

LOADS OF LOWER LIMB JOINTS DURING BICYCLE RIDE

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Abstract: This work features a geometric and static analysis of the lower limb during bicycle ride. Basic dimensions are indicated, which are necessary for the description of movement geometry. The simplified flat model was adopted for the analysis. Using the transformation of Denavit-Hartenberg frames, vectors were developed for the position of the rotation axis of the joints in immovable frame. The inverse kinematics problem was solved. The course of displacement changes in the ankle joint was adopted as an angle function for crankset position, based on experimental research results, published in professional literature. The course was approximated with fifth grade polynomial. Joint displacements and loads were established. A sample calculation is presented, illustrating the subject computational algorithm.

Key words: Bicycle, Lower Limb, Movement Geometry, Matrix Method, Static Analysis, Jacobian

1. INTRODUCTION

Bicycle riding is currently becoming more and more recognised and accessible form of recreation, physical well-being improvement and muscle building (Szczerek, 2012).

The course of displacement and joint and muscle loads depends on the assumed riding position, frame geometry, saddle and handlebars position, and the length of cranksets (Wanich et al., 2007). Another key factor is the crankset load, which depends on the gear ratio, configuration and type of the ground, air resistance and pedalling technique. Other important factors are bicycle weight, resistance of bicycle drivetrain systems (chain, bearing) and resistance of tyre rolling on the ground.

The correct selection of parameters influencing travel comfort and safety has been subject to many studies, with the literature dominated by the results of experimental studies. The studies concern the influence of bicycle element dimensions on selected values, e.g. the influence of the dimensions of steering system on stability during fast ride (Prince and Al-Jumaily, 2011), the influence of the dimensions of pedal crank on the knee joint load (Boyd et al., 1997), the influence of different geometry of the frame on the crank load (Gregor et al., 2002) and muscle load (Ricard et al., 2006).

The research also covered the kinematics and dynamics of the cyclist, e.g. the vector of angle (Cockcroft, 2011) and load (Park S.-Y. et al., 2012), (Li Li and Caldwell, 1998) change, in the hip, knee and ankle joints for the constant pedalling power using EMG measurement method. The study (Diefenthaler et al., 2008) also investigated trunk movement using a camera. Another elements subject to measurements are generated power, pulse and the level of lactic acid under isokinetic conditions (Koninckx et al., 2010), oxygen demand, blood pressure (Shimomura et al., 2009) and strength adaptation of muscles for changing speed (Neptune and Herzog, 2000).

The referred studies provide data for calculations as well as results of multi-faceted research inspiring the direction for theoretical works, furthermore they allow for verification of these works. In the context of theoretical publications, which utilise commercial software, the work (Apkarian et al., 1989) can be regarded

as trailblazing. The authors give mathematical formulas, using the matrix method of kinematics and estimate the load of lower limb joints on the basis of Newton-Euler equations.

The aim of this work is establishing the geometry of lower limb movement and indicating displacement and balancing torque in joints during bicycle ride. A preliminary analysis was introduced, in which a solution to the inverse kinematics problem was provided and the load of joints was estimated. The research used Jacobian matrix transforming the vector of loading force acting on pedal bearing into balancing torques in the joints. The work also provided an algorithm simplifying the estimation of Jacobi matrix. The estimations did not account for the issue of inertia. The analysis assumed that the torque applied to the crankset is of constant value during round full angle cycle. A simulation of movement was performed using own software.

2. ANALYSIS OF LIMB MOVEMENT DURING PEDALLING

Analysis of limb movement during pedalling comes down to indicating:

- the coordinates of the position of knee and ankle joint rotation axes in the frame $\{x_0, y_0\}$ adopted as immovable,
- inverse problem solution, i.e. indicating the value of angular positions in joints as coordinate functions for the pedal axis position.

Fig. 1a presents kinematic chain of lower limb during bicycle riding scaled in Denavit-Hartenberg frame. We have adopted the flat model of lower limb (Siemieniako et al., 2010) simplified as compared to that suggested in Zielińska and Trojnacki (2009). Together with the crankset, it creates an articulated pentagon, which constitutes a problematic issue in indicating a uniform inverse problem solution.

It is necessary to introduce dependencies between the two selected angles. The most preferable angle indicated as the function of the second angle is the position angle in the ankle joint β_p – Fig. 1b. The course of value changes for the angle $\beta_p = f(\alpha_k)$ depends on anatomical structure, individual riding style and there even occurs a differentiation between the course for left and right leg, which was confirmed by the Authors of the paper (Kusiak

and Winiarski, 2009) in their experimental research. In order to describe the course $\beta_p = f(\alpha_k)$ an equation of adopting fifth grade polynomial to a set of n points of the coordinates $(\alpha_{k,i}, \beta_{p,i})$.

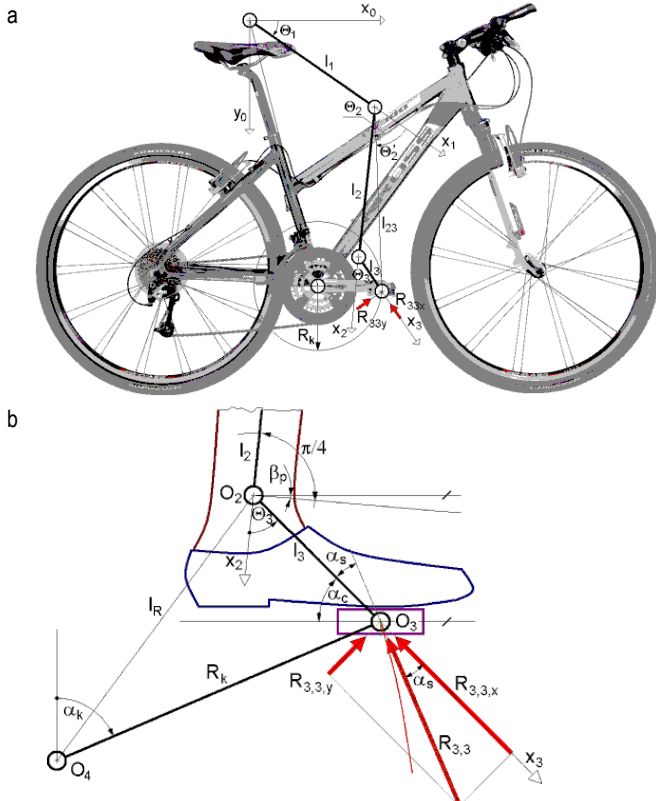


Fig. 1. Kinematic chain geometry; a – lower limb during bicycle ride, b – crankset system

A movement analysis of lower limb was performed for the simplified flat model. Movement trajectory of point O_3 was described using the parametric equations of the circle, thus the coordinates of the position of the pedal axis $(x_{O3,0}, y_{O3,0})$ in relation to the immovable frame $\{x_0, y_0\}$, according to Fig. 1a and 1b may be described as:

$$\begin{aligned} x_{O3,0} &= x_{O4,0} + R_K \sin \alpha_k, \\ y_{O3,0} &= y_{O4,0} + R_K \cos \alpha_k. \end{aligned} \quad (1)$$

The kinematics matrix method using the Denavit-Hartenberg frame transformation was used in order to transform the vectors of positions $r_{O_i,i}$ of the points marked "O*i*", determined in frames "i" into vectors of coordinates $(x_{O_i,0}, y_{O_i,0})$ determined in relation to the immovable frame $\{x_0, y_0\}$.

The matrices transforming coordinate frames from zero into first, from first into second and from second into third will have the form:

$$\mathbf{A}_i = \begin{bmatrix} c_i & -s_i & 0 & l_i c_i \\ s_i & c_i & 0 & l_i s_i \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \text{ where } i = 1, 2, 3, \quad (2)$$

whereas $s_i = \sin \theta_i, c_i = \cos \theta_i, l_i$ – distance between the axes of the turning pairs of the unit i, θ_i – rotation angle between the units $i-1$ and i .

Coordinates of the vectors for the kinematic pair centres

O_1, O_2 and O_3 in immovable frame, which will be used for further calculations and in the programme of movement animation, may be defined as follows:

$$\begin{bmatrix} x_{O1,0} \\ y_{O1,0} \\ z_{O1,0} \\ 1 \end{bmatrix} = \mathbf{A}_1 \mathbf{r}_{O1,1} = \begin{bmatrix} l_1 c_1 \\ l_1 s_1 \\ 0 \\ 1 \end{bmatrix}, \quad (3)$$

$$\begin{bmatrix} x_{O2,0} \\ y_{O2,0} \\ z_{O2,0} \\ 1 \end{bmatrix} = \mathbf{A}_1 \mathbf{A}_2 \mathbf{r}_{O2,2} = \begin{bmatrix} l_1 c_1 + l_2 c_{12} \\ l_1 s_1 + l_2 s_{12} \\ 0 \\ 1 \end{bmatrix}, \quad (4)$$

$$\begin{bmatrix} x_{O3,0} \\ y_{O3,0} \\ z_{O3,0} \\ 1 \end{bmatrix} = \mathbf{A}_1 \mathbf{A}_2 \mathbf{A}_3 \mathbf{r}_{O3,3} = \begin{bmatrix} l_1 c_1 + l_2 c_{12} + l_3 c_{13} \\ l_1 s_1 + l_2 s_{12} + l_3 s_{13} \\ 0 \\ 1 \end{bmatrix}. \quad (5)$$

Treating the variable distance $O_1 O_3 = l_{23}$ as known, the coordinates of the vector for the position of turning pair O_3 in relation to the frame $\{x_0, y_0\}$ may be also determined as:

$$\begin{bmatrix} x_{O3,0} \\ y_{O3,0} \\ z_{O3,0} \\ 1 \end{bmatrix} = \mathbf{A}_1 \mathbf{A}_{23} \mathbf{r}_{O3,3} = \begin{bmatrix} l_1 c_1 + l_{23} c_{123} \\ l_1 s_1 + l_{23} s_{123} \\ 0 \\ 1 \end{bmatrix}, \quad (6)$$

where: \mathbf{A}_{23} – matrix transforming the frame $\{x_1, y_1\}$ into frame $\{x_3, y_3\}, s_{12} = \sin(\theta_1 + \theta_2), c_{12} = \cos(\theta_1 + \theta_2), s_{123} = \sin(\theta_1 + \theta_2'), c_{123} = \cos(\theta_1 + \theta_2')$.

Inverse problem solution implies determining the angles $\theta_1, \theta_2', \theta_2$ and angle θ_3 (Fig. 1a). For the purposes of inverse problem solution we must determine the following:

- the value of the angle θ_3 based on Fig. 1b according to dependency:

$$\theta_3(\beta_p) = 0.5\pi + \beta_p - \alpha_c, \quad (7)$$

- the variable distance $O_1 O_3 = l_{23}$, which may be calculated according to dependency:

$$l_{23}(\theta_3) = \sqrt{l_2^2 + l_3^2 + 2l_2 l_3 \cos \theta_3}, \quad (8)$$

- the value of the angle θ_1 and θ_2' as dimension function $l_{23}(\theta_3)$, by using the dependency (6) and solving the simultaneous equations:

$$x_{O3,0} = l_1 c_1 + l_{23} c_{123}, \quad y_{O3,0} = l_1 s_1 + l_{23} s_{123}, \quad (9)$$

we obtain the value of angle θ_1 as the function $l_{23}(\theta_3)$:

$$\theta_1 = 2 \operatorname{atan}2 \left(B_1 + \sqrt{A_1^2 + B_1^2 - D_1^2}, A_1 + D_1 \right), \quad (10)$$

where: $A_1 = 2x_{O3,0} l_1, B_1 = 2y_{O3,0} l_1, D_1 = x_{O3,0}^2 + y_{O3,0}^2 + l_1^2 - l_{23}^2(\theta_3)$;

- the value of the angle θ_2' as the dimension function $l_{23}(\theta_3)$:

$$\theta_2' = \operatorname{atan}2 \left(\sqrt{1 - A_2^2}, A_2 \right), \quad (11)$$

where: $A_2 = \frac{x_{O3,0}^2 + y_{O3,0}^2 + l_1^2 - l_{23}^2(\theta_3)}{2l_1 l_{23}(\theta_3)}$.

From the triangle $O_1 O_2 O_3$, based on Fig. 1b, we can derive

the value of angle Θ_2 :

$$\Theta_2 = \text{atan2}\left(\sqrt{1-A_3^2}, A_3\right) + \Theta_2', \quad (12)$$

where: $A_2 = \frac{l_2^2 - l_3^2 + l_{23}^2(\theta_3)}{2l_2l_{23}(\theta_3)}$;

– the value of the angle α_s , may be determined based on the dependency:

$$\alpha_s = \text{atan2}\left(\sqrt{1-A_4^2}, A_4\right), \quad (13)$$

where: $A_4 = \frac{R_k^2 + l_3^2 - (x_{04,0} - x_{02,0})^2 - (y_{04,0} - y_{02,0})^2}{2l_3R_k}$,

whilst the coordinates $(x_{02,0}, y_{02,0})$ are determined based on the dependency (4).

3. LIMB STATIC ANALYSIS

Static analysis serves to determine the load of joints for imparted motion along the trajectory, which comprises mainly values of the torque for balancing the load of the foot during pedalling.

The load $R_{3,3}$ of the pedal determines the dependency:

$$R_{3,3} = \frac{M_K}{R_k}, \quad (14)$$

where: M_K is the imparted torque for loading the turning pair of the crankset during the ride.

The values of the coordinates of the loading force vector, determined within a frame of coordinates with the starting point O_3 determine the dependencies:

$$R_{3,3,x} = R_{3,3} \sin \alpha_s, \quad R_{3,3,y} = R_{3,3} \cos \alpha_s. \quad (15)$$

The vector of the loading force acting on the foot transforms into vectors of torques for driving forces in joints, according to the dependency:

$$\mathbf{M}_R = \mathbf{J}_{3,3}^T \mathbf{R}_{3,3}, \quad (16)$$

where: \mathbf{M}_R – the vector of driving torques in the joints, $\mathbf{R}_{3,3}$ – the vector of loading force acting on pedal bearing, determined in the frame $\{x_3, y_3\}$, $\mathbf{J}_{3,3}^T$ – transposed Jacobian matrix transforming the vector of loading force acting on pedal bearing into balancing torques in the joints.

Transposed Jacobian matrix $3 \times n$ is specified as follows:

$$\mathbf{J}_{3,3}^T = \begin{bmatrix} \beta_{1,3,x} & \beta_{1,3,y} & \beta_{1,3,z} \\ \beta_{2,3,x} & \beta_{2,3,y} & \beta_{2,3,z} \\ \beta_{3,3,x} & \beta_{3,3,y} & \beta_{3,3,z} \end{bmatrix}, \quad (17)$$

whose elements may be determined according to an algorithm simplifying the method of their derivation, presented in Stępniewski (2008):

$$\begin{bmatrix} \beta_{i,3,x} \\ \beta_{i,3,y} \\ \beta_{i,3,z} \end{bmatrix} = \begin{bmatrix} \beta_{i+1,3,x} \\ \beta_{i+1,3,y} \\ \beta_{i+1,3,z} \end{bmatrix} + l_i \begin{bmatrix} l_{i+1,3,y} \\ m_{i+1,3,y} \\ n_{i+1,3,y} \end{bmatrix}, \quad (18)$$

whereas:

$$\begin{bmatrix} \beta_{3,3,x} \\ \beta_{3,3,y} \\ \beta_{3,3,z} \end{bmatrix} = \begin{bmatrix} 0 \\ l_3 \\ 0 \end{bmatrix}, \quad (19)$$

where: $[l_{i+1,3,y} \ m_{i+1,3,y} \ n_{i+1,3,y}]^T$ the matrix composed of elements from the first, second and third column of the second line of the matrix \mathbf{A}_3 ($i = 2$) and $\mathbf{A}_2\mathbf{A}_3$ ($i = 1$), thus for ($i = 2$):

$$\begin{bmatrix} \beta_{2,3,x} \\ \beta_{2,3,y} \\ \beta_{2,3,z} \end{bmatrix} = \begin{bmatrix} 0 \\ l_3 \\ 0 \end{bmatrix} + l_2 \begin{bmatrix} s_3 \\ c_3 \\ 0 \end{bmatrix} = \begin{bmatrix} l_2s_3 \\ l_2c_3 + l_3 \\ 0 \end{bmatrix}. \quad (20)$$

Based on the second line of the matrix $\mathbf{A}_2\mathbf{A}_3$:

$$\begin{bmatrix} \beta_{1,3,x} \\ \beta_{1,3,y} \\ \beta_{1,3,z} \end{bmatrix} = \begin{bmatrix} l_2s_3 \\ l_3 + l_2c_3 \\ 0 \end{bmatrix} + l_1 \begin{bmatrix} s_{23} \\ c_{23} \\ 0 \end{bmatrix} = \begin{bmatrix} l_1s_{23} + l_2s_3 \\ l_1c_{23} + l_2c_3 + l_3 \\ 0 \end{bmatrix}. \quad (21)$$

The value of torques in joints, balancing the load acting on foot, according to the dependency (16) is determined as follows:

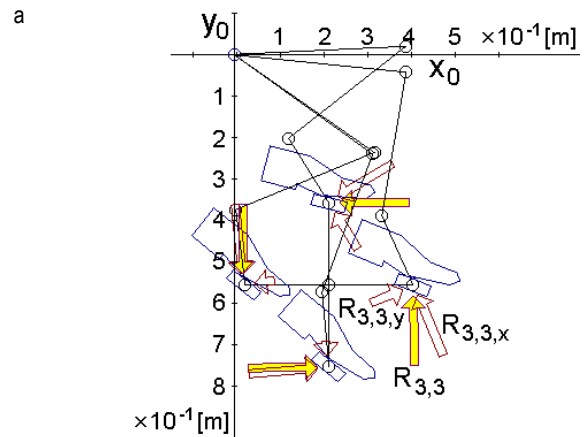
$$\begin{bmatrix} M_{R1} \\ M_{R2} \\ M_{R3} \end{bmatrix} = \mathbf{J}_{3,3}^T \mathbf{R}_{3,3} = \begin{bmatrix} l_1s_{23} + l_2s_3 & l_1c_{23} + l_2c_3 + l_3 & 0 \\ l_2s_3 & l_2c_3 + l_3 & 0 \\ 0 & l_3 & 0 \end{bmatrix} \begin{bmatrix} R_{3,3,x} \\ R_{3,3,y} \\ 0 \end{bmatrix},$$

$$\begin{bmatrix} M_{R1} \\ M_{R2} \\ M_{R3} \end{bmatrix} = \begin{bmatrix} (l_1s_{23} + l_2s_3)R_{3,3,x} + (l_1c_{23} + l_2c_3 + l_3)R_{3,3,y} \\ l_2s_3R_{3,3,x} + (l_2c_3 + l_3)R_{3,3,y} \\ l_3R_{3,3,y} \end{bmatrix}. \quad (22)$$

4. NUMERICAL EXAMPLE

The calculation involved the following data: length: $l_1 = 0.40\text{m}$, $l_2 = 0.35\text{m}$, $l_3 = 0.18\text{m}$, $R_k = 0.195\text{m}$, coordinates of the position of crankset turning pair in the hip joint frame: $(x_{04,0}, y_{04,0}) = (0.22, -0.55)$, constant angle $\alpha_c = 45^\circ$, nine coordinates $(\alpha_{k,i}, \beta_{p,i})$: $(0, 40^\circ)$, $(45^\circ, 5^\circ)$, $(90^\circ, 0)$, $(135^\circ, -15^\circ)$, $(180^\circ, -55^\circ)$, $(225^\circ, 25^\circ)$, $(270^\circ, 30^\circ)$, $(315^\circ, 35^\circ)$, $(360^\circ, 40^\circ)$. The calculation adopts constant torque for loading the crankset $M_K = 10\text{ N}\cdot\text{m}$ and constant force of a foot of one leg on the pedal during the full cycle (pedal with gear).

The results of the above calculations are presented in Fig. 2.



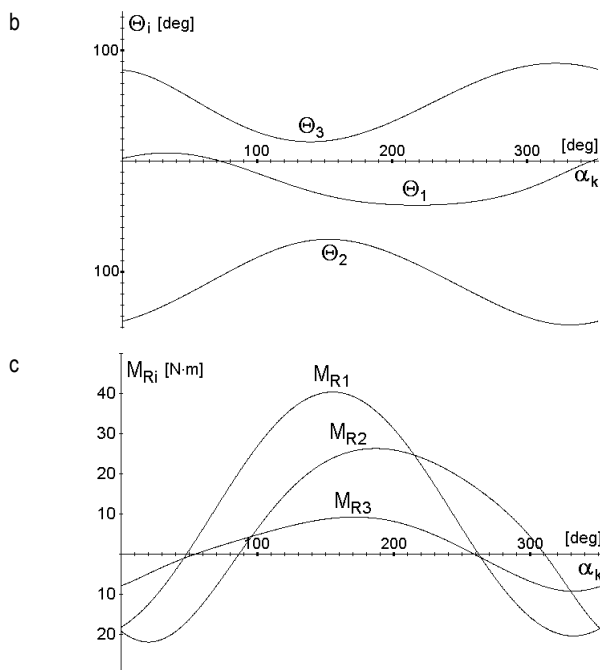


Fig. 2. Calculation results; a – simulation of limb movement during bicycle ride, b – courses of displacements in joints, c – courses of balancing torques in joints

5. CONCLUSIONS

On the basis of the performed geometrical and static analyses, the following conclusions are to be made:

- Lower limb together with flat version crankset creates an articulated pentagon with three drives.
- Movement in the ankle joint is not strictly determined, realisation of movement is possible for several different courses of the joint angle changes.

Based on the acquired results it can be stated that:

- The ranges of relative angular displacements in the joints are: hip 55° , knee 75° and ankle 65° .
- The extreme values of relative angular displacements in the joints are: hip 10° with $\alpha_k=35^\circ$ and -45° with $\alpha_k=220^\circ$, knee -70° with $\alpha_k=150^\circ$ and -145° with $\alpha_k=330^\circ$, ankle 20° with $\alpha_k=140^\circ$ and 85° with $\alpha_k=320^\circ$.
- The torques are of similar character and reach the maximum values in the joints: hip 40 N·m with $\alpha_k=155^\circ$, knee 25 N·m with $\alpha_k=190^\circ$ and ankle 10 N·m with $\alpha_k=170^\circ$.
- During the full cycle there occur two positions, where torques in the joints change their sense: in hip and ankle joint with $\alpha_k = 50^\circ$ and $\alpha_k = 260^\circ$, in knee joint with $\alpha_k = 85^\circ$ and $\alpha_k = 310^\circ$.

The above values were obtained by means of adopted geometrical measurements of the limb and bicycle, assuming constant torque for loading the crankset. Further analysis involves: changeability of the load torque, inertia interaction and optimisation of the geometrical measurements. It seems preferable to develop a comprehensive limb model, including the anatomical structure of the joints, which would allow for estimating the contact load of bones and ligaments. Obtained results of the estimation would give ground for clarification of practical guidelines concerning the use of a bike, e.g. for knee arthroscopy rehabilitation, where "movement therapy" is advisable.

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