piersol, 1980). To provide for a new application, the cross-correlation function has been utilized to correlate real-measured signal and a single harmonic signal generated by a software. Also, the Hilbert transform has been used for obtaining the envelope of the cross-correlation function (Thrane, 1984) where the envelope removes the oscillations (Thrane et al., 1999). In particular cases, experimental results have shown a linear shape of the envelope. It is observed when correlated signals have a common frequency value (Kotowski, 2010). This effect is well noted and very sensitive to generated single harmonic signal frequency. Thus, the paper presents the method of reading the particular frequency harmonic developed on the base of cross-correlation function and its envelope.

It is obviously known that after signal recording there is no way to have the longer one. This case couses the fixed frequency resolution as inverse of period of signal duration when using FFT. This case is especially noted for very short-time signals, e.g., from impulse tests. For avoiding that limitation, Cawley and Adams (1979) investigated the problem mentioned aboved and showed to be possible to obtain frequency resolution of one-tenth of the spacing between the frequency points produced by the Fourier transform. Also, it is commonly used zero-padding for improving frequency estimation (Quinn, 2009; Dunne, 2002). Zero-padding means that an array of zeros is appended to the end (or beginning) of analysed signal. Using the method presented in the paper there also is possible to obtain different frequency resolution than that fixed using FFT. Frequency resolution can be variable adjusted by user of the method starting from reference value of 1 Hz to up or down.

2. METHODOLOGY

The cross-correlation function $R_{xy}(\tau)$ between two processes, x(t) and y(t), is calculating by the expression (Bendat, Piersol, 1980):

$$R_{xy}(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_0^T x(t) y(t+\tau)\, dt \tag{1}$$

where: T – signal record length, τ – argument of cross-correlation function (time delay).

Then, the cross-correlation function $R_{xy}(\tau)$ is transformed into the envelope by Hilbert transform. The Hilbert transform of a real time signal, $x(t)$, is defined as follows (Thrane, 1984)

$$\mathrm{H}[x(t)] = \widetilde{x}(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} x(\tau) \frac{1}{t-\tau}\, d\tau \tag{2}$$

Thus, the Hilbert transform of the cross-correlation $R_{xy}(\tau)$ is given by:

$$\mathrm{H}[R_{xy}(\tau)] = \widetilde{R}_{xy}(\tau) = \frac{1}{\pi} \int_{-\infty}^{\infty} R_{xy}(\tau') \frac{1}{\tau-\tau'}\, d\tau' \tag{3}$$

The Hilbert transform enables calculation of the envelope of the signal x(t) as follows (Thrane et al., 1999):

$$|\overset{\nabla}{x}(t)| = \sqrt{x^2(t) + \widetilde{x}^2(t)} \tag{4}$$

where $|\overset{\nabla}{x}(t)|$ is the envelope. Similarly, we can calculate the envelope of the function $R_{xy}(\tau)$ as:

$$|\overset{\nabla}{R}_{xy}(\tau)| = \sqrt{R_{xy}{}^2(\tau) + \widetilde{R}_{xy}{}^2(\tau)} \tag{5}$$

The method also needs series of harmonic signals generated as follows:

$$y_i = \sin(2\pi \cdot (f_s + w \cdot i) \cdot t) \quad (6)$$

where: i - an integer value (index), f_s – starting frequency, w – factor as frequency resolution parameter.

Frequency f_s is fixed as a start point value and is increasing by i = 1, 2, 3, … , n. Also, the factor w is applied for changing the resolution of the harmonic frequency reading. This way, a form of the envelope points the case of detection and finally identification of harmonic. The harmonic frequency value equals one of the harmonics existing within the signal y_i(t). Preliminary studies have shown that envelope of the cross-correlation function is in the form of a straight line when input signal x(t) and the signal y(t) have in common one frequency determined as (f_s+w·i). This phenomenon is easy to detect and determination of the common frequency is fast. For that reason, plot of the envelope can be effectively used to identify harmonics included in recorded signals without Fourier transform.

There are a hundred of plots of the cross-correlation function envelope to illustrate four particular cases of the straight line effect mentioned above in Fig. 1. It has been used the four-harmonic signal x(t) generated as follows

$$x(t) = \sum_{k=1}^{4} \sin(2\pi \cdot f_k \cdot t) \quad (7)$$

where: f_1=64 Hz, f_2=85 Hz, f_3=130 Hz, f_4=150 Hz. The signal y_i is calculated in the way determined in Eq. 6, where f_s and w are constant and equal 60.0 and 1.0 respectively. The index i varies in the range from 1 to 100.

The value of frequency of harmonic included in the input signal x(t) is determined on the base of the plot of the envelope. When observing straight-line effect, we know the f_s value, w value and the i index value of the signal y_i(t) which was used for calculations (Eq. 6). This way, a formula (f_s +w·i) indicates the frequency of recognized harmonic.

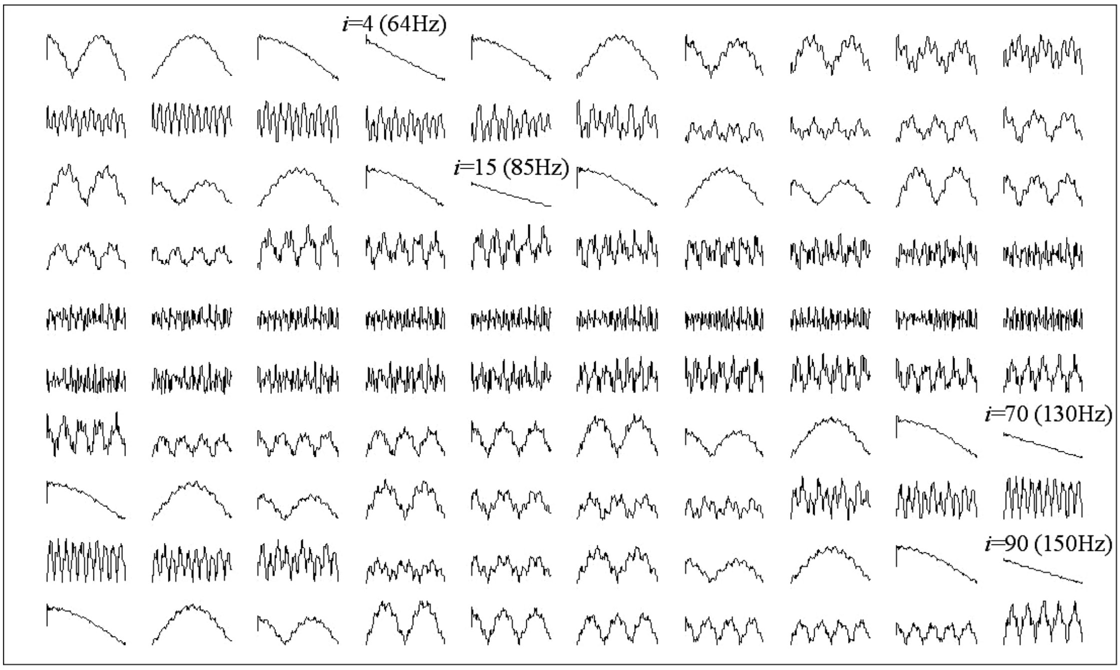

Fig. 1. The cross-correlation envelopes

3. RESULTS FOR STATIONARY SIGNAL

The exemplary experimental results have been based on signal of vibration presented in Fig. 2. The signal has been recorded by sampling frequency of 4096 Hz and over time of one second. The spectrum shown in Fig. 3 has a lot of well-observed harmonics. As shown in Fig. 4, two cases have been detected between 210 Hz and 310 Hz where envelopes of the cross-correlation function are almost in the form of a straight line. Thise situation has occurred for i=17 and i=92 by f_s=210Hz and w=1 (Eq. 6). Hence, it has been for 227 and 302 Hz with frequency resolution fixed by w as 1Hz (w=1).

Apart from detection based on cross-correlation envelope image, an indicator L_e has been used to express in numbers deviation of cross-correlation envelope from linearity. This way, it was possible to present a plot of changes in straight line overlay for all frequency span of recorded vibration signal. L_e is described as follows

$$L_e = \sum_{n=1}^{N} \left| y_{ref} - y_{env} \right| \quad (8)$$

where: y_{ref} – reference straight line, y_{env} – cross-correlation envelope, N – number of points for calculation.

This way, a plot of changes of indicator L_e has been prepared and presented in Fig. 5. It seems to be no difference between spectrun presented in Fig. 3 and the plot of L_e but if zooming the plot there are local minimas in places of dominant frequency appearance (spectrum peaks). If having the plots of L_e, it is possible to readout frequencies being under consideration (Figs. 6-9).

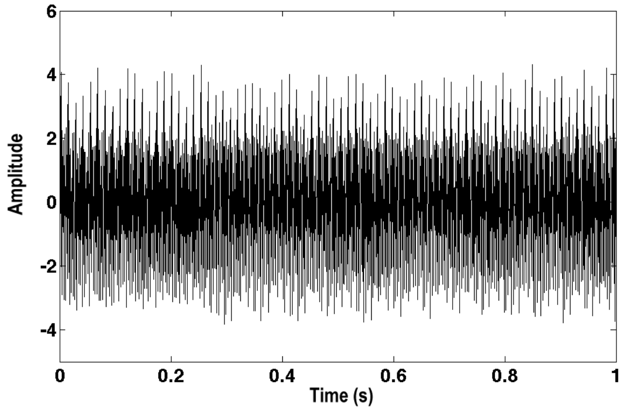

Fig. 2. Signal of vibration

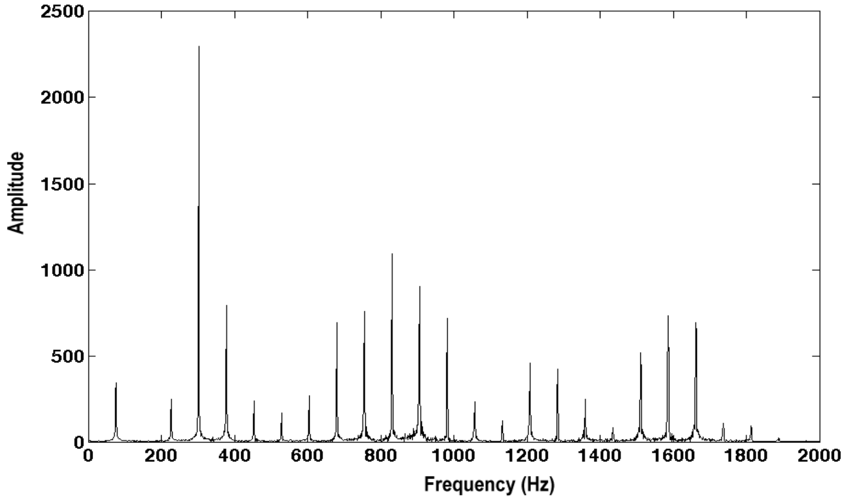

Fig. 3. Signal spectrum

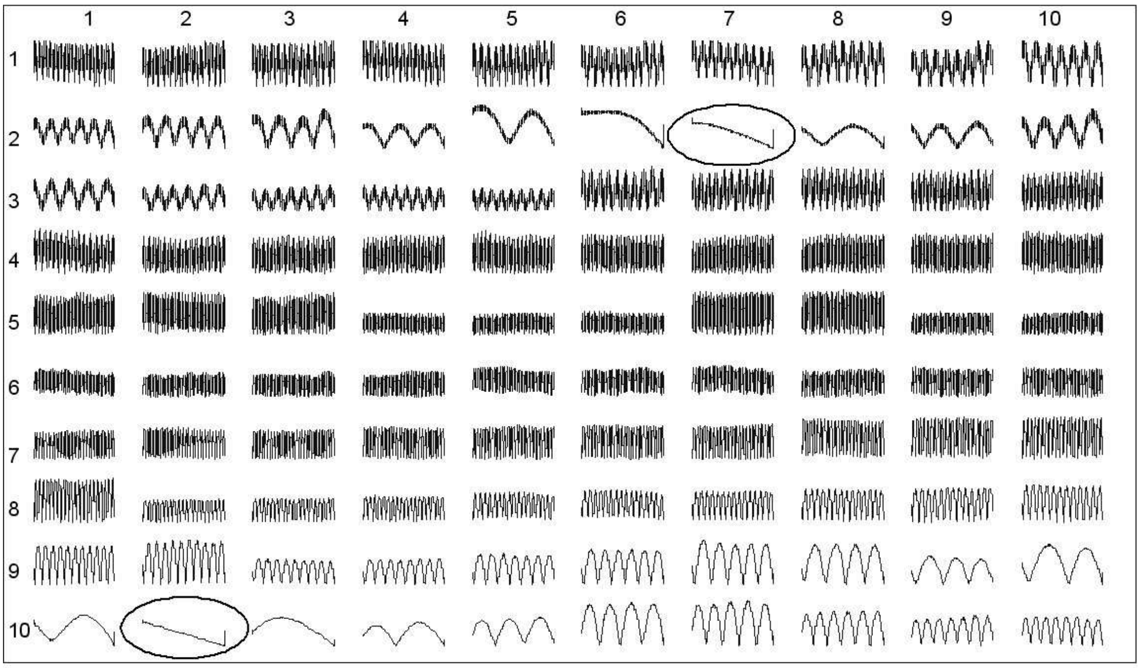

Fig. 4. Envelopes of the cross-correlation functions

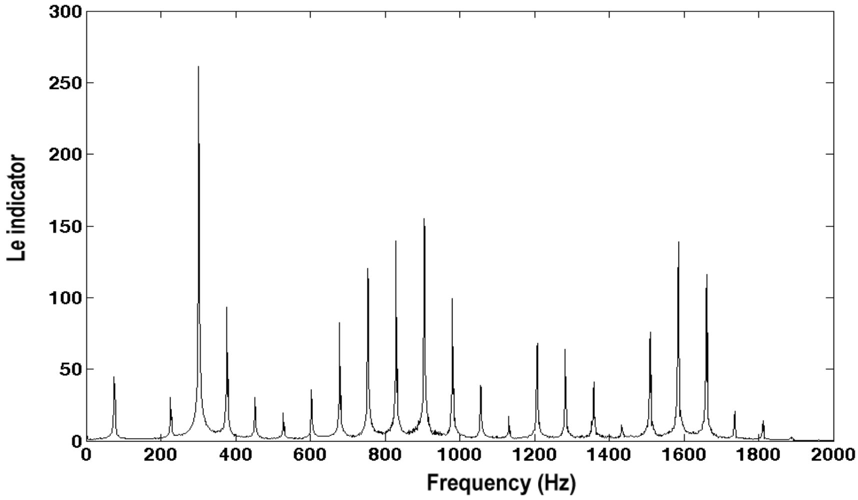

Fig. 5. L_e indicator plot for all frequency span of recorded vibration signal

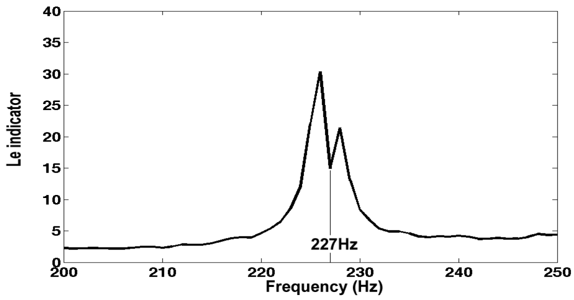

Fig. 6. Enlargement of L_e indicator plot around 227 Hz

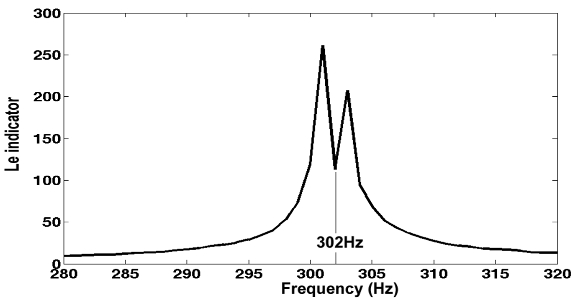

Fig. 7. Enlargement of L_e indicator plot around 302 Hz

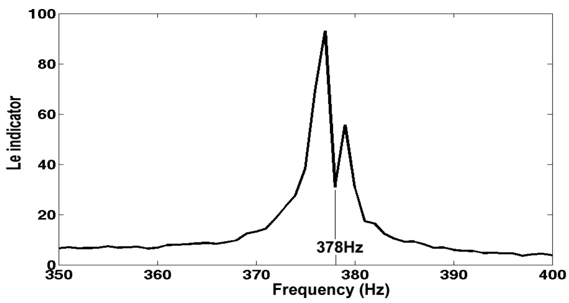

Fig. 8. Enlargement of L_e indicator plot around 378 Hz

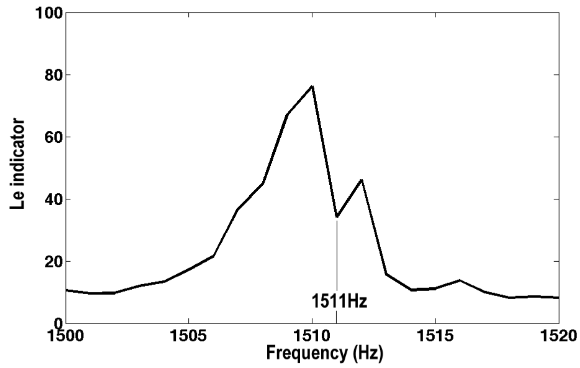

Fig. 9. Enlargement of L_e indicator plot around 1511 Hz

4. RESULTS FOR NONSTATIONARY SIGNAL

Frequency identification presented in section 3 can be applied for nonstationary signals where commonly used Fourier transform relies on a stationarity assumption and it is difficult to guarantee, in practice, the stationarity over a long signal time horizon (Benko and Juričić, 2008). A typical nonstationary signal can be the signal of response from impulse test. Exemplary impulse response under consideration is shown in Fig. 10. Laboratory software tool using curvefitting procedure have been utilized to obtain values of two frequencies at two highest amplitudes. It have resulted the frequency of 3794 and 13714 Hz.

In this case, impulse response analysis have shown that frequency readout is based on some different form of L_e indicator plot than obtained for vibration signal in section 3. It is well-observed an push-up effect presented in Figs. 11-12. This effect revealed the frequency of harmonics really existing in impulse response, i.e., 3794 and 13714 Hz.

Results presented previously have been obtained by the frequency resolution of 1 Hz. By changing the *w* parameter, it is possible to get greater resolution e.g. 0.2 Hz (*w*=0.2). As show in Fig. 13, the plot of L_e indicator has the same character but readout of frequency is more exact. In this case it is 3794.2 Hz – frequency at maximum of push-up effect.

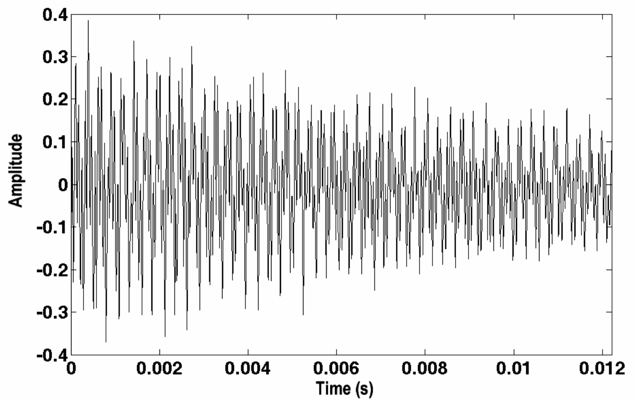

Fig. 10. Impulse response

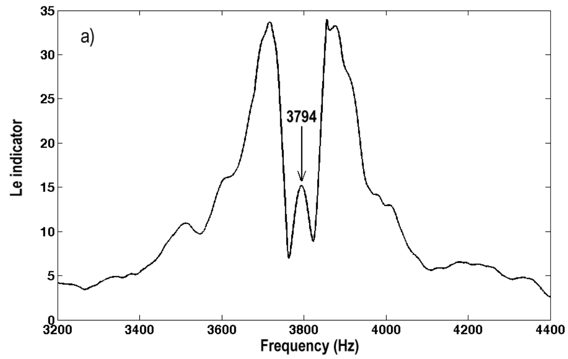

Fig. 11. L_e indicator plot for impulse response around 3794 Hz

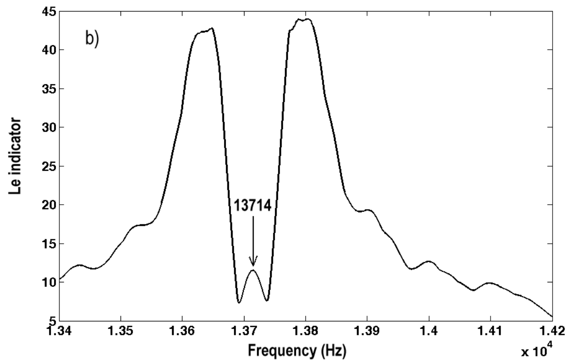

Fig. 12. L_e indicator plot for impulse response around 13714 Hz

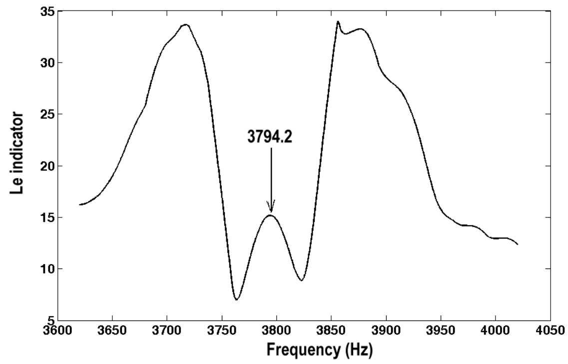

Fig. 13. L_e indicator plot for impulse response around 3794 Hz by the frequency resolution of 0.2 Hz

It is also able to obtain quasi-stationary signal from nonstationary by deviding the nonstationary signal into several sections and then use FFT. But this way, the Fourier spectrum resolution is going down. By deviding the signal presented in Fig. 10 into two parts, the spectrum resolution equals 164Hz (duration is 6.10 milisecond). After splitting into four sections, the spectrum resolution equals 328Hz (duration is 3.05 milisecond). However using the indicator L_e the method have its own spectrum resolution independent of duration of analysed signal or a fragment of analysed signal.

5. CONCLUSIONS

A general view of the use of cross-correlation function and its envelope for frequency analysis has been presented in the paper. That approach brings in the method for reading the frequency both for stationary and for nonstationary signals. For stationary signals, new possibility is based on the cross-correlation envelope straight-line effect observed for two signals (input signal and reference signal) when having one harmonic in common. The approach proposed in the paper shows a possibility to detect and finally to identify frequencies being within the input signal without use of Fourier transform, thus, without limitation in frequency resolution. The frequency resolution of proposed frequency analysis is determined over the factor used for generating reference signal. The method proposed in the paper gives a possibility to have the spectrum resolution controlled and independent of period of signal recording, e.g. signals lasting much less than one second always have Fourier spectrum resolution much over than 1 Hz and using the proposed methos it is able to obtain spectrum resolution 1Hz or even below 1Hz.

The cross-correlation function and its envelope can be a complementary method for frequency analysis, e.g. for accurate detection of natural frequencies using impulse tests.

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